# Comparison of Two Low Complexity Multiple Access Schemes

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***Abstract*— This paper studies multiple access channels with additive Gaussian noise in the low signal to noise ratio (SNR) regime. We compare the spectral efficiencies of the optimal superposition channel sharing scheme and two simple alternatives: the time division multiple access (TDMA) scheme and the parallel multiple access scheme with single user decoding (PMAS). We consider the situation when the receiver has multiple receive antennas while each transmitter only has single antenna. We show that, due to TDMA's inefficiency in exploiting the multiuser multiplex gain, the relative spectral efficiency of PMAS over TDMA grows drastically as the number of receive antennas increase. The relative spectral efficiency of PMAS over the optimal scheme is approximately 1/2, irrespective of the number of receive antennas. Since the simplicities of PMAS and TDMA are similar, our results suggest that PMAS is a better alternative to TDMA for multiple access system in the low SNR regime.** [1]

## I. Introduction

In multiple access systems, superposition strategies, where terminals transmit simultaneously in both time and frequency, offer higher information capacity, in general, than the orthogonal strategies such as the time-division multiple access (TDMA) [1]. However, the excessive encoding and decoding complexity of the optimal channel sharing scheme often makes it infeasible for practical implementation. The TDMA scheme, despite being suboptimal in common scenarios, remains the dominant channel sharing scheme in most of the wireless systems for multipoint-to-point and point-to-multipoint links. From a cross-layer point of view, since the simplicity of TDMA enables the tractability of many cross-layer optimizations, which consequently brings overall performance gain, the value of maintaining a simple channel sharing scheme extends much beyond the complexity consideration. The dominance of TDMA is indeed due to the fact that its suboptimality is often not significant enough to offset its advantage of simple system design.

It has been well recognized in the past decade that the use of multiple antennas is a cost effective way to boost the capacities of wireless multiple access and broadcast channels [2][3][4]. Although the capacity achieving schemes in multiple antenna systems can be excessively complex, capacity benefits brought by the use of multiple antennas can be easily exploited (although not fully) with simple suboptimal channel sharing schemes. In this paper, we consider multiple access systems with multiple-antenna receiver and single-antenna transmitters working in the low signal to noise ratio (SNR) regime. Our purpose is to show that, in such a scenario, TDMA is no longer the ideal channel sharing scheme due to its inefficiency in exploiting the multiuser multiplex gain. There is another equally simple (if not simpler) channel sharing scheme, the parallel multiple access with single user decoding (PMAS), that significantly outperforms TDMA.

We compare the spectral efficiencies of the optimal superposition (OPT) channel sharing scheme and those of the two simple schemes: PMAS and TDMA. The analyses are based on the tools introduced by Verdú and Caire in [5][1]. We derive the close-form expressions of the wideband system slopes (see detailed definition in Section II) for all three channel sharing schemes. Analyses on the asymptotic behaviors are carried out under the assumption of flat Rayleigh fading channels. We show the relative spectral efficiency, defined as the ratio of the system slopes of PMAS over TDMA, scales linearly in the number of receive antennas asymptotically[2], irrespective of whether the transmitters know the channel state information (CSI) or they only know the channel distribution information (CDI). Although the results are derived for asymptotics, we demonstrate via computer simulations that the superiority of PMAS over TDMA appears in systems with small number of receive antennas. In addition, we show the relative spectral efficiency of PMAS over the OPT scheme is always above $1/2$.

## II. Preliminaries

In order to simplify the notations, we denote information in nats instead of bits. All the logarithms are natural based. The proofs of all the theorems presented in this paper are given in [6].

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[2]In the paper, we consider two scenarios: either fixing the ratio between the number of receive antennas and the number of terminals, or simply fixing the number of antennas at the receiver. Asymptotics are taken by letting the number of terminals approach infinity.

For a single user channel, define $E$ as the transmitted energy per information nat. Let $P$ be the transmitted power per second. The information rate $R$ in nats per second satisfies $R\frac{E}{N_0} = \frac{P}{N_0}$, where $N_0$ is the one-side noise spectral density level. Given the bandwidth of the system, $B$, the spectral efficiency of a single user system is defined in [5] as $\mathsf{C} = \frac{R}{B}$, which is a function of $\frac{E}{N_0}$. Usually, in comparing systems with the same reserved bandwidth, there is no loss of generality in letting $\mathsf{C} = R$; the maximum value of $R$ can be computed theoretically via the Shannon formula as a function of the SNR. It was shown in [5] that, when SNR is low, the minimum transmitted energy per nat, $\frac{E}{N_0}_{\min} = \lim_{\text{SNR}\to 0} \frac{E}{N_0}$, and the wideband slope $\mathcal{S}_0 = \lim_{\frac{E}{N_0}\downarrow\frac{E}{N_0}_{\min}} \frac{\mathsf{C}}{\log\frac{E}{N_0} - \log\frac{E}{N_0}_{\min}}$ are two key performance measures characterizing the spectral efficiency and energy efficiency tradeoff of the single user communication system. If two systems have the same $\frac{E}{N_0}_{\min}$, then under the constraint of an equal energy efficiency, the ratio of their spectral efficiencies is given by the ratio of their wideband slops in the low SNR regime [5].

To analyze a multiuser system, it is suggested in [7][1] that one should fix the ratios among the information rates of the users. For example, given $\boldsymbol{\theta}$ with $\theta_i \geq 0, \forall i$ and $\sum_i \theta_i = 1$, we analyze the system in the low SNR regime while maintaining the following equality.

$$R_i = \theta_i \sum_j R_j, \qquad \forall i \tag{1}$$

where $R_i$ is the information rate of user $i$. Consequently, we can derive, for each individual user, the minimum energy per nat and the wideband slope, both are functions of $\boldsymbol{\theta}$. Similar to the single user system case, if two multiuser systems have equal minimum energy per nat for all $i$ and $\boldsymbol{\theta}$, their spectral efficiencies can be compared via the comparison of their wideband slop regions [7][1].

Let the sum transmitted energy per nat of a multiuser system be $E_{\text{sum}}$. Let the sum transmit power per second be $P_{\text{sum}}$. The sum information rate satisfies

$$R_{\text{sum}}\frac{E_{\text{sum}}}{N_0} = \frac{P_{\text{sum}}}{N_0} \tag{2}$$

Given the overall bandwidth of the system, $B$, we define the spectral efficiency of the multiuser system as

$$\mathsf{C} = \frac{R_{\text{sum}}}{B} \tag{3}$$

Similar to the single user case, when comparing different multiuser systems with the same total reserved bandwidth, we can simply let $\mathsf{C} = R_{\text{sum}}$; the right hand side is a function of the individual SNRs.

It is easy to see that, if Equality (1) is always true, the normalized minimum sum transmitted energy per nat of the system is the weighted sum of the corresponding individual limits. Namely,

$$\frac{E_{\text{sum}}}{N_0}_{\min} = \sum_j \theta_j \frac{E_j}{N_0}_{\min} \tag{4}$$

While fixing the ratios among the information rates of the terminals, we define the wideband system slope, which is a function of $\boldsymbol{\theta}$, as follows.

$$\mathcal{S}_0 = \lim_{\frac{E_{\text{sum}}}{N_0}\downarrow\frac{E_{\text{sum}}}{N_0}_{\min}} \frac{\mathsf{C}}{\log\frac{E_{\text{sum}}}{N_0} - \log\frac{E_{\text{sum}}}{N_0}_{\min}} \tag{5}$$

Let $P_i^r$ be the received power per second of user $i$. Define $\text{SNR}_i = \frac{P_i^r}{N_0}$ and $\text{SNR} = \sum_i \text{SNR}_i$. Asymptotically as SNR goes to zero, we have $\text{SNR}_i \to \theta_i\text{SNR}$. Due to the constraint of (1), when SNR goes to zero, the convergences on the ratios among the individual SNRs are uniform. Hence by following the analysis on individual slopes presented in [5], we obtain

$$\mathcal{S}_0 = \frac{2(\dot{R}_{\text{sum}}(\text{SNR})|_{\text{SNR}=0})^2}{\ddot{R}_{\text{sum}}(\text{SNR})|_{\text{SNR}=0}} \tag{6}$$

where the derivatives are taken with respect to $\text{SNR} = \sum_i \text{SNR}_i$.

In the rest of the paper, we study the OPT, PMAS and TDMA channel sharing schemes in the low SNR regime by comparing their minimum transmitted energies per nat and their maximum system slopes, which are denoted by $\mathcal{S}_{\max}^{\text{OPT}}(\boldsymbol{\theta})$, $\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta})$, and $\mathcal{S}_{\max}^{\text{TDMA}}(\boldsymbol{\theta})$, respectively. Since the system has zero spectral efficiency when SNR equals zero, if two channel sharing schemes have the same minimum sum transmitted energy per nat, the ratio between their system slopes characterizes the ratio between their system spectral efficiencies in the low SNR regime. Hence we define

$$\eta_{\text{PMAS}|\text{TDMA}}(\boldsymbol{\theta}) = \frac{\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\text{TDMA}}(\boldsymbol{\theta})} \tag{7}$$

as the relative spectral efficiency of PMAS over TDMA. We also term $\eta_{\text{PMAS}|\text{OPT}}(\boldsymbol{\theta})$ and $\eta_{\text{TDMA}|\text{OPT}}(\boldsymbol{\theta})$ the normalized spectral efficiencies of PMAS and TDMA, respectively.

## III. System Model

We assume there are $K$ users transmitting signals to a common receiver. The receiver is equipped with $M$ antennas, while the transmitters have only one antenna each. The received signal is given by a $M$-component complex-valued column vector,

$$\boldsymbol{y} = \sum_{i=1}^{K} \boldsymbol{h}_i x_i + \boldsymbol{n} \tag{8}$$

Here $x_i$ is the complex-valued symbol from user $i$; $\boldsymbol{h}_i$ is the channel gain vector from user $i$ to the receive antennas; $\boldsymbol{n}$ is an additive complex Gaussian noise with zero mean and covariance matrix $N_0\boldsymbol{I}$. Suppose the transmit power of terminal $i$ satisfies

$$E[|x_i|^2] \leq P_i = \text{SNR}_i N_0 \tag{9}$$

where $\text{SNR}_i$ is the normalized transmit power per receive antenna of user $i$.

## IV. Fading Channels with CDI at The Transmitters

If the transmitters only know the CDI, the capacity region of the multiple access system is given by

$$\boldsymbol{C}_{\text{OPT}} = \left\{ \boldsymbol{R} \left| \sum_{i \in J} R_i \leq E\left[ \log \left| \boldsymbol{I} + \sum_{i \in J} \text{SNR}_i \boldsymbol{h}_i \boldsymbol{h}_i^H \right| \right] \right. \right\} \tag{10}$$

where $\boldsymbol{h}_i^H$ denotes the conjugate transpose of $\boldsymbol{h}_i$. The boundary of the capacity region can be achieved by successive decoding.

The information rate region achieved by PMAS is,

$$\boldsymbol{C}_{\text{PMAS}} = \left\{ \boldsymbol{R} \left| R_i \leq E\left[ \log \frac{\left| \boldsymbol{I} + \sum_j \text{SNR}_j \boldsymbol{h}_j \boldsymbol{h}_j^H \right|}{\left| \boldsymbol{I} + \sum_{j \neq i} \text{SNR}_j \boldsymbol{h}_j \boldsymbol{h}_j^H \right|} \right] \right. \right\} \tag{11}$$

The information rate region achieved by TDMA, denoted by $\boldsymbol{C}_{\text{TDMA}}$, equals

$$\bigcup_{\substack{\xi_k \geq 0 \\ \sum \xi_k = 1}} \left\{ \boldsymbol{R} \left| R_i \leq \xi_i E\left[ \log\left( 1 + \frac{\text{SNR}_i \|\boldsymbol{h}_i\|^2}{\xi_i} \right) \right] \right. \right\} \tag{12}$$

where $\xi_i$ is the proportion of time when terminal $i$ is scheduled to transmit signal to the receiver.

From (10), (11) and (12), we obtain the following theorem.

**Theorem 1:** If the ratios among the information rates are fixed as in (1), then for any user $i$, the OPT, PMAS and TDMA channel sharing schemes achieve the same minimum transmitted energy per information nat, which is given by $\frac{E_i}{N_0}_{\min} = \frac{1}{E[\|\boldsymbol{h}_i\|^2]}$.

If the transmitters only know the CDI, the maximum system slope achieved by the three channel sharing schemes are, respectively,

$$\mathcal{S}_{\max}^{\text{OPT}}(\boldsymbol{\theta}) = \frac{2}{E\left[ \left\| \sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]} \boldsymbol{h}_i \boldsymbol{h}_i^H \right\|_F^2 \right]} \tag{13}$$

$$\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta}) = \frac{2}{2E\left[ \left\| \sum_i \frac{\theta_i}{E[\|\boldsymbol{h}_i\|^2]} \boldsymbol{h}_i \boldsymbol{h}_i^H \right\|_F^2 \right] - \sum_i \frac{E[\|\boldsymbol{h}_i\|^4]\theta_i^2}{E[\|\boldsymbol{h}_i\|^2]^2}} \tag{14}$$

$$\mathcal{S}_{\max}^{\text{TDMA}}(\boldsymbol{\theta}) = \frac{2}{\left( \sum_i \theta_i \frac{\sqrt{E[\|\boldsymbol{h}_i\|^4]}}{E[\|\boldsymbol{h}_i\|^2]} \right)^2} \tag{15}$$

where $\|.\|_F$ denotes the Frobenius norm. ■

Compare (14) with (13), we can see PMAS is not far from optimal in the following sense.

$$\eta_{\text{PMAS|OPT}}(\boldsymbol{\theta}) = \frac{\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\text{OPT}}(\boldsymbol{\theta})} \geq \frac{1}{2} \tag{16}$$

For many popular channel fading models, the maximum system slopes given in Theorem 1 can be evaluated explicitly. However, in order to avoid advanced random matrix analyses such as those introduced in [8], in this paper, we assume the channels are independently Rayleigh-faded. The following theorem gives the system slopes of the three channel sharing schemes and their asymptotic behaviors.

**Theorem 2:** Assume flat Rayleigh fading channels; the channel parameters of different transmitters are independently distributed; and the channel gains of each transmitter are i.i.d. Gaussian with zero mean. Then, the maximum system slopes of the OPT, PMAS and TDMA channel sharing schemes are, respectively,

$$\begin{aligned}
\mathcal{S}_{\max}^{\text{OPT}}(\boldsymbol{\theta}) &= \frac{2M}{1 + M\sum_i \theta_i^2} \\
\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta}) &= \frac{2M}{2 + (M-1)\sum_i \theta_i^2} \\
\mathcal{S}_{\max}^{\text{TDMA}}(\boldsymbol{\theta}) &= \frac{2M}{M+1}
\end{aligned} \tag{17}$$

Assume that there exists a real-valued non-negative function $f(x)$ defined on $x \in [0,1]$, with $\int_0^1 f(x)dx = 1$ and $\int_0^1 f(x)^2 dx < \infty$. Let the number of terminals go to infinity, and let $\theta_i$ converge to $\frac{1}{K} f\left(\frac{i}{K}\right)$ in the sense of $\lim_{K\to\infty} \frac{\theta_i}{K} f\left(\frac{i}{K}\right) = 1$. If we fix $\frac{M}{K} = \beta$, asymptotically, the normalized spectral efficiencies of PMAS and TDMA satisfy

$$\lim_{K\to\infty} \eta_{\text{PMAS|OPT}}(\boldsymbol{\theta}) = \frac{1 + \beta\int_0^1 f(x)^2 dx}{2 + \beta\int_0^1 f(x)^2 dx} \tag{18}$$

$$\lim_{K\to\infty} M\eta_{\text{TDMA|OPT}}(\boldsymbol{\theta}) = 1 + \beta\int_0^1 f(x)^2 dx \tag{19}$$

■

According to Theorem 2, if we consider the relative spectral efficiency of PMAS over TDMA, given by

$$\eta_{\text{PMAS|TDMA}}(\boldsymbol{\theta}) = \frac{\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\text{TDMA}}(\boldsymbol{\theta})} = \frac{M+1}{2 + (M-1)\sum_i \theta_i^2} \tag{20}$$

As $K \to \infty$ with $\frac{M}{K} = \beta$, we get

$$\lim_{K\to\infty} \frac{1}{M}\eta_{\text{PMAS|TDMA}}(\boldsymbol{\theta}) = \frac{1}{2 + \beta\int_0^1 f(x)^2 dx} \tag{21}$$

Hence $\eta_{\text{PMAS|TDMA}}(\boldsymbol{\theta})$ scales linearly in $M$. This indicates that PMAS achieves a system slope significantly larger than TDMA if the system has large number of receive antennas.

Interestingly, for Rayleigh fading channels with only CDI at the transmitters, the superiority of PMAS over TDMA holds even for single antenna systems, since

$$\eta_{\text{PMAS|TDMA}}(\boldsymbol{\theta}) = \frac{M+1}{2 + (M-1)\sum_i \theta_i^2} \geq 1, \quad \forall M \tag{22}$$

In such a case, sharing the communication channel via TDMA is even worse than simply letting all the users use the channel simultaneously with no additional control on the interuser interference.

If the traffic of the users are extremely unbalanced, in the sense that $\sum_i \theta_i^2$ converges to a constant rather than decreases in $\frac{1}{K}\int_0^1 f(x)^2 dx$, then, for large $M$, $\eta_{\text{TDMA|OPT}}(\boldsymbol{\theta})$ converges to $\sum_i \theta_i^2$ while $\eta_{\text{PMAS|OPT}}(\boldsymbol{\theta})$ converges to 1. Such an asymptotic behavior can be explained as follows: the fact that $K$ goes to infinity with $\sum_i \theta_i^2$ converging to a constant can be interpreted as the effective number of active users is fixed. Consequently, with a large number of receive antennas and a rich scattering environments, the signals from different terminals can be completely separated via receiver beamforming. Since each transmitter sees a single user channel asymptotically, it is waste of resources to limit the number of active transmitters at any moment.

## V. Static Channel with CSI at the Transmitters

Suppose the channel parameters are randomly generated once and are fixed thereafter. Assume the CSI is available both at the transmitters and the receiver. The close-form expressions on the minimum energy per information nat and the wideband system slopes can be obtained directly from Section IV.

If the ratios among the information rates are fixed as in (1), for any terminal $i$, the three channel sharing schemes achieve the same minimum transmitted energy per information nat, which equals $\frac{E_i}{N_0}_{\min} = \frac{1}{\|\boldsymbol{h}_i\|^2}$.

Given $\boldsymbol{\theta}$, the system slopes of the three channel sharing schemes are given respectively by,

$$\mathcal{S}_{\max}^{\text{OPT}}(\boldsymbol{\theta}) = \frac{2}{\left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2}$$

$$\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta}) = \frac{2}{2\left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2 - \sum_i \theta_i^2}$$

$$\mathcal{S}_{\max}^{\text{TDMA}}(\boldsymbol{\theta}) = 2 \quad (23)$$

As in Section IV, we have $\eta_{\text{PMAS|OPT}}(\boldsymbol{\theta}) = \frac{\mathcal{S}_{\max}^{\text{PMAS}}(\boldsymbol{\theta})}{\mathcal{S}_{\max}^{\text{OPT}}(\boldsymbol{\theta})} \geq \frac{1}{2}$.

The normalized spectral efficiencies of PMAS and TDMA are, respectively,

$$\eta_{\text{PMAS|OPT}}(\boldsymbol{\theta}) = \frac{\left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2}{2\left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2 - \sum_i \theta_i^2}$$

$$\eta_{\text{TDMA|OPT}}(\boldsymbol{\theta}) = \left\|\sum_i \frac{\theta_i}{\|\boldsymbol{h}_i\|^2}\boldsymbol{h}_i\boldsymbol{h}_i^H\right\|_F^2 \quad (24)$$

We now compare the performance of PMAS and TDMA in the low SNR regime in terms of their relative system spectral efficiency, as a function of $\boldsymbol{\theta}$ and as a function of the number of receive antennas, respectively. In the first example, the number of terminals is fixed at 2. The channel gains are independently generated according to the complex normal distribution with zero mean and unit variance. This corresponds to the flat Rayleigh fading case. Note that the unit variance assumption does not cause any loss of generality since the slopes are determined only by the normalized channel vectors. Figure 1 plots the median of the relative spectral efficiency of PMAS over TDMA as a function of $\boldsymbol{\theta}$ for 1, 2 and 3 receive antennas cases, computed through 20000 Monte-Carlo runs. For the single receive antenna case, TDMA achieves the optimal normalized spectral efficiency of 1, and the relative spectral efficiency of PMAS over TDMA is strictly less than 1 except when $\theta_1 = 0$ or $\theta_2 = 0$. However, as wee can see from Figure 1 that, with 3 receive antennas at the base, the probability of PMAS outperforming TDMA is above $50\%$ over all $\theta$ values.

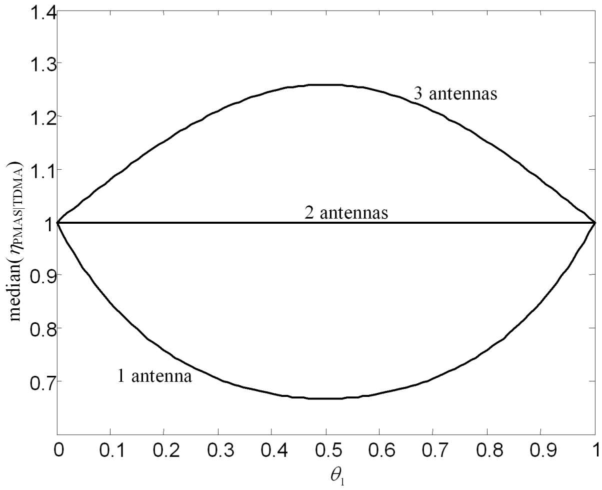


Fig. 1. Average relative spectral efficiency of PMAS over TDMA as a function of $\theta_1$. 2 terminals, static multiple access channel, 20000 Monte-Carlo runs.

In the second example, we set the information rates among the terminals to be equal, i.e., $\theta_i = \frac{1}{K}$, $\forall i$. While letting the number of users grow, we fix the ratio between the $K$ and $M$ at $\beta = \frac{M}{K} = \frac{1}{3}$. The median of the relative efficiency of PMAS over TDMA is illustrated in Figure 2. The $99.5\%$ quantile, $\overline{\eta_{\text{PMAS|OPT}}}$ ($P(\eta_{\text{PMAS|OPT}} < \overline{\eta_{\text{PMAS|OPT}}}) = 99.5\%$), and the $0.5\%$ quantile, $\underline{\eta_{\text{PMAS|OPT}}}$ ($P(\eta_{\text{PMAS|OPT}} < \underline{\eta_{\text{PMAS|OPT}}}) = 0.5\%$), are also shown. It is clearly seen that the relative spectral efficiency scales linearly in the number of receive antennas. For the system with 3 receive antennas, PMAS achieves a system slope larger than TDMA in over $99.5\%$ of the channel realizations.

Indeed, if the channel parameters are generated according to flat Rayleigh fading, the asymptotic behavior of the relative spectral efficiency can be shown theoretically. We have the following theorem.

**Theorem 3:** Assume the channel gains are independently generated. Also assume the channel gains of each terminal are identically distributed with the density function being symmetric around the origin. Assume there exists a non-negative function $f(x)$ defined on $x \in [0, 1]$, with $\int_0^1 f(x)dx = 1$ and $\int_0^1 f(x)^2 dx < \infty$. Let the number of users go to infinity, and let $\theta_i$ converge to $\frac{1}{K} f\left(\frac{i}{K}\right)$ in the sense of $\lim_{K\to\infty} \frac{\theta_i}{K} f\left(\frac{i}{K}\right) = 1$. If we fix $M$ and let $K$ go to infinity, then almost surely,

$$\eta_{\text{TDMA|OPT}}(\boldsymbol{\theta}) \to \frac{1}{M}, \qquad \eta_{\text{PMAS|OPT}}(\boldsymbol{\theta}) \to \frac{1}{2} \quad (25)$$

■

## VI. General Discussions

For multiple access systems working in the low SNR regime, not exploiting successive cancellation in PMAS results

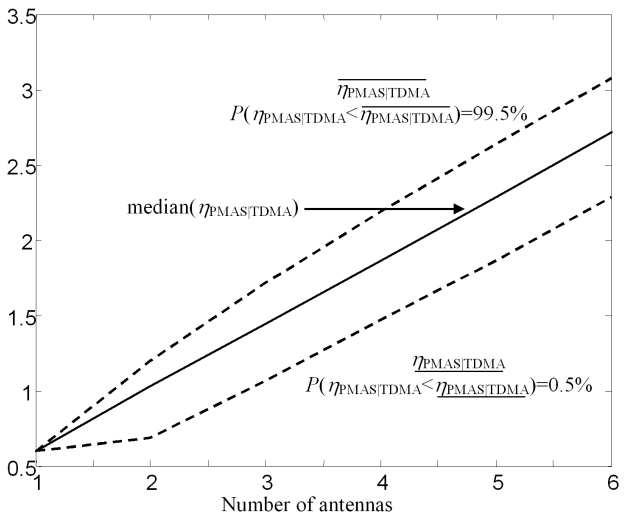


Fig. 2. Illustration on the relative spectral efficiency of PMAS over TDMA as a function of number of antennas, $M$. $\frac{K}{M} = 3$, static multiple access channel, 20000 Monte-Carlo runs.

in a penalty no worse than doubling the bandwidth requirement. Such cost on the spectral efficiency or energy efficiency are not easily avoidable if we insist on our strict requirement of simple transceiver design. However, it is important to note that with a simple system design, the bandwidth efficiency of PMAS can be improved by simply adding receive antennas, which is often more cost effective than requiring higher frequency bandwidth.

Having multiple receive antennas allows the receiver to distinguish signals spatially via beamforming, and consequently increases the multiplex gain. If the transmitters know the CSI perfectly, since beamforming is optimal in the low SNR regime, the signal of any user only occupies one spatial dimension at the base. Therefore, the difference between the spectral efficiencies of PMAS and TDMA can be significant due to the fact that TDMA exploits only one spatial dimension at the receiver at any moment while PMAS exploits the full spatial dimension by transmitting the signals in parallel.

When the transmitters only know the CDI, since the low SNR slope of the single user channel takes the form of $\frac{2M}{1+M}$, the slope of the multiuser system achieved by TDMA is only an average over the single user slopes. The optimal slope of the multiuser system, however, is in the form of $\frac{2KM}{K+M}$. The inefficiency of TDMA comes from the fact that it does not fully exploit the multiplex gain of the multiuser system at any moment. Although the slopes of both the single user and the multiuser channels in the low SNR regime take different forms from their high SNR regime correspondences (the high SNR slope of the single user channel is $\min(1, M)$ and the high SNR slope of the multiuser channel is $\min(K, M)$ [9]), the inefficiency of TDMA on not fully exploiting the multiplex gain at any moment remains essentially the same.

The understanding on the inefficiency of TDMA channel sharing can also be extended to orthogonal channel sharing schemes other than TDMA.

## VII. Conclusion

The dominance of TDMA channel sharing is due to its simplicity. However, letting one transmitter access the channel at a time is inefficient in exploiting the multiplex gain. In this paper, we show that, for multiple access channels with multiple receive antennas, the parallel multiple access channel sharing scheme with single user decoding (PMAS) is a more attractive alternative to TDMA in the low SNR regime, in the sense that PMAS achieves a significantly higher spectral efficiency than TDMA in practical scenarios while the simplicities of the two schemes are similar.

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