Superiority of Superposition Multiaccess With Single-User Decoding Over TDMA in the Low SNR Regime

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***Abstract*—This paper studies the Gaussian multiaccess channel with multiantenna basestation in the low signal to noise ratio (SNR) regime. We compare the spectral efficiencies of the optimal superposition channel sharing scheme and two simple alternatives: the time division multiaccess (TDMA) scheme and superposition multiaccess with single-user decoding (SSD). Due to the fact that SSD, but not TDMA, exploits the multiuser multiplexing gain, the relative spectral efficiency of SSD over TDMA grows drastically as the number of antennas at the basestation increases. The results suggest that, in the low SNR regime with multiple antenna basestation, TDMA's suboptimality can no longer be offset by its simplicity since SSD can achieve much higher spectral efficiency while the simplicities of the two channel sharing schemes are similar.**

***Index Terms*—Low SNR regime, multiaccess, multiantenna, multiplexing gain, spectral efficiency.**

I. Introduction

In multiaccess channels, superposition strategies, where users transmit simultaneously in both time and frequency, offer higher information capacity than the orthogonal strategies such as the time-division multiple access (TDMA) [1], [2]. However, despite being suboptimal in common scenarios, TDMA remains the dominant channel sharing scheme in many wireless systems for multipoint-to-point and point-to-multipoint links. From a cross-layered networking point of view, maintaining a simple channel sharing scheme such as TDMA is beneficial since simplicity can bring overall performance gain by enabling the tractability of many cross-layered optimizations [3], [4]. The dominance of TDMA is indeed due to the fact that its suboptimality is often not significant enough to offset its advantage of simple system design.

Manuscript received August 04, 2008; revised October 02, 2009. Date of current version October 20, 2010. This work was supported by the Collaborative Technology Alliance for Communication & Networks, sponsored by the U.S. Army Laboratory under Cooperative Agreement DAAD19-01-2-0011 and by the National Science Foundation Grant ANI 02-05330. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Aeronautics and Space Administration or the Army Research Laboratory of the U.S. Government. Parts of the material in this paper were presented at the IEEE International Symposium on Information Theory, Seattle, WA, July 2006.

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Communicated by R. A. Berry, Associate Editor for Communication Networks.

Digital Object Identifier 10.1109/TIT.2010.2068830

In this paper, we consider Gaussian multiaccess channels with multiple antenna basestation and single antenna users in the low signal to noise ratio (SNR) regime. We compare spectral efficiencies of the optimal superposition (OPT) channel sharing scheme and those of the two simple schemes: superposition with single-user decoding (SSD) and TDMA, in terms of their wideband slope regions [5], [2], [6] and their system slopes (defined in Section II). Under various channel conditions, we show that, asymptotically[1], the relative spectral efficiency, defined as the ratio between the system slopes, of SSD over TDMA scales linearly in the number of receiving antennas. Although the results are derived for asymptotics, we demonstrate via computer simulations that the superiority of SSD over TDMA holds for systems with small number of receiving antennas. In addition, the relative spectral efficiency of SSD over the OPT scheme is no less than $1/2$, and this is consistent to single receiving antenna case shown in [7], [8]. On one hand, inefficiency of TDMA compared with SSD is essentially unbounded[2]. On the other hand, TDMA channel sharing is no simpler than SSD. Therefore, in the studied scenarios, TDMA's suboptimality can no longer be offset by its simplicity.

It has been well recognized in the past decade that the use of multiple antennas can bring multiplexing gain and can therefore significantly boost the capacities of wireless multiaccess and broadcast channels in the high SNR regime [9]–[14]. It has also been shown in [5], [15], [16] that multiplexing gain can significantly improve the spectral efficiencies of multiple antenna systems in the low SNR regime. Multiplexing gain can be efficiently exploited when the number of transmitting antennas is equal to the number of receiving antennas [9], [5]. In a multiaccess system with multiple antenna basestation and single antenna users, in terms of multiplexing gain exploitation, the lack of multiple transmitting antennas can be compensated by allowing multiple users to transmit in parallel [14]. The multiplexing gain in this case is called the multiuser multiplexing gain. The key inefficiency of TDMA is that it does not exploit multiuser multiplexing gain. Consequently, capacity loss in the high SNR regime and spectral efficiency loss in the low SNR regime due to TDMA can be arbitrarily large. Meanwhile, although capacity achieving schemes in multiple antenna systems are overly complex for practical systems [17], multiplexing gain

[1]In the paper, we consider two scenarios: either fixing the ratio between the number of antennas at the basestation and the number of users, or simply fixing the number of antennas at the basestation. Asymptotics are taken by letting the number of users go to infinity.

[2]In the sense that the relative spectral efficiency of SSD over TDMA can grow to infinity.

can be easily exploited using simple suboptimal channel sharing schemes. As shown in [18], transmitter preprocessing schemes such as interference alignment can be used in the high SNR regime to exploit multiplexing gain with only single user signal detection at the receiver. In the low SNR regime, parallel transmission with single user detection at the receiver is a cost-effective way to exploit multiuser multiplexing gain, as shown in this paper. Therefore, TDMA is not an ideal channel sharing scheme in systems where multiuser multiplexing gain is a significant factor.

II. Preliminaries

We denote information in nats instead of bits. All the logarithms are natural-based.

In a multiuser communication system, the sum transmitted energy per nat, E_{sum}, and the sum power per symbol, P_{sum}, are related through the sum information rate as

$$R_{\text{sum}}\frac{E_{\text{sum}}}{N_0} = \frac{P_{\text{sum}}}{N_0} \tag{1}$$

where N_0 is the spectral density of the white Gaussian noise. Similarly, the sum received energy per nat, E^r_{sum}, and the sum received power per symbol, P^r_{sum}, satisfy

$$R_{\text{sum}}\frac{E^r_{\text{sum}}}{N_0} = \frac{P^r_{\text{sum}}}{N_0}. \tag{2}$$

We define the spectral efficiency of a multiuser system by

$$\mathsf{C} = \frac{R_{\text{sum}}}{B} \tag{3}$$

where B is the overall frequency bandwidth of the system. Without loss of generality, we let $\mathsf{C} = R_{\text{sum}}$ when comparing different multiuser systems with the same total reserved bandwidth.

It was suggested in [19], [2] that one should fix the ratios between information rates of the users when analyzing a multiuser system. Particularly, given a real-valued vector $\boldsymbol{\theta}$ whose non-negative elements satisfy $\sum_i \theta_i = 1$, system analysis is carried out by fixing the ratios between information rates at

$$R_i = \theta_i \sum_j R_j, \qquad \forall i. \tag{4}$$

Under condition (4), both the normalized minimum sum received energy per nat and the normalized minimum sum transmitted energy per nat are weighted sums of the corresponding individual limits. Namely,

$$\begin{aligned} \frac{E^r_{\text{sum}}}{N_0}_{\text{min}} &= \sum_j \theta_j \frac{E^r_j}{N_0}_{\text{min}} \\ \frac{E_{\text{sum}}}{N_0}_{\text{min}} &= \sum_j \theta_j \frac{E_j}{N_0}_{\text{min}}. \end{aligned} \tag{5}$$

We define the wideband system slope[3], which is a function of $\boldsymbol{\theta}$, as

$$\begin{aligned} \mathcal{S}_0 &= \lim_{\frac{E_{\text{sum}}}{N_0} \downarrow \frac{E_{\text{sum}}}{N_0}_{\text{min}}} \frac{\mathsf{C}}{\log \frac{E_{\text{sum}}}{N_0} - \log \frac{E_{\text{sum}}}{N_0}_{\text{min}}} \\ &= \lim_{\frac{E^r_{\text{sum}}}{N_0} \downarrow \frac{E^r_{\text{sum}}}{N_0}_{\text{min}}} \frac{\mathsf{C}}{\log \frac{E^r_{\text{sum}}}{N_0} - \log \frac{E^r_{\text{sum}}}{N_0}_{\text{min}}}. \end{aligned} \tag{6}$$

Define $\text{SNR}^r_i = \frac{P^r_i}{N_0}$ and $\text{SNR}^r = \sum_i \text{SNR}^r_i$. From (4) and the fact that $\frac{E^r_i}{N_0} = \frac{\text{SNR}^r_i}{R_i}$, if the minimum received energy per nat of all the users are equal, as SNR^r goes to zero, we have $\text{SNR}^r_i \to \theta_i \text{SNR}^r$. The convergence on the ratios between individual SNRs is uniform due to the constraint of (4) [2]. Following the analysis on wideband slopes of individual users presented in [5], we obtain

$$\mathcal{S}_0 = \frac{2(\dot{R}_{\text{sum}}(\text{SNR}^r)|_{\text{SNR}^r=0})^2}{\ddot{R}_{\text{sum}}(\text{SNR}^r)|_{\text{SNR}^r=0}} \tag{7}$$

where $\dot{R}$ and $\ddot{R}$ denote respectively the first and the second order derivatives of R taken with respect to $\text{SNR}^r = \sum_i \text{SNR}^r_i$. Note that $\mathcal{S}_0$ depends on the $R_{\text{sum}}(\text{SNR}^r)$ function. As in [5], we are interested in deriving the maximum achievable system slope, denoted by $\mathcal{S}_{\text{max}}$, which is obtained by maximizing $\mathcal{S}_0$ over the R_{sum} function under various constraints.

In the rest of the paper, we study the OPT, SSD and TDMA channel sharing schemes in the low SNR regime in terms of their slope regions (defined in [2]) and their maximum system slopes, denoted by $\mathcal{S}^{\text{OPT}}_{\text{max}}(\boldsymbol{\theta})$, $\mathcal{S}^{\text{SSD}}_{\text{max}}(\boldsymbol{\theta})$, and $\mathcal{S}^{\text{TDMA}}_{\text{max}}(\boldsymbol{\theta})$, respectively. Since when SNR equals zero the system has zero spectral efficiency, if two channel sharing schemes have the same minimum sum transmitted energy per nat, the ratio between their system slopes characterizes the ratio between their spectral efficiencies in the low SNR regime. Hence we define

$$\eta_{\text{SSD}|\text{TDMA}}(\boldsymbol{\theta}) = \frac{\mathcal{S}^{\text{SSD}}_{\text{max}}(\boldsymbol{\theta})}{\mathcal{S}^{\text{TDMA}}_{\text{max}}(\boldsymbol{\theta})} \tag{8}$$

as the relative spectral efficiency between SSD over TDMA. We also term $\eta_{\text{SSD}|\text{OPT}}(\boldsymbol{\theta})$ and $\eta_{\text{TDMA}|\text{OPT}}(\boldsymbol{\theta})$ the normalized spectral efficiencies of SSD and TDMA, respectively.

III. The Gaussian Multiaccess Channel

Assume there are K users transmitting signals to a common receiver. The receiver (or the basestation) is equipped with M antennas, while the transmitters (or the users) have only one antenna each. The received signal at the basestation is given by a M-component complex-valued column vector

$$\boldsymbol{y} = \sum_{i=1}^{K} \boldsymbol{h}_i x_i + \boldsymbol{n}. \tag{9}$$

[3]A similar definition was originally given in [5], [20] to define the wideband slope of a multiuser system. In this paper, we use the term *system slope* in order to avoid possible confusion with the slope region and the slope of individual users introduced in [19], [2].

Here, x_i is the complex-valued symbol from user i; $\boldsymbol{h}_i$ is the channel gain vector from user i to the receiving antennas; $\boldsymbol{n}$ is an additive complex Gaussian noise with zero mean and covariance matrix $N_0\boldsymbol{I}$. Suppose the transmission power of user i satisfies

$$E[|x_i|^2] \leq P_i = \mathrm{SNR}_i N_0 \tag{10}$$

where SNR_i is the normalized transmission power per receiving antenna of user i.

A. Fading Channels With Channel Distribution Information at the Transmitters

Assume the channels experience fading and the transmitters only have channel distribution information (CDI). Assume the receiver has perfect channel state information (CSI) which enables coherent signal reception. Note that although obtaining CSI at the receiver can be difficult for low SNR communications, results obtained based on the coherent reception assumption can still provide valuable insight to practical system design.

The ergodic capacity region of the multiaccess channel is given by

$$\boldsymbol{C}_{\mathrm{OPT}} = \left\{ \boldsymbol{R} \middle| \sum_{i\in J} R_i \leq E \times \left[\log \left| \boldsymbol{I} + \sum_{i\in J} \mathrm{SNR}_i \boldsymbol{h}_i \boldsymbol{h}_i^H \right| \right], \quad \forall J \right\} \tag{11}$$

where $\boldsymbol{h}_i^H$ denotes the conjugate transpose of $\boldsymbol{h}_i$. Since each vertex of the multiaccess capacity region $\boldsymbol{C}_{\mathrm{OPT}}$ can be achieved using successive decoding in a particular order [1], $\boldsymbol{C}_{\mathrm{OPT}}$ can be written in the following equivalent form. Define $\boldsymbol{\pi}_k, k = 1, 2, \ldots, K!$, as one of the $K!$ permutations of the users. Let π_{kj} denotes the order of user j in permutation $\boldsymbol{\pi}_k$. $\boldsymbol{C}_{\mathrm{OPT}}$ can be represented by

$$\boldsymbol{C}_{\mathrm{OPT}} = \bigcup_{\substack{\alpha_k\geq 0\\ \sum \alpha_k = 1}} \left\{ \boldsymbol{R} \middle| R_i \leq \sum_k \alpha_k E \times \left[\log \frac{\left| \boldsymbol{I} + \sum_{j,\pi_{kj}\geq \pi_{ki}} \mathrm{SNR}_j \boldsymbol{h}_j \boldsymbol{h}_j^H \right|}{\left| \boldsymbol{I} + \sum_{j,\pi_{kj}> \pi_{ki}} \mathrm{SNR}_j \boldsymbol{h}_j \boldsymbol{h}_j^H \right|} \right], \forall i \right\}. \tag{12}$$

The information rate region achieved by SSD is represented by

$$\boldsymbol{C}_{\mathrm{SSD}} = \left\{ \boldsymbol{R} \middle| R_i \leq E \left[\log \frac{\left| \boldsymbol{I} + \sum_j \mathrm{SNR}_j \boldsymbol{h}_j \boldsymbol{h}_j^H \right|}{\left| \boldsymbol{I} + \sum_{j\neq i} \mathrm{SNR}_j \boldsymbol{h}_j \boldsymbol{h}_j^H \right|} \right], \forall i \right\}. \tag{13}$$

The information rate region achieved by TDMA is ■

$$\boldsymbol{C}_{\mathrm{TDMA}} = \bigcup_{\substack{\xi_k\geq 0\\ \sum \xi_k = 1}} \left\{ \boldsymbol{R} \middle| R_i \leq \xi_i E \times \left[\log \left(1 + \frac{\mathrm{SNR}_i \|\boldsymbol{h}_i\|^2}{\xi_i} \right) \right], \forall i \right\} \tag{14}$$

where ξ_i is the time proportion when user i is scheduled to transmit its signal to the receiver.

Following the derivations presented in [2], it can be shown that the three channel sharing schemes achieve both the same minimum sum received energy per information nat and the same minimum sum transmitted energy per information nat. In addition, the minimum received energy per information nat of all the users are identical.

The slope regions, which were firstly introduced in [19], of the three channel sharing schemes are given by the following theorem.

Theorem 1: If the transmitters only know about CDI, given $\boldsymbol{\theta}$, the slope regions achieved by the OPT, SSD, and TDMA channel sharing schemes are given respectively by

$$\begin{aligned}
\boldsymbol{\mathcal{S}}^{\mathrm{OPT}}(\boldsymbol{\theta}) &= \left\{ \boldsymbol{\mathcal{S}} \middle| 0 \leq \mathcal{S}_i \leq \frac{2E[\|\boldsymbol{h}_i\|^2]^2}{E[\|\boldsymbol{h}_i\|^4]}, \forall i, \sum_i \frac{\theta_i^2}{\mathcal{S}_i} \geq \frac{1}{2} E\left[\left\| \sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]} \boldsymbol{h}_i \boldsymbol{h}_i^H \right\|_F^2 \right] \right\}\\
\boldsymbol{\mathcal{S}}^{\mathrm{SSD}}(\boldsymbol{\theta}) &= \left\{ \boldsymbol{\mathcal{S}} \middle| 0 \leq \mathcal{S}_i \leq \frac{2\theta_i^2}{\frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2}\theta_i^2 + \sum_{j\neq i} \frac{2\theta_i\theta_j E[|\boldsymbol{h}_i^H \boldsymbol{h}_j|^2]}{E[\|\boldsymbol{h}_i\|^2] E[\|\boldsymbol{h}_j\|^2]}}, \forall i \right\}\\
\boldsymbol{\mathcal{S}}^{\mathrm{TDMA}}(\boldsymbol{\theta}) &= \left\{ \boldsymbol{\mathcal{S}} \middle| \mathcal{S}_i \geq 0, \forall i, \sum_i \frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2} \mathcal{S}_i \leq 2 \right\}
\end{aligned} \tag{15}$$

where $\|.\|_F$ denotes the Frobenius norm. ■

The proof of Theorem 1 is given in Appendix A.

The maximum system slopes (obtained by maximizing $\mathcal{S}_0$ over the R_{sum} function) of the three channel sharing schemes are given in the following theorem.

Theorem 2: Given $\boldsymbol{\theta}$, if the transmitters only know the CDI, the system slopes achieved by the three channel sharing schemes are, respectively

$$\begin{aligned}
\mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta}) &= \frac{2}{E\left[\left\| \sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]} \boldsymbol{h}_i \boldsymbol{h}_i^H \right\|_F^2 \right]}\\
\mathcal{S}_{\max}^{\mathrm{SSD}}(\boldsymbol{\theta}) &= \frac{2}{2E\left[\left\| \sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]} \boldsymbol{h}_i \boldsymbol{h}_i^H \right\|_F^2 \right] - \sum_i \frac{E[\|\boldsymbol{h}_i\|^4]\theta_i^2}{E[\|\boldsymbol{h}_i\|^2]^2}}\\
\mathcal{S}_{\max}^{\mathrm{TDMA}}(\boldsymbol{\theta}) &= \frac{2}{\left(\sum_i \theta_i \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]} \right)^2}.
\end{aligned} \tag{16}$$

■

The proof of Theorem 2 is given in Appendix B.

According to (16), the SSD channel sharing is not far from optimal in the following sense.

$$\eta_{\mathrm{SSD|OPT}}(\boldsymbol{\theta}) = \frac{\mathcal{S}_{\max}^{\mathrm{SSD}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta})} \geq \frac{1}{2}. \quad (17)$$

Note that this result is consistent to the single antenna case studied in [7], [8].

B. The Flat Rayleigh Fading Case

Assume flat Rayleigh fading with the channel gains being i.i.d. complex Gaussian[4]. The maximum system slopes and their asymptotic behaviors are characterized by the following theorem.

Theorem 3: Assume flat Rayleigh fading with channel gains being zero mean i.i.d. Gaussian. The maximum system slopes of the OPT, SSD and TDMA channel sharing schemes are given, respectively, by

$$\begin{aligned} \mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta}) &= \frac{2M}{1 + M\sum_i \theta_i^2} \\ \mathcal{S}_{\max}^{\mathrm{SSD}}(\boldsymbol{\theta}) &= \frac{2M}{2 + (M-1)\sum_i \theta_i^2} \\ \mathcal{S}_{\max}^{\mathrm{TDMA}}(\boldsymbol{\theta}) &= \frac{2M}{M+1}. \end{aligned} \quad (18)$$

Consequently, the normalized spectral efficiencies of SSD and TDMA are given by

$$\begin{aligned} \eta_{\mathrm{SSD|OPT}}(\boldsymbol{\theta}) &= \frac{\mathcal{S}_{\max}^{\mathrm{SSD}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta})} \\ &= \frac{1 + M\sum_i \theta_i^2}{2 + (M-1)\sum_i \theta_i^2} \\ \eta_{\mathrm{TDMA|OPT}}(\boldsymbol{\theta}) &= \frac{\mathcal{S}_{\max}^{\mathrm{TDMA}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta})} = \frac{1 + M\sum_i \theta_i^2}{M+1}. \end{aligned} \quad (19)$$

■

The proof of Theorem 3 is presented in Appendix C.

In order to characterize the asymptotic behavior of $\eta_{\mathrm{SSD|OPT}}(\boldsymbol{\theta})$ and $\eta_{\mathrm{TDMA|OPT}}(\boldsymbol{\theta})$, we first specify the asymptotic behavior of $\boldsymbol{\theta}$ by letting $\theta_i = \frac{1}{K} f(\frac{i}{K})$ for all i, where $f(x)$ is assumed to be a real-valued non-negative function defined on $x \in [0,1]$, with $\int_0^1 f(x)dx = 1$ and $\int_0^1 f(x)^2 dx < \infty$.

Following from Theorem 3, if we fix $\frac{M}{K} = \beta$, then the normalized spectral efficiencies of SSD and TDMA satisfy

$$\begin{aligned} \lim_{K\to\infty, \frac{M}{K}=\beta} \eta_{\mathrm{SSD|OPT}}(\boldsymbol{\theta}) &= \frac{1 + \beta\int_0^1 f(x)^2 dx}{2 + \beta\int_0^1 f(x)^2 dx} \\ \lim_{K\to\infty, \frac{M}{K}=\beta} M\eta_{\mathrm{TDMA|OPT}}(\boldsymbol{\theta}) &= 1 + \beta\int_0^1 f(x)^2 dx. \end{aligned} \quad (20)$$

[4]Although flat Rayleigh fading is a typical channel fading model for a *narrowband* system, *wideband* results [5], [2] still apply when SNR goes to zero.

The relative spectral efficiency of SSD over TDMA satisfies

$$\begin{aligned} \eta_{\mathrm{SSD|TDMA}}(\boldsymbol{\theta}) &= \frac{M+1}{2 + (M-1)\sum_i \theta_i^2} \geq 1, \quad \forall M \\ \lim_{K\to\infty, \frac{M}{K}=\beta} \frac{1}{M}\eta_{\mathrm{SSD|TDMA}}(\boldsymbol{\theta}) &= \frac{1}{2 + \beta\int_0^1 f(x)^2 dx}. \end{aligned} \quad (21)$$

Equation (21) indicates that SSD achieves a larger system slope than TDMA for all M, and the relative spectral efficiency of SSD over TDMA scales linearly in M for large M. In other words, sharing the communication channel via TDMA is even worse (and can be significantly worse) than simply letting all the users transmit simultaneously with no additional control on interuser interference.

C. The Time-Invariant Channel Case

Assume the channels are time-invariant throughout the communication and channel gains are known to both the transmitters and the receiver. Whether SSD channel sharing is superior to TDMA in this case depends on the actual channel realization. Assume channel realization belongs to a given ensemble, and the transmitters choose the communication scheme *after knowing the channel realization*. In this section, we characterize the probability that the relative spectral efficiency of SSD over TDMA exceeds certain threshold. Note that we consider an ensemble of channel realizations each being time-invariant, and this differs from the case of a single multiaccess channel with time-varying channel gains.

Consider a particular channel realization, over which we can compare the channel sharing schemes in terms of the achievable information rates and the wideband slopes. Since the channel gains are time-invariant, the slope regions of the OPT, SSD and TDMA channel sharing schemes can be obtained from Theorem 1 by removing the expectation operation in (15):

$$\begin{aligned} \boldsymbol{\mathcal{S}}^{\mathrm{OPT}}(\boldsymbol{\theta}) &= \left\{ \boldsymbol{\mathcal{S}} \,\middle|\, \begin{array}{l} 0 \leq \mathcal{S}_i \leq 2, \forall i, \\ \sum_i \frac{\theta_i^2}{\mathcal{S}_i} \geq \frac{1}{2}\left\| \sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H \right\|_F^2 \end{array} \right\} \\ \boldsymbol{\mathcal{S}}^{\mathrm{SSD}}(\boldsymbol{\theta}) &= \left\{ \boldsymbol{\mathcal{S}} \,\middle|\, 0 \leq \mathcal{S}_i \leq \frac{2\theta_i^2}{\theta_i^2 + \sum_{j\neq i} \frac{2\theta_i\theta_j |\boldsymbol{h}_i^H\boldsymbol{h}_j|^2}{\|\boldsymbol{h}_i\|^2\|\boldsymbol{h}_j\|^2}}, \forall i \right\} \\ \boldsymbol{\mathcal{S}}^{\mathrm{TDMA}}(\boldsymbol{\theta}) &= \left\{ \boldsymbol{\mathcal{S}} \,\middle|\, \mathcal{S}_i \geq 0, \forall i, \sum_i \mathcal{S}_i \leq 2 \right\}. \end{aligned} \quad (22)$$

The maximum system slopes are obtained from Theorem 2 as

$$\begin{aligned} \mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta}) &= \frac{2}{\left\| \sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H \right\|_F^2} \\ \mathcal{S}_{\max}^{\mathrm{SSD}}(\boldsymbol{\theta}) &= \frac{2}{2\left\| \sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H \right\|_F^2 - \sum_i \theta_i^2} \\ \mathcal{S}_{\max}^{\mathrm{TDMA}}(\boldsymbol{\theta}) &= 2. \end{aligned} \quad (23)$$

Consequently, the relative spectral efficiency of SSD over TDMA is given by

$$\eta_{\text{SSD}|\text{TDMA}}(\boldsymbol{\theta}) = \frac{1}{2\left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2 - \sum_i \theta_i^2}. \tag{24}$$

The following theorem shows that, if channel realization is drawn randomly from certain ensemble, the probability of $\eta_{\text{SSD}|\text{TDMA}}(\boldsymbol{\theta}) \leq 1$ is small.

Theorem 4: Assume the channel realization is randomly drawn from an ensemble of multiaccess channels, denoted by U. The channel gains of members in U are samples of i.i.d. complex random variables whose density function is symmetric around the origin. Let $\theta_i = \frac{1}{K}f(\frac{i}{K})$ for all i, where we assume $f(x)$ is a non-negative function defined on $x \in [0,1]$ with $\int_0^1 f(x)dx = 1$ and $\int_0^1 f(x)^2 dx < \infty$. On one hand, if we fix $\frac{M}{K} = \beta$ and let K go to infinity, for any $\Delta > 0$, we have

$$\lim_{K\to\infty, \frac{M}{K}=\beta} MP\{\eta_{\text{SSD}|\text{TDMA}} \leq \Delta\} \leq \Delta\left[\beta\int_0^1 f(x)^2 dx + 2\right]. \tag{25}$$

On the other hand, if we fix M and let K go to infinity, for any $\Delta > 0$, we have

$$\lim_{K\to\infty} P\{\eta_{\text{SSD}|\text{TDMA}} \leq \Delta\} \leq \frac{2\Delta}{M}. \tag{26}$$

■

The proof of Theorem 4 is given in Appendix D.

Theorem 4 demonstrated that, if the number of receiving antennas is not small, for most of the multiaccess channels in U, SSD channel sharing achieves a higher spectral efficiency than TDMA. Indeed, according to the following computer simulations, with a high probability, the spectral efficiency of SSD channel sharing can be *significantly* higher than TDMA.

We set information rates of the users to be equal, i.e., $\theta_i = \frac{1}{K}, \forall i$. The channel gains are independently generated according to the complex normal distribution with zero mean and unit variance. While letting the number of antennas grow, we fix the ratio between the K and M at $\beta = \frac{M}{K} = \frac{1}{3}$. The median, the 99.5% quantile, $\overline{\eta_{\text{SSD}|\text{OPT}}}(P\{\eta_{\text{SSD}|\text{OPT}} < \overline{\eta_{\text{SSD}|\text{OPT}}}\} = 99.5\%)$, and the 0.5% quantile, $\underline{\eta_{\text{SSD}|\text{OPT}}}(P(\eta_{\text{SSD}|\text{OPT}} < \underline{\eta_{\text{SSD}|\text{OPT}}}) = 0.5\%)$ of the relative spectral efficiency of SSD over TDMA are shown in Fig. 1. Each data point is obtained based on 20 000 Monte-Carlo runs. It is clearly seen that the median of the relative spectral efficiency scales linearly in the number of receiving antennas. For the system with three receiving antennas, SSD achieves a system slope larger than TDMA in over 99.5% of the channel realizations.

IV. Discussion

In the low SNR regime, since interuser interference is a minor factor compared with the ambient noise, single-user decoding loses less than half of the spectral efficiency. Under a complexity constraint, simplicity of the single user decoding enables the use of large number of receiving antennas at the basestation. Hence single user decoding can be an ideal channel sharing scheme for complexity-constrained systems.

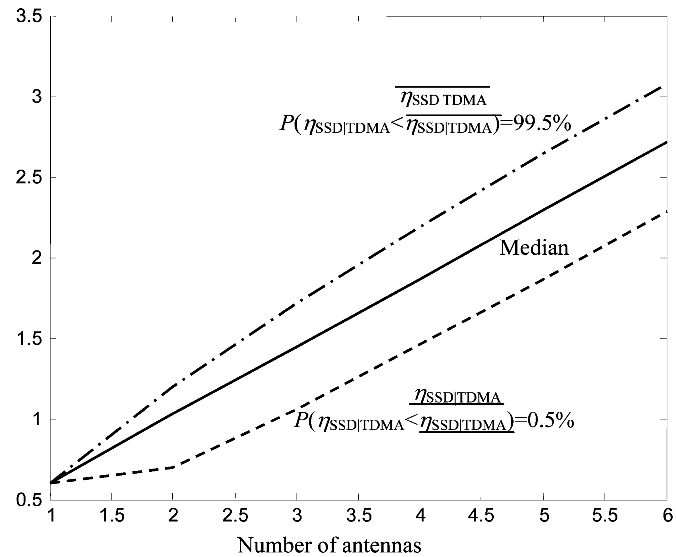

Fig. 1. Illustration on the relative spectral efficiency of SSD over TDMA as a function of number of antennas, M. $\frac{K}{M} = 3$. Data obtained from 20 000 random realizations of time-invariant multiaccess channels.

Having multiple antenna at the basestation enables the receiver to spatially distinguish signals and consequently introduces the multiplexing gain. A necessary condition for exploiting such multiplex gain is to let multiple users communicate simultaneously over each time, frequency or coding dimension. According to this understanding, it is easily seen that the inefficiency of TDMA (i.e., not being able to exploit multiuser multiplexing gain) also applies to other orthogonal channel sharing schemes such as the orthogonal frequency division multiaccess (FDMA) and the orthogonal code-division multiaccess (CDMA).

Appendix A

Note that the uniform convergence property is widely used in the derivations presented in the appendices. Most derivations are carried out using a two step procedure. In the first step, we assume that some parameters such as the normalized signal correlation matrices or the time sharing coefficients are given. We term these parameters the *secondary parameters*. The wideband slope regions are obtained as functions of the secondary parameters. Then, in the second step, we obtain the wideband slope regions by taking the union over all possible values of the secondary parameters. The justification of such a two step procedure is the fact that the convergence of the results on the secondary parameters is uniform, as demonstrated in the proof of [2, Th. 1].

A. *Proof of Theorem 1*

Proof: By definition, we have

$$\frac{E_i^r}{N_0} = \frac{E[\|\boldsymbol{h}_i\|^2]\text{SNR}_i}{R_i}. \tag{27}$$

It can be shown that $\frac{E_i^r}{N_0} \to 1$ when SNR_i goes to zero, for all i. Hence, (27) implies, for all i

$$E[\|\boldsymbol{h}_i\|^2]\mathrm{SNR}_i = \theta_i \sum_j E[\|\boldsymbol{h}_j\|^2]\mathrm{SNR}_j. \tag{28}$$

Define

$$\mathrm{SNR} = \sum_j E[\|\boldsymbol{h}_j\|^2]\mathrm{SNR}_j. \tag{29}$$

It can be shown that (7) holds with SNR^r being replaced by SNR.

1) The Slope Region of Optimal Superposition: Fix $\boldsymbol{\alpha}$. From (12), we know that the maximum information rate of user i in the low SNR regime can be written asymptotically as a function of the SNR by

$$R_i(\mathrm{SNR}) = \sum_k \alpha_k E \times \left[\log \frac{\left|\boldsymbol{I} + \mathrm{SNR}\sum_{j,\pi_{kj} \geq \pi_{ki}} \frac{\theta_j}{E[\|\boldsymbol{h}_j\|^2]}\boldsymbol{h}_j\boldsymbol{h}_j^H\right|}{\left|\boldsymbol{I} + \mathrm{SNR}\sum_{j,\pi_{kj} > \pi_{ki}} \frac{\theta_j}{E[\|\boldsymbol{h}_j\|^2]}\boldsymbol{h}_j\boldsymbol{h}_j^H\right|}\right]. \tag{30}$$

According to the following formula [5]

$$\frac{d}{du}\log|\boldsymbol{I} + u\boldsymbol{A}|\Big|_{u=0} = \mathrm{trace}(\boldsymbol{A})$$
$$\frac{d^2}{du^2}\log|\boldsymbol{I} + u\boldsymbol{A}|\Big|_{u=0} = -\mathrm{trace}(\boldsymbol{A}^2) \tag{31}$$

we have

$$\dot{R}_i(0) = \theta_i$$
$$-\ddot{R}_i(0) = \frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2}\theta_i^2 + 2\sum_k \alpha_k \sum_{j,\pi_{kj} > \pi_{ki}} \frac{\theta_i\theta_j E\left[\left|\boldsymbol{h}_i^H\boldsymbol{h}_j\right|^2\right]}{E[\|\boldsymbol{h}_i\|^2]E[\|\boldsymbol{h}_j\|^2]}. \tag{32}$$

Because the slope of user i must satisfy $\mathcal{S}_i \leq \frac{2\dot{R}_i(0)^2}{-\ddot{R}_i(0)}$ [5], [2], combining with (32) and $\alpha_k \geq 0$ for all k, we get

$$\mathcal{S}_i \leq \frac{2\dot{R}_i(0)^2}{-\ddot{R}_i(0)} \leq \frac{2E[\|\boldsymbol{h}_i\|^2]^2}{E[\|\boldsymbol{h}_i\|^4]}. \tag{33}$$

Meanwhile

$$\sum_i \frac{\theta_i^2}{\mathcal{S}_i} \geq \sum_i \frac{-\theta_i^2\ddot{R}_i(0)}{2\dot{R}_i(0)^2} = -\frac{1}{2}\sum_i \ddot{R}_i(0)$$
$$= \frac{1}{2}\sum_i \frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2}\theta_i^2 + \sum_k \alpha_k \sum_{i,j,\pi_{kj} > \pi_{ki}} \frac{\theta_i\theta_j E\left[\left|\boldsymbol{h}_i^H\boldsymbol{h}_j\right|^2\right]}{E[\|\boldsymbol{h}_i\|^2]E[\|\boldsymbol{h}_j\|^2]}$$
$$= \frac{1}{2}\sum_i \frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2}\theta_i^2 + \sum_{i,j,i>j} \frac{\theta_i\theta_j E\left[\left|\boldsymbol{h}_i^H\boldsymbol{h}_j\right|^2\right]}{E[\|\boldsymbol{h}_i\|^2]E[\|\boldsymbol{h}_j\|^2]}$$
$$= \frac{1}{2}E\left[\left\|\sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2\right]. \tag{34}$$

Since both (33) and (34) can be achieved with equality, the slope region of optimal superposition must be given by the first equality in (15).

2) The Slope Region of SSD: From (13), we know that the maximum information rate of user i in the low SNR regime can be written as

$$R_i(\mathrm{SNR}) = \sum_k \alpha_k E \times \left[\log \frac{\left|\boldsymbol{I} + \mathrm{SNR}\sum_j \frac{\theta_j}{E[\|\boldsymbol{h}_j\|^2]}\boldsymbol{h}_j\boldsymbol{h}_j^H\right|}{\left|\boldsymbol{I} + \mathrm{SNR}\sum_{j,j\neq i} \frac{\theta_j}{E[\|\boldsymbol{h}_j\|^2]}\boldsymbol{h}_j\boldsymbol{h}_j^H\right|}\right]. \tag{35}$$

Hence,

$$\dot{R}_i(0) = \theta_i$$
$$-\ddot{R}_i(0) = \frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2}\theta_i^2 + 2\sum_{j,j\neq i} \frac{\theta_i\theta_j}{E[\|\boldsymbol{h}_i\|^2]E[\|\boldsymbol{h}_j\|^2]}E\left[\left|\boldsymbol{h}_i^H\boldsymbol{h}_j\right|^2\right]. \tag{36}$$

From $\mathcal{S}_i \leq \frac{2\dot{R}_i(0)^2}{-\ddot{R}_i(0)}$, we obtain the slope region of SSD as the second equality in (15).

3) The Slope Region of TDMA: For TDMA, fix $\boldsymbol{\xi}$, the maximum information rate of user i in the low SNR regime is given by

$$R_i(\mathrm{SNR}) = \xi_i E\left[\log\left(1 + \mathrm{SNR}\frac{\theta_i\|\boldsymbol{h}_i\|^2}{\xi_i E[\|\boldsymbol{h}_i\|^2]}\right)\right]. \tag{37}$$

Consequently, the slope of user i must satisfy

$$\mathcal{S}_i \leq \frac{2\dot{R}_i(0)^2}{-\ddot{R}_i(0)} = 2\xi_i\frac{E[\|\boldsymbol{h}_i\|^2]^2}{E[\|\boldsymbol{h}_i\|^4]}. \tag{38}$$

Since (38) can be achieved with equality, taking the union of the right hand side of (38) over all $\boldsymbol{\xi}$ gives the third equality in (15). ■

B. Proof of Theorem 2

Proof: Note that the maximum system slope is obtained by maximizing $\mathcal{S}_0$ over the R_{sum} function. Following a similar analysis presented in [5], it can be shown that the maximum system slope is achieved when R_{sum} equals the sum capacity of the multiaccess system. This part of the proof is skipped.

For optimal superposition, we write the maximum sum information rate in the low SNR regime as a function of the SNR, defined in (29), as

$$R_{\mathrm{sum}}(\mathrm{SNR}) = E\left[\log\left|\boldsymbol{I} + \mathrm{SNR}\sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]}\boldsymbol{h}_i\boldsymbol{h}_i^H\right|\right]. \tag{39}$$

The maximum wideband system slope is then obtained by

$$\mathcal{S}_{\max}^{\mathrm{OPT}} = \frac{2\dot{R}_{\mathrm{sum}}(0)^2}{-\ddot{R}_{\mathrm{sum}}(0)} = \frac{2}{E\left[\left\|\sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2\right]}. \tag{40}$$

For SSD, the sum information rate is given by

$$R_{\mathrm{sum}}(\mathrm{SNR}) = \sum_i E \times \left[\log \frac{\left|\boldsymbol{I} + \mathrm{SNR}\sum_j \frac{\theta_j}{E[\|\boldsymbol{h}_j\|^2]}\boldsymbol{h}_j\boldsymbol{h}_j^H\right|}{\left|\boldsymbol{I} + \mathrm{SNR}\sum_{j\neq i} \frac{\theta_j}{E[\|\boldsymbol{h}_j\|^2]}\boldsymbol{h}_j\boldsymbol{h}_j^H\right|}\right]. \tag{41}$$

Consequently

$$\mathcal{S}_{\max}^{\mathrm{SSD}} = \frac{2\dot{R}_{\mathrm{sum}}(0)^2}{-\ddot{R}_{\mathrm{sum}}(0)} = \frac{2}{2E\left[\left\|\sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2\right] - \sum_i \frac{E[\|\boldsymbol{h}_i\|^4]\theta_i^2}{E[\|\boldsymbol{h}_i\|^2]^2}}. \tag{42}$$

For TDMA, the sum rate is given by

$$R_{\mathrm{sum}}(\mathrm{SNR}) = \sum_i \xi_i E\left[\log\left(1 + \mathrm{SNR}\frac{\theta_i\|\boldsymbol{h}_i\|^2}{\xi_i E[\|\boldsymbol{h}_i\|^2]}\right)\right]. \tag{43}$$

Therefore, the wideband system slope equals

$$\mathcal{S}_{\max}^{\mathrm{TDMA}} = \frac{2\dot{R}_{\mathrm{sum}}(0)^2}{-\ddot{R}_{\mathrm{sum}}(0)} = \frac{2}{\sum_i \frac{\theta_i^2 E[\|\boldsymbol{h}_i\|^4]}{\xi_i E[\|\boldsymbol{h}_i\|^2]^2}}. \tag{44}$$

Note that

$$\sum_i \frac{\theta_i^2 E[\|\boldsymbol{h}_i\|^4]}{\xi_i E[\|\boldsymbol{h}_i\|^2]^2} = \sum_i \theta_i \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]} \frac{1}{\frac{\xi_i E[\|\boldsymbol{h}_i\|^2]}{\theta_i\sqrt{E[\|\boldsymbol{h}_i\|^4]}}}. \tag{45}$$

Define $p_i = \frac{\theta_i \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]}}{\sum_j \theta_j \frac{\sqrt{E[\|\boldsymbol{h}_j\|^4]}}{E[\|\boldsymbol{h}_j\|^2]}}$ and regard p_i as a probability variable. Due to the inequality that $E[\frac{1}{x}] \geq \frac{1}{E[x]}$ for $x > 0$, we get from (45)

$$\begin{aligned}\sum_i \frac{\theta_i^2 E[\|\boldsymbol{h}_i\|^4]}{\xi_i E[\|\boldsymbol{h}_i\|^2]^2} &= \left(\sum_j \theta_j \frac{\sqrt{E[\|\boldsymbol{h}_j\|^4]}}{E[\|\boldsymbol{h}_j\|^2]}\right)\sum_i p_i \frac{1}{\frac{\xi_i E[\|\boldsymbol{h}_i\|^2]}{\theta_i\sqrt{E[\|\boldsymbol{h}_i\|^4]}}} \\ &\geq \left(\sum_j \theta_j \frac{\sqrt{E[\|\boldsymbol{h}_j\|^4]}}{E[\|\boldsymbol{h}_j\|^2]}\right)\frac{1}{\sum_i p_i \frac{\xi_i E[\|\boldsymbol{h}_i\|^2]}{\theta_i\sqrt{E[\|\boldsymbol{h}_i\|^4]}}} \\ &= \left(\sum_j \theta_j \frac{\sqrt{E[\|\boldsymbol{h}_j\|^4]}}{E[\|\boldsymbol{h}_j\|^2]}\right)^2.\end{aligned} \tag{46}$$

Consequently, we have

$$\mathcal{S}_{\max}^{\mathrm{TDMA}} \leq \frac{2}{\left(\sum_i \theta_i \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]}\right)^2}. \tag{47}$$

Since (47) holds with equality when $\xi_i = \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]}\theta_i$ for all i, we have

$$\mathcal{S}_{\max}^{\mathrm{TDMA}} = \frac{2}{\left(\sum_i \theta_i \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]}\right)^2}. \tag{48}$$

Note that if $\frac{E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2}$ are identical for all i, the system slope is maximized when the received energy per nat of the users are identical. ■

C. Proof of Theorem 3

Proof: Denote the kth element of $\boldsymbol{h}_i$ by h_{ik}. Since the entries of $\boldsymbol{h}_i$ are i.i.d. Gaussian with zero mean, denote the variance of h_{ik} by σ^2. We have

$$\begin{aligned}E[\|\boldsymbol{h}_i\|^4] &= \sum_i E[|h_{ii}|^4] + \sum_{k,l,k\neq l} E[|h_{ik}|^2|h_{il}|^2] \\ &= (M^2+M)\sigma^4.\end{aligned} \tag{49}$$

According to (48) and (49), we have

$$\mathcal{S}_{\max}^{\mathrm{TDMA}}(\boldsymbol{\theta}) = \frac{2}{\left(\sum_i \theta_i \frac{\sqrt{M^2+M}}{M}\right)^2} = \frac{2M}{M+1}. \tag{50}$$

To get the maximum system slope of optimal superposition and SSD, we first obtain

$$\begin{aligned}E&\left[\left\|\sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2\right] \\ &= \sum_i \frac{\theta_i^2 E[\|\boldsymbol{h}_i\|^4]}{E[\|\boldsymbol{h}_i\|^2]^2} + \sum_{i,j,i\neq j} \frac{\theta_i\theta_j \sum_{k=1}^M E[|h_{ik}|^2]E[|h_{jk}|^2]}{E[\|\boldsymbol{h}_i\|^2]E[\|\boldsymbol{h}_j\|^2]} \\ &= \frac{M+1}{M}\sum_i \theta_i^2 + \frac{1}{M}\sum_{i\neq j}\theta_i\theta_j \\ &= \sum_i \theta_i^2 + \frac{1}{M}.\end{aligned} \tag{51}$$

According to (40), (42) and (51), we get

$$\begin{aligned}\mathcal{S}_{\max}^{\mathrm{OPT}}(\boldsymbol{\theta}) &= \frac{2M}{1+M\sum_i \theta_i^2} \\ \mathcal{S}_{\max}^{\mathrm{SSD}}(\boldsymbol{\theta}) &= \frac{2M}{2+(M-1)\sum_i \theta_i^2}.\end{aligned} \tag{52}$$

■

D. Proof of Theorem 4

Proof: Denote the kth element of $\boldsymbol{h}_i$ by h_{ik}. Since the entries of $\boldsymbol{h}_i$, for all i, are i.i.d. with zero mean, we have

$$\begin{aligned} E&\left[\left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2\right] \\ &= \sum_i\left(\theta_i^2 E\left[\frac{\left\|\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2}{\|\boldsymbol{h}_i\|^4}\right]\right) \\ &\quad + \sum_{i,j,i\neq j} E\left[\frac{\theta_i\theta_j\sum_{k=1}^M |h_{ik}|^2|h_{jk}|^2}{\|\boldsymbol{h}_i\|^2\|\boldsymbol{h}_j\|^2}\right] \\ &= \sum_i \theta_i^2 + \frac{1}{M}\sum_{i,j,i\neq j}\theta_i\theta_j \\ &= \frac{M-1}{M}\sum_i \theta_i^2 + \frac{1}{M}. \end{aligned} \tag{53}$$

Consequently

$$E[\eta_{\mathrm{TDMA|SSD}}(\boldsymbol{\theta})] = \frac{M-2}{M}\sum_i \theta_i^2 + \frac{2}{M}. \tag{54}$$

Note that $\eta_{\mathrm{TDMA|SSD}}(\boldsymbol{\theta}) \geq 0$. If we fix $\frac{M}{K} = \beta$ and let $K \to \infty$, we obtain from (54) and Markov's inequality,

$$\begin{aligned} &\lim_{K\to\infty,\frac{M}{K}=\beta} MP\{\eta_{\mathrm{SSD|TDMA}} \leq \Delta\} \\ &= \lim_{K\to\infty,\frac{M}{K}=\beta} MP\left\{\eta_{\mathrm{TDMA|SSD}} \geq \frac{1}{\Delta}\right\} \\ &\leq \lim_{K\to\infty,\frac{M}{K}=\beta} M\Delta E[\eta_{\mathrm{TDMA|SSD}}(\boldsymbol{\theta})] \\ &= \lim_{K\to\infty,\frac{M}{K}=\beta} \Delta\left[(M-2)\sum_i \theta_i^2 + 2\right] \\ &= \Delta\left[\beta\int_0^1 f(x)^2 dx + 2\right]. \end{aligned} \tag{55}$$

If we fix M and let $K \to \infty$, we have

$$\begin{aligned} &\lim_{K\to\infty} P\{\eta_{\mathrm{SSD|TDMA}} \leq \Delta\} \\ &= \lim_{K\to\infty} P\left\{\eta_{\mathrm{TDMA|SSD}} \geq \frac{1}{\Delta}\right\} \\ &\leq \lim_{K\to\infty} \Delta E[\eta_{\mathrm{TDMA|SSD}}(\boldsymbol{\theta})] \\ &= \lim_{K\to\infty} \Delta\left[\frac{M-2}{M}\sum_i \theta_i^2 + \frac{2}{M}\right] = \frac{2\Delta}{M}. \end{aligned} \tag{56}$$

■

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