On the Throughput, Capacity and Stability Regions of Random Multiple Access over Standard Multi-Packet Reception Channels

Jie Luo, Anthony Ephremides
ECE Dept. Univ. of Maryland
College Park, MD 20742
Email: {rockey, tony}@eng.umd.edu

Abstract— **This paper studies finite-terminal random multiple access over a standard multipacket reception (MPR) channel. In the first part of the paper, we show that if the MPR channel is standard, the throughput region of random multiple access is co-ordinate convex. We then study the information capacity region of multiple access without code synchronization and feedback. We show that the asymptotic capacity region is identical to the throughput region. In the second part of the paper, we study the stability region of ALOHA multiple access. For a class of packet arrival distributions, we show that the stationary distribution of the queues possesses the positive and strong positive correlation properties; and this consequently gives an outer bound to the stability region. We also show that if a conjectured "sensitivity monotonicity" property can be shown for the stationary distribution of the queues, then the equivalence between the closure of the stability region and the throughput region follows as a direct consequence, irrespective of the packet arrival distributions.**[1]

I. Introduction

In this paper, we address the relationship among stability, throughput and capacity regions in radom multiple access, all over a "standard" MPR channel (see detailed definition in section II). First, we derive the throughput region of random multiple access and show that if the MPR channel is standard, the throughput region is co-ordinate convex. Second, we derive both inner and outer bounds to the information capacity region of random multiple access without code synchronization and feedback. We show that both bounds approach the throughput region asymptotically as the packet size approaches infinity. Consequently, we define the asymptotic capacity region, and show that the asymptotic capacity region equals to the throughput region, in number of packets per slot. Such a result explains, in a general channel model, the relationship between the results of [1][2] and the throughput region (see definition in section III).

Next, we study the stability region of finite-terminal slotted ALOHA multiple access over a standard MPR channel. We follow the framework presented by Anantharam in [3] and show that, if the ALOHA system is stable, for a class of packet arrival distributions, the stationary distribution of the queue status satisfies the positive and strong positive correlation properties. Our results extend the ones obtained in [3], not only in the sense that we consider the general standard MPR channel, but also in the sense that our results no longer depend on *correlated* packet arrivals, which has been considered in [4] as the "unrealistic" part of the results obtained in [3]. Nevertheless, unlike the collision channel case, for slotted ALOHA system over a standard MPR channel, the strong positive correlation property only gives an outer bound to the stability region. The major challenge in obtaining the closure of the stability region is due to the lack of sensitivity analysis results with respect to the transmission probabilities. To justify our comment, we give an additional property of the stationary distribution of the queues, together with a conjectured "sensitivity monotonicity" property. Under the validity of the conjectured sensitivity monotonicity, we show that the closure of the stability region equals the throughput region irrespective of the packet arrival distributions. Further discussions on the stability issues are also presented. Although theoretical support to the conjectured property is not available, we hope the discussion can serve as a brief survey that shows both the interesting and the challenging parts of the open stability problem, and can also serve as a guidance to future research efforts.

II. System Model

The schematic system model we study in this paper is illustrated in Figure 1.

We assume there are N terminals. Each terminal occasionally transmits packet to its assigned receiver[2]. Assume that all packets are of the same length. Time is slotted with

[1]This work was supported by National Aeronautics and Space Administration award No. NCC8-235 and Collaborative Technology Alliance for Communication & Networks sponsored by the U.S. Army Laboratory under Cooperative Agreement DAAD19-01-2-0011. Any opinions findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Aeronautics and Space Administration or the Army Research Laboratory of the U.S. Government.

[2]Note that multiple terminals can be assigned to the same receiver.

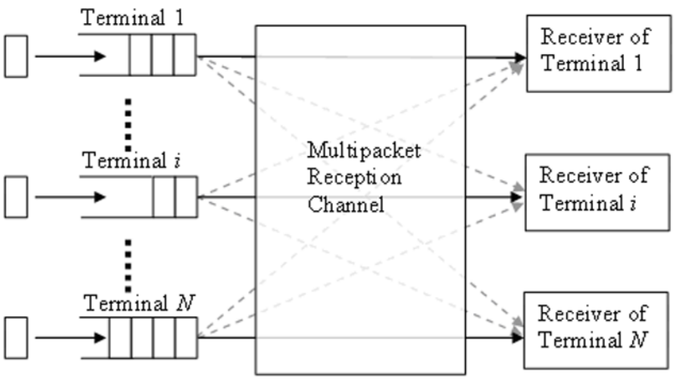

Fig. 1. Finite-terminal random multiple access over a MPR channel.

each slot equals one packet duration, and packet transmissions start only at the slot edges. In each slot, when one or more packets are transmitted, each of them has certain probability of being received successfully, or not being received by its receiver, depending on the channel model.

For a particular slot, let $t_i \in \{0, 1\}$ be the transmission indicator of terminal i; $t_i = 1$ indicates that terminal i transmits a packet in the slot. Let $r_i \in \{0, 1\}$ be the reception indicator at the receiver of terminal i; $r_i = 1$ indicates that a packet from terminal i is received successfully. Let $\mathcal{S}$ and $\mathcal{R}$ be two groups of terminals such that $\mathcal{R} \subseteq \mathcal{S}$. A MPR channel is specified by the complete set of parameters $\mathsf{q}_{\mathcal{R},\mathcal{S}}$, for all $\mathcal{S}$ and $\mathcal{R}$; and $\mathsf{q}_{\mathcal{R},\mathcal{S}}$ is defined as in [4] by

$$\mathsf{q}_{\mathcal{R},\mathcal{S}} = P\left(r_{i\in\mathcal{R}} = 1, r_{i\notin\mathcal{R}} = 0 | t_{i\in\mathcal{S}} = 1, t_{i\notin\mathcal{S}} = 0\right) \quad (1)$$

Suppose $\mathcal{U}$, $\mathcal{S}$, $\hat{\mathcal{S}}$ are three groups of terminals. We say that the MPR channel is "standard", if for all $\mathcal{U} \subseteq \mathcal{S} \subseteq \hat{\mathcal{S}}$, we have

$$\sum_{\mathcal{R},\ \mathcal{U}\subseteq\mathcal{R}\subseteq\mathcal{S}} \mathsf{q}_{\mathcal{R},\mathcal{S}} \geq \sum_{\mathcal{R},\ \mathcal{U}\subseteq\mathcal{R}\subseteq\hat{\mathcal{S}}} \mathsf{q}_{\mathcal{R},\hat{\mathcal{S}}} \quad (2)$$

In other words, if the MPR channel is standard, for the reception of any particular groups of packets, simultaneous packet transmissions are unhelpful.

III. Throughput Region of Random Multiple Access

Suppose each terminal always has packets to transmit to its receiver. In each time slot, terminal i transmits a packet with probability p_i; and with probability $1 - p_i$, terminal i keeps silent. Let $r_i \in \{0, 1\}$ be the reception indicator at the receiver of terminal i. Suppose packet transmissions are independent both among different terminals and among different time slots. Define $\boldsymbol{p}$ as the transmission probability vector, whose ith component is p_i. Given the channel parameters, the throughput of terminal i, in number of packets per slot, is defined as $T_i(\boldsymbol{p}) = P(r_i = 1)$, which is given by,

$$T_i(\boldsymbol{p}) = \sum_{\substack{\mathcal{S},\,\mathcal{R},\\ i\,\in\,\mathcal{R}\,\subseteq\,\mathcal{S}}} \mathsf{q}_{\mathcal{R},\mathcal{S}} \prod_{j\in\mathcal{S}} p_j \prod_{k\in\bar{\mathcal{S}}} (1 - p_k) \quad (3)$$

Let $\boldsymbol{T}$ be the throughput vector, whose ith component is T_i. The "throughput region", $\mathcal{C}_T$, is defined by,

$$\mathcal{C}_T = \left\{\tilde{\boldsymbol{T}} | \tilde{T}_i = T_i(\boldsymbol{p}), 0 \leq p_i \leq 1, \forall i\right\} \quad (4)$$

Theorem 1: If the MPR channel is standard, the throughput region $\mathcal{C}_T$ is co-ordinate convex, i.e., for any vector $\tilde{\boldsymbol{T}}$, if there exists a transmission vector $\boldsymbol{p}$ such that $0 \leq \tilde{T}_i \leq T_i(\boldsymbol{p})$, $\forall i$, then $\tilde{\boldsymbol{T}} \in \mathcal{C}_T$.

The proof of Theorem 1 is given in [5].

IV. Capacity Region of Random Multiple Access

In this section, we study the information capacity region of a standard MPR channel in random multiple access. In this context, we use the term "random multiple access" to refer to the assumption of no code synchronization between the terminals and no feedback from the receivers, as assumed in [2]. In such a scenario, we say a rate vector $\boldsymbol{R}$ is inside the capacity region, if the information rate R_i from terminal i to its receiver can be achieved simultaneously for all i. Unfortunately, to obtain the exact expression of the information capacity is not a trivial task. Alternatively, we first derive an outer bound and an inner bound to the capacity region in section IV-A and IV-B, respectively. We later show in section IV-C that these bounds are asymptotically equal.

A. An Outer Bound to The Capacity Region

We first construct an "enhanced" system whose capacity region contains the capacity region of the original system. We assume that all the packets are transmitted to a single receiver. The code symbol alphabet for transmitter (or terminal) i is $X_i \in \{0, 1, \ldots, 2^m\}$, where the symbol 0 represents an idle and symbols $1 \sim 2^m$ each represent a packet of m-bit length. We further assume that when a source symbol is generated by a terminal, an "enhanced" symbol is formed automatically by attaching the transmitter ID to the data symbol. If the source symbol is $X_i = 0$, terminal i idles in the slot, while if $X_i \neq 0$, the enhanced symbol that contains both the data symbol X_i and the transmitter ID is transmitted to the receiver through the MPR channel. The output of the receiver is represented by a N-length column vector $\boldsymbol{Y}$, whose ith component Y_i takes value in $\{0, 1, \ldots, 2^m\}$. If a non-zero symbol X_i from terminal i is received, we have $Y_i = X_i$, otherwise, $Y_i = 0$.

Given the input symbols X_i, $1 \leq i \leq N$, define the set of transmitters $\mathcal{S}$ such that $X_{i\in\mathcal{S}} \neq 0$ and $X_{i\notin\mathcal{S}} = 0$. Suppose the output symbol is $\boldsymbol{Y}$. Define the set $\mathcal{R}$ such that $Y_{i\in\mathcal{R}} \neq 0$ and $Y_{i\notin\mathcal{R}} = 0$. The conditional probability of the output symbol $\boldsymbol{Y}$ is given by

$$P(\boldsymbol{Y}|X_{1\leq i\leq N}) = \begin{cases} \mathsf{q}_{\mathcal{R},\mathcal{S}} & \mathcal{R} \subseteq \mathcal{S}, Y_{i\in\mathcal{R}} = X_i, Y_{i\notin\mathcal{R}} = 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Suppose the MPR channel, defined by the parameter set $\mathsf{q}_{\mathcal{R},\mathcal{S}}$, is standard. In the rest of this section, we analyze the information capacity region of the enhanced system.

As shown in [1], due to the lack of code synchronization and treating signals from other terminals as memoryless noise, reliable communication for each terminal i at rate $\mathcal{R}_i$, in m bits per slot, is achievable if and only if for some input distribution $P_{X_j}(X_j)$, $\forall j$, and

$$P(\boldsymbol{Y}, X_{1\leq i\leq N}) = P(\boldsymbol{Y}|X_{1\leq i\leq N})\prod_{j=1}^{N} P_{X_j}(X_j) \qquad (6)$$

we have, for all i, $R_i \leq \frac{1}{m}I(X_i;\boldsymbol{Y})$.

Let $\tilde{X}_i \in \{1,\ldots,2^m\}$ be the non-zero symbols of terminal i. Given the source distribution $P(X_i)$, let the distribution of $\tilde{X}_i$ be $P(\tilde{X}_i) = P(X_i|X_i \neq 0)$. Let $t_i \in \{0,1\}$ be the indicator of $X_i \neq 0$, the source symbol X_i can be written as the product of two independent random variables, $X_i = t_i\tilde{X}_i$. Let $r_i \in \{0,1\}$ be the indicator that $Y_i \neq 0$. The following theorem gives the information capacity region of the enhanced system.

Theorem 2: For the enhanced random multiple access system, given the source distributions, $P(X_i)$, let $\boldsymbol{p}$ be the probability vector whose ith component, p_i, is the probability that $P(X_i \neq 0)$. The mutual information between X_i and $\boldsymbol{Y}$ is given by

$$I(X_i;\boldsymbol{Y}) = I(t_i;\boldsymbol{r}) + H(\tilde{X}_i)T_i(\boldsymbol{p}) \qquad (7)$$

where $I(t_i;\boldsymbol{r})$ is the mutual information between t_i and $\boldsymbol{r}$; $H(\tilde{X}_i)$ is the entropy of $\tilde{X}_i$; and $T_i(\boldsymbol{p})$ is defined as in (3).

Given $\boldsymbol{p}$ and m, define $I_i^E(\boldsymbol{p},m) = I(t_i;\boldsymbol{r}) + mT_i(\boldsymbol{p})$. The information capacity region of the enhanced system is given by

$$\mathcal{C}_I^E(m) = \left\{\boldsymbol{R}\middle| R_i \leq \frac{1}{m}I_i^E(\boldsymbol{p},m), 0 \leq p_i \leq 1, 1 \leq i \leq N\right\} \qquad (8)$$

The proof of Theorem 2 is given in [5].

B. An Inner Bound to The Capacity Region

To obtain an inner bound to the capacity region, we construct a "constrained" system, whose capacity region is contained in the capacity region of the original system. We assume that no information exchange is allowed between the receivers corresponding to different transmitters. Let $\lceil\log N\rceil$ be the smallest integer that is greater than or equal to $\log N$, and assume that $\tilde{m} = m - \lceil\log N\rceil \geq 0$. Instead of considering $X_i \in \{0,1,\ldots,2^m\}$, we assume that the source symbol of terminal i, X_i, is constrained within the set $\{0,(i-1)2^{\tilde{m}}+1,\ldots,i2^{\tilde{m}}\}$. Let $\tilde{X}_i \in \{(i-1)2^{\tilde{m}}+1,\ldots,i2^{\tilde{m}}\}$ be the non-zero symbols of terminal i, and $P(\tilde{X}_i) = P(X_i|X_i \neq 0)$. We can see that the symbol sets of $\tilde{X}_i$ and $\tilde{X}_j$, for $i \neq j$, are mutually exclusive.

At the receiver side, if a non-zero symbol $\tilde{X}_i$ within the set $\{(i-1)2^{\tilde{m}}+1,\ldots,i2^{\tilde{m}}\}$ is received by the receiver of terminal i, then $Y_i = \tilde{X}_i = X_i$; otherwise $Y_i = 0$. Let $\boldsymbol{Y}$ be a column vector whose ith symbol is Y_i. The conditional distribution of $\boldsymbol{Y}$ given the input symbols is given by (5).

Let t_i, r_i and $\boldsymbol{p}$ be defined as in section IV-A, the following theorem gives the capacity region of the constrained random multiple access system.

Theorem 3: For the constrained random multiple access system, given $\boldsymbol{p}$, $\tilde{m}$ and the source distributions, $P(X_i)$, the mutual information between X_i and Y_i is given by

$$I(X_i;Y_i) = I(t_i;r_i) + H(\tilde{X}_i)T_i(\boldsymbol{p}) \qquad (9)$$

where $I(t_i;r_i)$ is the mutual information between t_i and r_i; $H(\tilde{X}_i)$ is the entropy of $\tilde{X}_i$; and $T_i(\boldsymbol{p})$ is defined as in (3).

Note that $m = \tilde{m}+\lceil\log N\rceil$, define $I_i^C(\boldsymbol{p},m) = I(t_i;\boldsymbol{r}) + (m-\lceil\log N\rceil)T_i(\boldsymbol{p})$. The information capacity region of the constrained system is given by

$$\mathcal{C}_I^C(m) = \left\{\boldsymbol{R}\middle| R_i \leq \frac{1}{m}I_i^C(\boldsymbol{p},m), 0 \leq p_i \leq 1, 1 \leq i \leq N\right\} \qquad (10)$$

The proof of Theorem 3 is presented in [5].

C. Asymptotic Capacity Region of Random Multiple Access

For random multiple access over a standard MPR channel, we say a set of "asymptotic rates" R_i^∞ from terminal i to its receiver, $\forall i$, is achievable if and only if we can find a sequence of rate sets $\{R_i^{(m)}\}$ such that, given the packet size m, the rate $R_i^{(m)}$ (in m bits per slot) is achievable from terminal i to its receiver, simultaneously for all i; and $\lim_{m\to\infty} R_i^{(m)} = R_i^\infty$, $\forall i$.[3] Define the "asymptotic capacity" region as the closure of the union of all achievable asymptotic rate vectors. We have

Theorem 4: For random multiple access over a standard MPR channel, the asymptotic capacity region is given by,

$$\mathcal{C}_C = \left\{\tilde{\boldsymbol{R}}|R_i \leq T_i(\boldsymbol{p}), 0 \leq p_i \leq 1, \forall i\right\} \qquad (11)$$

which equals the throughput region $\mathcal{C}_T$ according to Theorem 1.

The proof of Theorem 4 is presented in [5].

V. Stability Region of ALOHA Multiple Access

In this section, we study the stability region of slotted finite-terminal ALOHA system over a standard MPR channel.

[3] Note that in practical systems, enlarging the packet size may consequently result in a change in the MPR channel parameters. Therefore, the asymptotic capacity region obtained in Theorem 4 should be interpreted as an approximation to the actual capacity region in the situation of large m.

A. ALOHA Multiple Access

We assume packet arrivals at the N terminals are stationary, with the average packet arrival rate at terminal i being λ_i packets per slot. We assume that each terminal has a buffer of infinite capacity to store the incoming packets. The buffer of each terminal forms a queue of packets. At the beginning of each slot, if terminal i's buffer is not empty, with a probability of p_i, terminal i transmits the first packet in the buffer; and with probability $1-p_i$, terminal i keeps silent. The decision whether a terminal transmits its packet (given its queue being non-empty) is made independently from other terminals. When packets are transmitted, each of them can be either received successfully, or not received, with a probability depending on the MPR channel model, as described in section II. We assume that the information of a successful transmission is fed back to the source terminal instantly. If a transmission is successful, the corresponding packet is removed from the queue; otherwise, it stays in the queue. Define $q_i(n)$ as the number of packets in the queue of terminal i at the beginning of time slot n. Given fixed packet arrival rates and transmission probabilities, queue i of the system is *stable* if

$$\lim_{n\to\infty} P\{q_i(n) \leq x\} = F(x) \quad \text{and} \quad \lim_{x\to\infty} F(x) = 1 \tag{12}$$

Given a fixed packet arrival rate vector $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_N]^T$, we say $\boldsymbol{\lambda}$ is *stable* if one can find a transmission probability vector $\boldsymbol{p} = [p_1, \dots, p_N]^T$ such that all the queues in the corresponding system are stable. The union of all stable $\boldsymbol{\lambda}$ vectors is defined as the stability region of the ALOHA system.

B. Positive Correlation in Stationary Queue Status of ALOHA Systems

Let $\boldsymbol{Q}$ be an arbitrary set of vectors. $\boldsymbol{q}_i, \boldsymbol{q}_j \in \boldsymbol{Q}$. Denote the kth element of $\boldsymbol{q}_i$ by q_{ik}. We define a partial order, such that $\boldsymbol{q}_i \leq \boldsymbol{q}_j$ if and only if $q_{ik} \leq q_{jk}$, $\forall k$.

Let $\{\boldsymbol{q}(n)\}$ be a discrete time Markov chain, whose state space $\boldsymbol{Q}$ is a partially ordered set with finite or countably infinite number of components. Assume that the Markov chain is irreducible and stationary, with stationary distribution denoted by U. Let $P\left(\boldsymbol{q}_i|\boldsymbol{q}_j\right)$ be the probability that the state changes to $\boldsymbol{q}_i$ in one transition, given that the previous state is $\boldsymbol{q}_j$. We say the transitions of the Markov chain is up or down, if $P\left(\boldsymbol{q}_i|\boldsymbol{q}_j\right) = 0$ unless either $\boldsymbol{q}_i \leq \boldsymbol{q}_j$ or $\boldsymbol{q}_j \leq \boldsymbol{q}_i$ holds true [6].

Suppose f is a real-valued function defined on the state space $\boldsymbol{Q}$. We say that f is *increasing* if for all $\boldsymbol{q}_i, \boldsymbol{q}_j \in \boldsymbol{Q}$, $\boldsymbol{q}_i \leq \boldsymbol{q}_j$ implies $f(\boldsymbol{q}_i) \leq f(\boldsymbol{q}_j)$.

If U is a probability function defined on $\boldsymbol{Q}$, where $U(\boldsymbol{q})$ is the probability of $\boldsymbol{q}$, we say that U has "positive correlation" if for all bounded increasing functions f_1, f_2, the following inequality holds true [6].

$$E_U[f_1(\boldsymbol{q})f_2(\boldsymbol{q})] \geq E_U[f_1(\boldsymbol{q})]E_U[f_2(\boldsymbol{q})] \tag{13}$$

Theorem 5: Let $\{\boldsymbol{q}(n)\}$ be a monotonic discrete time Markov chain, whose state space $\boldsymbol{Q}$ is a countable partially ordered set. Assume that the Markov chain is stationary, with only up or down transitions. Then the stationary distribution U of the Markov chain has positive correlation.

The proof of Theorem 5 is presented in [5].

Define the packet arrivals as follows. Assume "virtual packets" arrive at the system in each slot according to a geometric distribution with parameter Λ. In other words, the probability of the number of virtual packets ν in each slot satisfies,

$$P(\nu = 0) = \frac{1}{1+\Lambda}, \quad \dots, \quad P(\nu = k) = \frac{\Lambda^k}{(1+\Lambda)^{k+1}} \tag{14}$$

Upon the arrival of each virtual packet, we generate a vector of real packets $\Delta\boldsymbol{q}$, ($\Delta q_i \geq 0$), according to a joint distribution of $P_a(\Delta\boldsymbol{q})$. We then append Δq_i packets to terminal i. If k virtual packets arrive in one slot, we generate k vectors $\Delta\boldsymbol{q}_1$, ..., $\Delta\boldsymbol{q}_k$ independently, each according to $P_a(\Delta\boldsymbol{q})$; and then append $\sum_{m=1}^{k} \Delta\boldsymbol{q}_{mi}$ packets to terminal i. Suppose the resulting average packet arrival rate at terminal i is λ_i; and assume that the MPR channel is standard and $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_N]^T$ is stable.

Theorem 6: The stationary distribution of queues on the system described above, denoted by $U(\boldsymbol{q})$, which is measured right before the transmission moment in each slot, satisfies the positive correlation property.

The proof of Theorem 6 is presented in [5].

It is easy to see that if in the joint distribution $P_a(\Delta\boldsymbol{q})$, $P_a(\Delta q_i)$ and $P_a(\Delta q_j)$ are independent, then the corresponding packet arrival distributions of terminal i and j are also independent.

C. Strong Positive Correlation Property and Outer Bound to the Stability Region

Theorem 7: Suppose the ALOHA system is stable, the MPR channel is standard and the packet arrivals can be modeled as in Theorem 6. In each slot, we associate a binary-valued flag to each of the terminals; let $b_i \in \{0, 1\}$ be the flag associated to terminal i. The value of the flag vector $\boldsymbol{b}$ is generated according to a joint distribution B and the flag generation in any particular slot is independent of all other events. Let Φ be the empty set. Let $\mathcal{S}$ be a set of terminals and $|\boldsymbol{b}_\mathcal{S}|$ be the number of 1's in vector $\boldsymbol{b}_\mathcal{S}$. Assume that for all i, $\mathcal{S}$, $i \in \mathcal{S}$, the joint distribution B satisfies the following properties

$$\begin{aligned} &P_B(|\boldsymbol{b}_\mathcal{S}| \geq 2) \leq \mathsf{q}_{\Phi,\mathcal{S}} \\ &P_B(b_i = 1 \text{ or } |\boldsymbol{b}_\mathcal{S}| \geq 2) \leq \mathsf{q}_{\Phi,\mathcal{S}} + \mathsf{q}_{\{i\},\mathcal{S}} \end{aligned} \tag{15}$$

Denote $\boldsymbol{v}$ as the vector whose ith component equals $b_i t_i$. Then, for all groups of terminals $\mathcal{S}$, the stationary distribution of the queues conditioned on $\boldsymbol{v}_\mathcal{S} = \boldsymbol{0}$, denoted by $U(\boldsymbol{q}|\boldsymbol{v}_\mathcal{S} = \boldsymbol{0})$, has positive correlation.

The proof of Theorem 7 is presented in [5].

Since setting $\boldsymbol{b} \equiv \boldsymbol{0}$ leads us back to Theorem 6, for a standard MPR channel, the set of joint distributions of

B satisfying (15) is non-empty. In the specific situation when we have a collision channel, we can define B such that $P_B(\boldsymbol{b} = \boldsymbol{1}) = 1$. Consequently, Theorem 7 gives the strong positive correlation property presented in [3].

Theorem 8: Suppose the ALOHA system is stable, the packet arrivals can be modeled as in Theorem 6 and the MPR channel is standard. Let $\mathsf{q}_0 = \min_{\mathcal{S}} \mathsf{q}_{\Phi,\mathcal{S}}$. Define P_B as the set of probability vectors, where $\boldsymbol{p}_b \in P_B$ if and only if the following inequalities are satisfied,

$$\prod_{i\in\mathcal{S}}(1-p_{bi}) + \sum_{i\in\mathcal{S}} p_{b_i} \prod_{j\in\mathcal{S}, j\neq i}(1-p_{bj}) \geq 1 - \frac{\mathsf{q}_{\Phi,\mathcal{S}} - \mathsf{q}_0}{1-\mathsf{q}_0}, \qquad \forall \mathcal{S} \tag{16}$$

$$\prod_{j\in\mathcal{S}}(1-p_{bj}) + \sum_{j\in\mathcal{S}, j\neq i} p_{b_j} \prod_{k\in\mathcal{S}, k\neq j}(1-p_{bk}) \geq 1 - \left\{\frac{\mathsf{q}_{\Phi,\mathcal{S}} - \mathsf{q}_0}{1-\mathsf{q}_0} + \frac{\mathsf{q}_{\{i\},\mathcal{S}}}{1-\mathsf{q}_0}\right\}, \quad \forall i, \mathcal{S}, i \in \mathcal{S} \tag{17}$$

And define $\mathcal{C}(\boldsymbol{p}_b)$ as the region that

$$\mathcal{C}(\boldsymbol{p}_b) = \left\{ \text{vect}\left[(1-\mathsf{q}_0)p_i \prod_{j\neq i}(1-p_{bj}p_j)\right] : \begin{array}{c} 0 \leq p_i \leq 1 \\ 1 \leq i \leq N \end{array} \right\} \tag{18}$$

Then, the packet arrival rate vector $\boldsymbol{\lambda}$ is located inside $\hat{\mathcal{C}}_S = \bigcap_{\boldsymbol{p}_b \in P_B} \mathcal{C}(\boldsymbol{p}_b)$.

The proof of Theorem 8 is presented in [5].

On one hand, if we have a collision channel, since $\mathsf{q}_0 = 0$ and $\boldsymbol{p}_b \equiv \boldsymbol{1} \in P_B$, the outer bound becomes the closure of the stability region as given in [3]. On the other hand, if $\mathsf{q}_{\mathcal{S},\mathcal{S}} = 1$ for all $\mathcal{S}$ and packets never cause collision to each other, we have $\mathsf{q}_0 = 0$ and the only member in P_B is $\boldsymbol{p}_b \equiv \boldsymbol{0}$, in this case we have $\hat{\mathcal{C}}_S = \{\boldsymbol{\lambda} | \lambda_i \leq 1, \forall i\}$, which is again the closure of the stability region. However, for a general standard MPR channel, whether the outer bound given by Theorem 8 is a tight one depends on the channel parameters.

D. Conjecture on the Stability Region

In this section, we present an interesting property together with a "sensitivity monotonicity" conjecture on the stationary distribution of ALOHA systems. We show that if the sensitivity monotonicity conjecture is true, the equivalence between the stability region and the throughput region follows as a direct consequence.

For each slot, we define a_i as the flag of "channel availability associated to terminal i". The conditional probability of a_i given a transmission status vector $\boldsymbol{t}$, with $t_{j\in\mathcal{S}} = 1$ and $t_{j\notin\mathcal{S}} = 0$, for an arbitrary $\mathcal{S}$, is given by

$$P(a_i | t_{j\in\mathcal{S}} = 1, t_{j\notin\mathcal{S}} = 0) = \sum_{\mathcal{R}, i\in\mathcal{R}\subseteq\mathcal{S}_i} \mathsf{q}_{\mathcal{R},\mathcal{S}\cup\{i\}} \tag{19}$$

In other words, (19) can be interpreted as, given $\boldsymbol{t}$, *if terminal i transmits a packet in the slot, the probability that the packet from terminal i can be received successfully is given by $P(a_i | t_{j\in\mathcal{S}} = 1, t_{j\notin\mathcal{S}} = 0)$.*

We should first note that, given the transmission status of terminals other than i, the channel availability a_i is independent from the current transmission status of terminal i. It is easy to see that, for a standard MPR channel, $P(a_i|\boldsymbol{q})$ is a function of $q_{j\neq i}$, but not a function of q_i. In addition, $P(a_i|\boldsymbol{q})$ is a decreasing function in $q_{j\neq i}$.

Theorem 9: Suppose the ALOHA system is stable and the MPR channel is standard. Let p_i be the transmission probability and let λ_i be the average packet arrival rate, at terminal i. Let U be the stationary distribution of the queues and $P_U(a_i = 1)$ be the stationary probability that "channel is available to terminal i". Then $\lambda_i \leq p_i P_U(a_i = 1)$ must hold for all i. On the other hand, given an ALOHA system, let $P_U(a_i = 1)$ denote the stationary probability that $a_i = 1$[4]. If $\lambda_i < p_i P_U(a_i = 1)$ is satisfied for all i, then the ALOHA system is stable.

The proof of Theorem 9 is presented in [5].

Conjecture (Sensitivity monotonicity): Suppose A_1 and A_2 are two finite-terminal ALOHA systems with the same number of terminals, the same packet arrival distributions and over the same standard MPR channel. Let $\boldsymbol{p}^{(A1)}$ and $\boldsymbol{p}^{(A2)}$ be the transmission probability vectors of A_1 and A_2, respectively. Define $P_{A1}(a_i = 1)$ as the stationary probability that $a_i = 1$ in system A_1, and define $P_{A2}(a_i = 1)$ as the stationary probability that $a_i = 1$ in system A_2. Then $P_{A1}(a_i = 1) \geq P_{A2}(a_i = 1)$ if $\boldsymbol{p}^{(A1)} \leq \boldsymbol{p}^{(A2)}$.

Combined with Theorem 9, in the next Theorem, we show that the equivalence between the closure of the stability region and the throughput region follows as a direct consequence of the sensitivity monotonicity conjecture.

Theorem 10: Suppose the ALOHA system is stable and the MPR channel is standard. Suppose the conjectured sensitivity monotonicity holds true. Then the closure of the stability region of the ALOHA system is given by

$$\mathcal{C}_S = \{\boldsymbol{\lambda} | \lambda_i = T_i(\boldsymbol{p}), 0 \leq p_i \leq 1, \forall i\} = \mathcal{C}_T \tag{20}$$

The proof of Theorem 10 is presented in [5].

References

[1] J. Hui, "Multiple Accessing for the Collision Channel Without Feedback" *IEEE J. Sel. A. in Commun.*, Vol. SAC-2, No. 4, pp. 575-582, Jul. 1984.

[2] J. Massey and P. Mathys, "The Collision Channel Without Feedback", *IEEE Trans. Info. Theory*, Vol. IT-31, No. 2, pp. 192-204, Mar. 1985.

[3] V. Anantharam, "The Stability Region of the Finite-User Slotted ALOHA Protocol", *IEEE Trans. Info. Theory*, Vol. 37, No. 3, pp. 535-540, May 1991.

[4] V. Naware, G. Mergen and L. Tong, "Stability and Delay of Finite User Slotted ALOHA with Multipacket Reception", to appear in *IEEE Trans. Info. Theory.*

[5] J. Luo and A. Ephremides, "On the Throughput, Capacity and Stability Regions of Random Multiple Access", submitted to *IEEE Trans. Info. Theory.*

[6] T. Harris, "A Correlation Inequality for Markov Processes in Partially Ordered State Spaces", *Ann. Probab.*, Vol. 5, No. 3, pp. 451-454, 1977.

[4]Note that even if the queues are not stable, the distribution of a_i can still be stationary, and hence $P_U(a_i = 1)$ can still be computed.