On Rate Control of Packet Transmission over Fading Channels

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***Abstract*— In error controlled packet reception, a packet is received only if its error probability can be kept below a pre-determined level. Error probability control is achieved by posing a minimum signal to noise ratio (SNR) threshold with corresponding packet internal coding scheme, which upper-bounds the packet data rate. We first consider packet transmission over a single-user wireless fading channel with additive Gaussian noise. We derive the optimal SNR threshold that maximizes the communication throughput. We show under a set of generous conditions that the optimal SNR threshold in the low-SNR regime is proportional to the transmit power; the ratio depends neither on the packet internal coding scheme nor on the pre-determined error probability level. The result is then extended to packet multicasting where common information is transmitted to a group of receivers over fading channels.**

***Index Terms*— Error controlled reception, multicast, rate control, SNR threshold.**

I. INTRODUCTION

IN the layered network architecture, the data link layer is responsible for transforming the raw transmission facility into virtual error free logical links to the upper layers [1]. A key component of such transformation is the error controlled packet reception widely implemented at the medium access control (MAC) sublayer. In error controlled reception, a packet is accepted by the receiver only if its coding error probability can be maintained below a predetermined level [2]. A packet is erased by the receiver if the error probability requirement cannot be met.

Given a time-invariant channel and a finite packet length, the tradeoff between communication rate and decoding error probability is characterized by the error exponent of the packet internal coding scheme (as opposed to the upper layer processing that treats packets as fundamental units) [3][4]. The tradeoff determines the maximum number of data bits that can be packed into one packet if the error probability must be kept below the pre-determined level.

When packets are transmitted over a wireless fading channel, and the channel is time-invariant within a packet duration (or a time slot), the actual channel experienced by each packet transmission is a random realization from an ensemble of channels. Assume the transmitter only knows the channel distribution information (CDI) while the receiver knows the channel state information (CSI). Error controlled reception can be achieved by choosing packet internal coding to ensure low error probability for a subset of channels (in the ensemble) and requiring the receiver to erase the packet if the channel realization is outside the subset. The "rate control" problem is defined as finding the optimal channel subset to maximize the overall communication throughput, in bit per second.

In this letter, we first consider packet transmission over a single-user wireless fading channel with additive Gaussian noise. The transmitter chooses a channel subset by posing a minimum received signal to noise ratio (SNR) threshold. A packet is accepted (as opposed to being erased) by the receiver only if its received SNR is above the threshold. We define packet throughput, in number of packets per slot, as a function of the packet erasure probability. The communication throughput is defined as the packet throughput multiplies the packet data rate, in bit per second. Under a set of generous assumptions, we derive the optimal SNR threshold that maximizes the communication throughput. We show that the optimal SNR threshold is proportional to the transmit power in the low-SNR regime; the ratio is neither a function of the packet internal coding scheme nor a function of the pre-determined error probability level. We then show similar result also holds for packet multicasting systems where common information is transmitted from a transmitter to a group of receivers over fading channels.

II. SYSTEM MODEL

Consider packet transmission over a block fading channel with additive Gaussian noise. We assume the size of each packet is fixed at N symbols. We call a packet transmission duration a block or a time slot. The channel output symbols of a packet can be represented by an N-dimensional column vector $\boldsymbol{y}$,

$$\boldsymbol{y} = h\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{x}$ is an N-dimensional column vector whose elements are the transmitted symbols; $\boldsymbol{n}$ is the additive Gaussian noise with $E[\boldsymbol{n}] = \boldsymbol{0}$ and $E[\boldsymbol{n}\boldsymbol{n}^T] = \boldsymbol{I}$; h is the normalized channel gain[1]. We assume the channel gain is time-invariant within a packet duration, and may vary over different packet transmissions. Assume only CDI at the transmitter while the receiver knows the CSI. The transmitted symbols have an average power of $E[\|\boldsymbol{x}\|^2/N] = P$. Consequently, the received SNR of a packet is given by

$$\text{SNR} = |h|^2 P. \tag{2}$$

[1]Note that we assumed unit noise variance in the channel model. Therefore, h equals the channel gain normalized by the actual noise variance (or the actual interference plus noise variance in a multiuser environment).

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To control error probability in packet reception, the transmitter poses a minimum received SNR threshold T. A packet is accepted by the receiver if and only if its received SNR satisfies SNR $\geq T$. Since the coding error probability must be maintained below a pre-determined level, the error exponent of the packet internal coding must be maintained above a corresponding threshold. Consequently, the relation between the maximum packet data rate (in bit per second) R, and the SNR threshold T, is characterized by the error exponent curve of the packet internal coding scheme. This enables us to write the packet data rate $R(T)$ as a function of the SNR threshold. Note that $R(T)$ is nondecreasing in T. The actual expression of $R(T)$ depends on the coding scheme and the pre-determined error probability requirement.

Assumption 1: We assume $R(T)$ is continuously differentiable[2] with respect to T. We also assume $R(T)$ possess the follow three properties: $\lim_{T\to\infty}\frac{R(T)}{T}=0$, $R(0)=0$, and $d\frac{R(T)}{dT}\Big|_{T=0}>0$. ■

The three properties posed in Assumption 1 are generous in the sense that violation of these properties often suggests an ill system design. For example, $d\frac{R(T)}{dT}\Big|_{T=0}>0$ is satisfied by most of the popular coding schemes; $R(0)=0$ can be violated only if the error probability requirement can be satisfied without transmitting any data symbol to the receiver; $\lim_{T\to\infty}\frac{R(T)}{T}=0$ can be violated if the channel has infinite bandwidth.

Let the density function of $|h|^2$ be $f(|h|^2)$. Given the SNR threshold T, packet erasure probability ρ can be obtained by

$$\rho\left(\frac{T}{P}\right)=Pr\left\{|h|^2P<T\right\}=\int_0^{\frac{T}{P}}f(|h|^2)d|h|^2. \quad (3)$$

Note that $(1-\rho)$ is the average number of packets received by the receiver per slot, i.e., the packet throughput. Usually, due to upper layer data processing, different packets may not always carry independent information. We assume upper layer protocols treat packets as fundamental units in the sense that we can write the "effective" throughput $G=G_\rho(\rho)$ in packet per slot as a function of the packet erasure probability ρ. Note that $G_\rho(\rho)\leq 1-\rho$ since the effective packet throughput can never be larger than the packet throughput. Due to (3), effective packet throughput can be written as a function of $\frac{T}{P}$, denoted by $G\left(\frac{T}{P}\right)=G_\rho\left(\rho\left(\frac{T}{P}\right)\right)$.

Assumption 2: We assume the packet throughput function $G(\alpha)$ is continuously differentiable with respect to α. Assume there exists at least one $\alpha_0>0$, such that $G(\alpha_0)>0$. We also assume $\lim_{\alpha\to\infty}\alpha G(\alpha)=0$. ■

It is easily verified that Assumption 2 is satisfied if $|h|$ is Rayleigh distributed and $G_\rho(\rho)=1-\rho$.

We define the communication throughput r, in bit per second, as the effective packet throughput multiplies the packet data rate.

$$r(T)=G_\rho(\rho)R(T)=G\left(\frac{T}{P}\right)R(T). \quad (4)$$

[2]Packet data rate of a practical system often takes a finite number of possible values. In this case $R(T)$ is not continuous and therefore is not continuously differentiable with respect to T. Nevertheless, we can approximate a practical $R(T)$ using a continuous differentiable function. Such approximation enables us to obtain useful insight about practical system design.

The rate control problem is defined as finding the optimal SNR threshold T^* to maximize the communication throughput r.

III. The Unicasting Case

In the following theorem, we show that, if the system operates in the low-SNR regime, then T^* is proportional to P.

Theorem 1: Suppose Assumptions 1 and 2 hold. The optimal SNR threshold T^* that maximizes the communication throughput satisfies

$$\lim_{P\to 0}\frac{T^*}{P}=\alpha^*, \quad (5)$$

where α^* is given by

$$\alpha^*=\arg\max_\alpha \alpha G(\alpha). \quad (6)$$

Proof: We first show for any $P>0$, we have $T^*\neq 0$ and $T^*\neq\infty$.

Since $G\left(\frac{T}{P}\right)\leq 1$, we have, for any $P>0$, $r(0)=G(0)R(0)=0$. Meanwhile, according Assumptions 1 and 2,

$$\lim_{T\to\infty}r(T)=P\lim_{T\to\infty}\left[\frac{T}{P}G\left(\frac{T}{P}\right)\right]\frac{R(T)}{T}=0. \quad (7)$$

It is easy to show $r(T)$ does not stay 0 for all T. Therefore, we have $0<T^*<\infty$.

Next, we derive T^* under an auxiliary assumption that $P\to 0$ should imply $T^*\to 0$. We will later show that this auxiliary assumption *must* hold.

Since $r(T)$ is continuously differentiable with respect to T, and $0<T^*<\infty$, T^* must satisfy $d\frac{r(T^*)}{dT^*}=0$, i.e.,

$$\frac{1}{P}d\frac{G\left(\frac{T^*}{P}\right)}{d\frac{T^*}{P}}R(T^*)+G\left(\frac{T^*}{P}\right)d\frac{R(T^*)}{dT^*}=0. \quad (8)$$

Define

$$\alpha^*=\frac{T^*}{P}. \quad (9)$$

Substitute (9) into (8), we obtain

$$\alpha^*d\frac{G(\alpha^*)}{d\alpha^*}\frac{R(T^*)}{T^*}+G(\alpha^*)\,d\frac{R(T^*)}{dT^*}=0. \quad (10)$$

Define $\dot{R}(T)=d\frac{R(T)}{dT}$. According to Assumption 1,

$$\lim_{T\to 0}\frac{R(T)}{T}=\dot{R}(0)>0. \quad (11)$$

As $P\to 0$, which implies $T^*\to 0$ by the auxiliary assumption, α^* converges to a solution of the following equation, which is obtained by combining (10) and (11),

$$\alpha^*d\frac{G(\alpha^*)}{d\alpha^*}+G(\alpha^*)=0. \quad (12)$$

According to (9) and (12), the system throughput $r(T^*)$ satisfies

$$\lim_{P\to 0}\frac{1}{P}r(T^*)=\lim_{P\to 0}\frac{1}{P}G(\alpha^*)R(\alpha^*P)=\alpha^*G(\alpha^*)\dot{R}(0). \quad (13)$$

Clearly, if (12) has multiple solutions, the optimal α^* must be given by (6). Consequently,

$$\lim_{P\to 0}\frac{1}{P}r(T^*)=\alpha^*G(\alpha^*)\dot{R}(0)>0. \quad (14)$$

Finally, we show the auxiliary assumption must hold, i.e., $P \to 0$ does imply $T^* \to 0$.

Suppose there exists a constant $T_0 > 0$ such that as $P \to 0$, we have $T^* \geq T_0$. Since $R(T)$ is continuously differentiable and nondecreasing in T, $\lim_{T\to\infty} \frac{R(T)}{T} = 0$ implies that we can find a constant $U_0 < \infty$ to satisfy

$$\frac{R(T)}{T} \leq U_0, \qquad \forall T \geq T_0. \tag{15}$$

Consequently, assume $T^* \geq T_0$, we have according to Assumption 2,

$$\lim_{P\to 0} \frac{1}{P} G\left(\frac{T^*}{P}\right) R(T^*) \leq \lim_{P\to 0} \frac{T^*}{P} G\left(\frac{T^*}{P}\right) U_0 = 0. \tag{16}$$

Comparing (16) to (14), as $P \to 0$, the throughput achieved by $T = \alpha^* P$ with α^* being determined by (6) is larger than the one that keeps $T \geq T_0$. Therefore, $T^* \to 0$ as $P \to 0$ must be true. ■

The result of Theorem 1 is surprising in the following sense. Under the condition that Assumptions 1 and 2 hold, according to (6), the value of α^* only depends on the upper layer protocols that determine $G_\rho(\rho)$, and the channel statistics that determines $\rho\left(\frac{T}{P}\right)$; α^* is not a function of $R(T)$, and hence its value depends neither on the packet internal coding scheme nor on the error probability requirement.

IV. The Multicasting Case

Consider a packet multicasting system where a common set of packets are transmitted from the transmitter to a group of K receivers over block fading channels with additive Gaussian noise. Let the channel output symbols of a packet received by the kth receiver be

$$\boldsymbol{y}_k = h_k \boldsymbol{x} + \boldsymbol{n}_k \tag{17}$$

where h_k is the normalized channel gain from the transmitter to the kth receiver; $\boldsymbol{n}_k$ is the additive Gaussian noise with $E[\boldsymbol{n}_k] = \boldsymbol{0}$ and $E[\boldsymbol{n}_k \boldsymbol{n}_k^T] = \boldsymbol{I}$. We assume the transmitter only knows the CDI while each receiver knows the CSI of the corresponding channel.

Let $\boldsymbol{e}$ be a K-dimensional binary-valued "erasure vector" whose kth element e_k takes value 1 if the packet is erased by receiver k, i.e.,

$$e_k = \begin{cases} 1 & \text{if } |h_k|^2 P < T \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Note that since the noise distributions of all channels are identical and the transmitter only knows the CDI, the SNR thresholds T of all the receivers must be identical. Consequently, given the joint distribution of the channel gains, the distribution of $\boldsymbol{e}$, denoted by $\boldsymbol{\rho}(\boldsymbol{e}) = \boldsymbol{\rho}\left(\frac{T}{P}\right)$ can be written as a function of $\frac{T}{P}$.

Define the effective multicast throughput G as the rate in which the transmitter delivers common information to *all* the receivers. We assume upper layer protocols treat packets as fundamental units in the sense that we can write the effective multicast throughput in number of packets per slot as a function of the packet erasure distribution $\boldsymbol{\rho}(\boldsymbol{e})$, i.e., $G = G_{\boldsymbol{\rho}}(\boldsymbol{\rho})$. Discussions on example upper layer protocols can be found in [5]. Given the channel statistics, $G = G_{\boldsymbol{\rho}}(\boldsymbol{\rho}) = G\left(\frac{T}{P}\right)$ can be further written as a function of $\frac{T}{P}$. The effective multicast communication throughput, in bit per second, is defined as the rate in which the transmitter delivers common information to all the receivers,

$$r(T) = G\left(\frac{T}{P}\right) R(T). \tag{19}$$

Define the rate control problem as finding the optimal T^* that maximizes the effective multicast communication throughput r. It is easily seen that the optimal SNR threshold T^* is given by (8); and Theorem 1 holds with the same proof.

Note that comparing to the ergodic channel capacity, error controlled reception may cause significant rate loss in the low-SNR regime [5]. Even though the optimal SNR threshold T^* and hence the optimal packet throughput $G\left(\frac{T^*}{P}\right) = G(\alpha^*)$ does not depend on the packet internal coding scheme, the optimal communication throughput $r = G(\alpha^*)R(T^*)$ is affected by $R(T^*)$. Further discussions on moderate and high SNR cases can be found in [5].

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