A Channel Coding Approach for Random Access Communication with Bursty Sources

Jie Luo
Electrical & Computer Engineering Department
Colorado State University
Fort Collins, CO 80523
Email: rockey@engr.colostate.edu

Anthony Ephremides
Electrical & Computer Engineering Department
University of Maryland
College Park, MD 20742
Email: etony@umd.edu

***Abstract*— We extend Information Theoretic analysis to time-slotted packet random access communication with bursty sources. A new channel coding approach for coding within each packet is proposed with built-in support for bursty sources phenomena, such as message underflow, and for random access mechanisms, such as packet collision detection. The coding approach does not require joint communication rate determination either between the transmitters or between the transmitters and the receiver. Its performance limitation is characterized by an achievable region defined in terms of communication rates, such that reliable packet recovery is supported for all rates within the region and reliable collision detection is supported for all rates outside the region. For random access communication over a discrete-time memoryless channel using a class of random coding schemes, it is shown that the maximum achievable rate region of the introduced coding approach equals the Shannon information rate region without a convex hull operation.**

I. Introduction

Classical information theory regards each transmitter in a multiuser communication system as backlogged with an infinite reservoir of traffic [1]. To achieve reliable communication, transmitters first jointly determine their codebooks and information rates and share this information with the receiver. The encoded symbols are then transmitted to the receiver continuously over a long time duration. Channel capacity and channel coding theorems are proved using the standard argument of jointly typical sequences by taking the sequence (or codeword) length to infinity [1].

By allowing a small acceptable communication error probability, information theoretic results can be extended to channel coding within a long, but finite-length, time duration [2][3]. Consequently, in a time-slotted communication model, if information bits arrive stochastically and queue at the transmitters, transmitters can jointly adapt their information rates in each time slot to optimize certain system performance based on coding theoretic results and on the status of the message queues [4][5]. Determination of fundamental performance limitations, such as the throughput and queuing delay tradeoff, can therefore be obtained as in [5]. Although such an extension enabled the incorporation of stochastic traffic in information theoretic analysis, it inherited the assumption and, hence, also the limitation of joint information rate determination among users in each time slot [4][5].

For various practical reasons such as bursty traffic arrivals, timely data dissemination, and adaptive, i.e., cognitive, networking, random access operation, such as opportunistic packet transmission, is unavoidable in communication networks [6]. Random access leads to unavoidable packet collision [7]. For the efficient functioning of the upper layer protocols, when reliable packet recovery is not possible, reporting a packet collision is often much preferred than forwarding an unreliable packet estimate to the upper layers [7].

Due to the challenging nature of relaxing the joint rate determination assumption among transmitters and receivers, information theoretic analysis has not been fully and successfully extended to practical random access systems. Without the support of rigorous coding theorems, networking practice often focuses on throughput optimization using packet-based channel models [8]. The explicit connection of the packet-based channel models to the physical layer channel is usually not specified except through the limited means of packet erasure channels. Networking practice allows bursty traffic at the transmitters and does not require joint communication rate determination among transmitters. However, a fundamental performance characterization in that case, in a form similar to Shannon's coding theorem, is not available [6].

In this paper, we propose an approach that holds promise in extending information theoretic analysis to packet random access communication with bursty sources. The essence of our approach consists of using the classical foundation of coding for <u>each packet</u> and explicitly building-in the support of random access operations and bursty sources phenomena in the following sense. In our coding approach each transmitter determines its communication rate by choosing the number of data bits to encode in each packet. It requires neither joint communication rate determination among transmitters, nor pre-sharing the communication rate information with the receiver. It also enables collision detection at the receiver whenever reliable packet recovery is not possible. Although defined quite differently from classical channel coding, we find that the introduced coding approach does lead to a meaningful achievable region characterization that is consistent with current understanding and methodology of Information Theory. More specifically, we define an achievable region on the communication rates such that reliable packet recovery is supported for all communication rates within the region and reliable packet collision detection is supported for all communication rates outside the region. For random multiple access communication over a discrete-time memoryless channel using a class of random coding schemes, we show that the maximum achievable rate region of the introduced coding approach equals the Shannon information rate region without a convex hull operation. Although we only illustrate our results in single-user and simple multiple access systems, the general problem formulation shown in the paper can be extended to other random access scenaria.

Next, we start with a detailed explanation of the coding approach in a single user system (i.e., single transmitter-receiver pair) in Section II. We then extend it to a random multiple access system and prove the main coding theorem in Section III. Proofs of the results presented in this paper can be found in [10] together with further extensions and discussions.

II. A New Packet Coding Approach – The Single User Case

Let us first consider a single user communication system over a discrete-time memoryless channel. The channel is modeled by a conditional distribution function $P_{Y|X}$ where $X \in \mathcal{X}$ is the channel input symbol and $Y \in \mathcal{Y}$ is the channel output symbol and, $\mathcal{X}$, $\mathcal{Y}$ are the finite input and output alphabets. We assume that time is slotted with each slot equaling N symbol durations, which is also the length of a packet. Unless otherwise specified, we will confine our focus on block channel codes of length N that represent coding within each packet.

Recall the standard description of classical block channel coding with a pre-determined information rate R_0 and a codeword length of N symbols [1]. The transmitter first chooses a codebook $C^{(N)}$ with 2^{NR_0} codewords and shares the codebook information with the receiver. At the beginning of a time-slot, the transmitter maps a message $w \in \{1, \dots, 2^{NR_0}\}$ into a codeword $C^{(N)}(w) = \{x_{w1}, \dots, x_{wN}\}$ and sends it through the channel. The receiver outputs a message estimate $\hat{w}$ upon observing the channel output $\{y_1, \dots, y_N\}$. The error probability is defined as $P_e(C^{(N)}) = \max_w Pr\{w \neq \hat{w}\}$. Let $\mathcal{C}$ be the Shannon capacity of the channel. If $R_0 < \mathcal{C}$, there exists a sequence of codebooks $\{C^{(N)}\}$ such that $\lim_{N\to\infty} P_e(C^{(N)}) = 0$ [1]. The asymptotic result here should be interpreted as follows: given two small positive constants ϵ_1, ϵ_2, there exists a threshold $N(\epsilon_1, \epsilon_2)$, such that $N > N(\epsilon_1, \epsilon_2)$ and $R_0 < \mathcal{C} - \epsilon_2$ imply $P_e(C^{(N)}) \leq \epsilon_1$ [3]. Discussions on the tradeoff between N, ϵ_1, ϵ_2 is skipped. We will not repeat this well-known observation in the rest of the paper.

Now recall the standard practice of packet networking with bursty sources [7]. Depending on message availability, in each time slot, the transmitter will either stay idle or transmit a packet according to the MAC layer protocol. Suppose that the same codebook is used in multiple time slots, which means that when the codebook is designed, the transmitter does not know whether or not a message will be available in a particular time slot. To model the idle operation, we can regard "idle" as a specific channel input symbol and add a particular codeword $C^{(N)}(0) = \{\text{idle}, \dots, \text{idle}\}$ into the codebook. When no message is available, we say $w = 0$ and the transmitter sends $C^{(N)}(0)$ through the channel. It can be shown that reliable message recovery, including reliable idle status detection, can be achieved asymptotically, if $R_0 < \mathcal{C}$.

The above channel coding scheme is still a classical one. However, we will make a conceptual extension by introducing a communication rate parameter r. According to the usual practice, when the transmitter idles, we say the communication rate is $r = 0$, otherwise, the communication rate is $r = R_0$. We say the codebook has two classes of codewords. The first class contains one codeword $C^{(N)}(0)$ corresponding to $r = 0$. The second class contains 2^{NR_0} codewords corresponding to $r = R_0$. The transmitter can choose its communication rate by mapping a message and rate pair (w, r) into a codeword. Reliable recovery of (w, r) can be achieved asymptotically if $R_0 < \mathcal{C}$.

Now consider a more complicated situation when the channel is random and can take two possible values $P^{(1)}_{Y|X}$, $P^{(2)}_{Y|X}$. We assume the channel is time-invariant within each time slot but its state is unknown to the transmitter. This channel model is considered here for illustration purpose, and its significance will become clear when we study multiuser systems. To simplify the analysis, let us assume that the channels described by $P^{(1)}_{Y|X}$ and $P^{(2)}_{Y|X}$ have capacities $\mathcal{C}_1$ and $\mathcal{C}_2$, with $\mathcal{C}_1 < \mathcal{C}_2$, respectively, and that both capacities are achieved for the same input distribution (e.g. binary symmetric channels with different cross probabilities). Because channel coding is applied only within a packet, it can be beneficial if the transmitter occasionally communicates information at a rate higher than $\mathcal{C}_1$. Let us consider a codebook that contains three classes of codewords. The first class contains one codeword $C^{(N)}(0) = \{\text{idle}, \dots, \text{idle}\}$ corresponding to communication rate $r = 0$. The second class contains 2^{NR_0} codewords corresponding to $r = R_0 < \mathcal{C}_1$. The third class contains 2^{NR_1} codewords corresponding to $r = R_1$, s.t. $\mathcal{C}_1 < R_1 < \mathcal{C}_2$. The transmitter now has three rate options $r = 0, R_0, R_1$, which respectively correspond to idling, or encoding NR_0, or NR_1 data bits in a packet of N symbols. This extends the standard networking operation where only a fixed number of bits is encoded in each packet. Because the capacity $\mathcal{C}$ of the channel in a particular time slot is random, that is it can take either value $\mathcal{C}_1$ or $\mathcal{C}_2$, when $r = R_1 > \mathcal{C}_1$, reliable message recovery is not always possible. Without sharing r with the receiver, we require the receiver to identify whether the estimated communication rate $\hat{r}$ is above the channel capacity. If $\hat{r} < \mathcal{C}$, the receiver outputs the estimated $(\hat{w}, \hat{r})$ pair. If $\hat{r} > \mathcal{C}$, the receiver reports a packet collision (or an erasure) by letting $\hat{r} = \text{collision}$. Note that the term "collision" is used to maintain consistency with the networking terminology. Throughout the paper, "collision" means packet erasure, irrespective of whether it is caused by multi-packet interference or channel fading.

Given a channel capacity $\mathcal{C}$, define the error probability

$P_e(C^{(N)}, r)$ and the collision probability $P_c(C^{(N)}, r)$, both as functions of r, as follows

$$P_e(C^{(N)}, r) = \sup_w Pr\{w \neq \hat{w} \text{ or } \hat{r} \neq r\}$$
$$P_c(C^{(N)}, r) = \sup_w Pr\{\hat{r} = \text{collision}\}. \quad (1)$$

It can be shown that there exists a sequence of codebooks $\{C^{(N)}\}$ and decoding algorithms such that the following asymptotic results hold for both $\mathcal{C} = \mathcal{C}_1$ and $\mathcal{C} = \mathcal{C}_2$.

$$\lim_{N\to\infty} P_e(C^{(N)}, r) = 0, \qquad \text{for } r < \mathcal{C},$$
$$\lim_{N\to\infty} P_c(C^{(N)}, r) = 1, \qquad \text{for } r > \mathcal{C}. \quad (2)$$

When the random channel can take an infinite number of values $P^{(s)}_{Y|X}$, $s \in \{1, \ldots, \infty\}$, it is generally impossible to achieve (2) for all channel realizations with the same sequence of codebooks. Alternatively, if capacities of all channel realizations $P^{(s)}_{Y|X}$ are achieved for the same input distribution P_X, asymptotic properties similar to (2) can still be achieved using a sequence of random coding schemes, which we will define soon.

The reader may have noticed that the communication rate parameter introduced in the above discussion is a subtle concept. From a practical point of view, Nr represents the number of information bits the transmitter encodes in a packet. From a theoretic point of view, however, if the transmitter maps (w, r) into a codeword, one can index all possible values of (w, r), and regard the index W as a macro message. Consequently, the communication rate $r = r(W)$ is an arbitrarily defined function of W. Properties of achievable rates are dependent on the particular form of the $r(W)$ function. This motivated us to consider the following *standard* $r(W)$ function. Assume codebook $C^{(N)}$ has $2^{NR_{\max}}$ codewords, where $R_{\max}$ is an arbitrary large constant. Let the codewords be indexed by $W \in \{1, \ldots, 2^{NR_{\max}}\}$. We define the *standard communication rate* parameter by $r = r(W) = \frac{1}{N}\log_2 W$. Because the standard rate function is invertible, achievable rates of any other rate function can be derived from achievable rates of the standard rate function.

Now we define a random coding scheme as follows. The definition is motivated by a similar presentation given in [9].

Definition 1: Let $\mathcal{L} = \{\mathcal{C}_\theta : \theta \in \Theta\}$ be a library of codebooks. Each codebook in the library contains $2^{NR_{\max}}$ codewords of length N, where $R_{\max}$ is an arbitrary large constant. Let the codebooks be indexed by a set Θ. Let the actual codebook chosen by the transmitter be $\mathcal{C}_\theta$ where the index θ is a random variable following distribution γ. We define $(\mathcal{L}, \gamma)$ as a *random coding scheme* following distribution P_X, given that the random variables $X_{W,j} : \theta \to \mathcal{C}_\theta(W)_j, \forall j, W$, should be i.i.d. based on the input distribution P_X. ■

Let $(\mathcal{L}^{(N)}, \gamma^{(N)})$ be a random coding scheme following distribution P_X, where each codebook in library $\mathcal{L}^{(N)}$ contains $2^{NR_{\max}}$ codeword of length N. In each time slot, we assume the transmitter randomly generates a codebook index θ and use codebook $C^{(N)}_\theta$ to encode source message W. We assume $\mathcal{L}^{(N)}$, and the actual realization of θ is known to the receiver. Note that, in a practical system, this assumption only implies that a random codebook generation algorithm should be shared by the transmitter and the receiver. Given the channel $P_{Y|X}$, let $P_{e|\theta}(\mathcal{C}_\theta, W)$ be the probability, conditioned on θ, that the receiver is not able to recover message W, and let $P_{c|\theta}(\mathcal{C}_\theta, W)$ be the conditional probability that the receiver reports a collision. We define $P_e(W)$ and $P_c(W)$ as the unconditional error probability and the unconditional collision probability of message W, respectively, as follows

$$P_e(W) = E_\theta[P_{e|\theta}(\mathcal{C}_\theta, W)],\ P_{c|\theta}(W) = E_\theta[P_{c|\theta}(\mathcal{C}_\theta, W)]. \quad (3)$$

Let the standard rate function be $r(W) = \frac{1}{N}\log_2 W$. We say that a communication rate region $[0, R)$ is asymptotically achievable by the random coding scheme if there exists a sequence of decoding algorithms under which the following two conditions are satisfied. First, for all message sequences $\{W^{(N)}\}$ with $\lim_{N\to\infty} r(W^{(N)}) < R$, we have $\lim_{N\to\infty} P_e\{W^{(N)}\} = 0$. Second, for all message sequences $\{W^{(N)}\}$ with $\lim_{N\to\infty} r(W^{(N)}) > R$, we have $\lim_{N\to\infty} P_c\{W^{(N)}\} = 1$. We have the following theorem.

Theorem 1: For a discrete-time memoryless channel $P_{Y|X}$ with Shannon capacity $\mathcal{C}$ and optimal input distribution P_X, communication rate region $[0, \mathcal{C})$ is asymptotically achievable by a random coding scheme with input distribution P_X. ■

It is important to note that the definition of the achievable rate region introduced in this paper is significantly different from that of the Shannon capacity. For reliable communication in the classical Shannon sense, information rate is pre-determined and known to receiver. Collision or erasure detection is not needed [1]. In our coding approach, the codebook contains a large number of codewords corresponding to different communication rates. Reliable communication is only supported for rates within the achievable region. It is the receiver's responsibility to detect whether the random communication rate is within the achievable region or not.

III. A New Packet Coding Approach – Random Multiple Access Communication

Consider now a K-user, symbol-synchronous, random multiple access communication system over a discrete-time memoryless channel. The channel is modeled by a conditional distribution function $P_{Y|X_1,\ldots,X_K}$, where $X_i \in \mathcal{X}_i$ is the channel input symbol of user i with $\mathcal{X}_i$ being the input alphabet, and $Y \in \mathcal{Y}$ is the channel output symbol with $\mathcal{Y}$ being the output alphabet. To simplify the discussion, we assume $\mathcal{Y}$ and $\mathcal{X}_i$, for all i, are finite. Extending the results to continuous channels is straightforward. As in Section II, we assume time is slotted with each slot being equal to N symbol durations, which is also the length of a packet.

Before message transmission is started, each user, say user i, chooses a codebook $\mathcal{C}_i$, termed a random access codebook, with $2^{NR_{\max}}$ codewords of length N, where $R_{\max}$ is an arbitrary large constant whose particular value is not important. Then $\mathcal{C}_i$ is a mapping $\mathcal{C}_i : \{1, \ldots, 2^{NR_{\max}}\} \to \mathcal{X}_i^{(N)}$ that associates to each message W in $\{1, \ldots, 2^{NR_{\max}}\}$ a block of channel input symbols $\mathcal{C}_i(W) = (X_{iW1}, X_{iW2}, \ldots, X_{iWN})$.

We consider a class of random coding schemes defined in Definition 2. Compared to the random coding scheme defined in Section II, the following coding scheme allows different input distributions to be used at different communication rates.

Definition 2: Let $\mathcal{L}_i = \{\mathcal{C}_{i\theta_i} : \theta_i \in \Theta_i\}$ be a library of codebooks for user i. Each codebook in the library contains $2^{NR_{\max}}$ codewords of length N. Let the codebooks be indexed by a set Θ_i. Let the actual codebook chosen by user i be $\mathcal{C}_{i\theta_i}$ where the index θ_i is a random variable following distribution γ_i. We define $(\mathcal{L}_i, \gamma_i)$ as a *random coding scheme* following distribution $P_{X_i|W}$ for user i, given that the following conditions are satisfied. First, the random variables $X_{i,W,j} : \theta_i \to \mathcal{C}_{i\theta_i}(W)_j$, $\forall j, W$, are independent. Second, for given i and W, $X_{i,W,j}$ should be i.i.d. based on the conditional input distribution $P_{X_i|W}$. ■

We assume each user, say user i, is equipped with a random coding scheme $(\mathcal{L}_i, \gamma_i)$, which is known to the receiver. Before message transmission is started, user i, for all i, chooses its codebook by generating a codebook index θ_i according to distribution γ_i, independently from other users. We assume the actual value of θ_i, $\forall i$, is known to the receiver. The standard communication rate of message $W^{(N)}$ is defined as $r(W^{(N)}) = \frac{1}{N}\log_2(W^{(N)})$.

Definition 3: Given a sequence of random coding schemes $\{(\mathcal{L}_i^{(N)}, \gamma_i^{(N)})\}$ of user i, where $(\mathcal{L}_i^{(N)}, \gamma_i^{(N)})$ is a random coding scheme with each codebook in library $\mathcal{L}_i^{(N)}$ containing $2^{NR_{\max}}$ codewords of length N, $R_{\max}$ is the maximum rate. Denote the conditional input distribution associated with $(\mathcal{L}_i^{(N)}, \gamma_i^{(N)})$ and message $W^{(N)}$ by $P^{(N)}_{X_i|W^{(N)}}$. Let $P_{X_i|r}$ be a conditional input distribution function defined for all rates $r \le R_{\max}$. We say that $\{(\mathcal{L}_i^{(N)}, \gamma_i^{(N)})\}$ follows an asymptotic conditional input distribution $P_{X_i|r}$ if for all $\hat{r} \le R_{\max}$ and for all message sequence $\{W^{(N)}\}$ with $\lim_{N\to\infty} r(W^{(N)}) = \hat{r}$, the following limits exist, and satisfy

$$\lim_{N\to\infty} P^{(N)}_{X_i|W^{(N)}} = \lim_{N\to\infty} P_{X_i|r(W^{(N)})}. \tag{4}$$

Note that since we do not assume $P_{X_i|r}$ is continuous in r at $\hat{r}$, we may not have $\lim_{N\to\infty} P_{X_i|r(W^{(N)})} = P_{X_i|\hat{r}}$. ■

We use bold-font characters to denote vectors whose ith elements are the corresponding variables of user i. For example, $\boldsymbol{\mathcal{L}}$ represents the vector of code libraries of the users. Also, $\boldsymbol{\theta}$ denotes the random index vector, $\boldsymbol{\mathcal{C}_\theta}$ denotes the codebook vector, $\boldsymbol{W}$ denotes the message vector, $\boldsymbol{r}(\boldsymbol{W})$ denotes the standard rate vector, and $\boldsymbol{P_{X|r}}$ denotes the asymptotic input distributions of the random coding schemes, etc.

Assume message $\boldsymbol{W}$ is transmitted over the multiple access channel using codebook $\boldsymbol{\mathcal{C}_\theta}$. Assume a decoding algorithm is given. We define $P_{e|\boldsymbol{\theta}}(\boldsymbol{\mathcal{C}_\theta}, \boldsymbol{W})$ as the probability, conditioned on $\boldsymbol{\theta}$, that the receiver is not able to recover the message vector $\boldsymbol{W}$. Define $P_{c|\boldsymbol{\theta}}(\boldsymbol{\mathcal{C}_\theta}, \boldsymbol{W})$ as the conditional probability that the receiver reports a collision. Assume random coding schemes $(\boldsymbol{\mathcal{L}}, \boldsymbol{\gamma})$. Let $\boldsymbol{\theta}$ be drawn independently according to $\boldsymbol{\gamma}$. We define $P_e(\boldsymbol{W})$ and $P_c(\boldsymbol{W})$ as the unconditional error probability and the unconditional collision probability of message $\boldsymbol{W}$, respectively.

$$P_e(\boldsymbol{W}) = E_{\boldsymbol{\theta}}[P_{e|\boldsymbol{\theta}}(\boldsymbol{\mathcal{C}_\theta}, \boldsymbol{W})],\ P_c(\boldsymbol{W}) = E_{\boldsymbol{\theta}}[P_{c|\boldsymbol{\theta}}(\boldsymbol{\mathcal{C}_\theta}, \boldsymbol{W})]. \tag{5}$$

Definition 4: Consider a discrete-time memoryless multiple access channel $P_{Y|X_1,\ldots,X_K}$ and a sequence of random coding schemes $\{(\boldsymbol{\mathcal{L}}^{(N)}, \boldsymbol{\gamma}^{(N)})\}$, where $(\boldsymbol{\mathcal{L}}^{(N)}, \boldsymbol{\gamma}^{(N)})$ is a random coding scheme with each codebook in $\boldsymbol{\mathcal{L}}^{(N)}$ containing $2^{NR_{\max}}$ codewords of length N, and $R_{\max}$ is the maximum rate for all users[1]. Let $\boldsymbol{R}$ be a region of standard rate vectors. Let $\boldsymbol{R}_c$ be the closure of $\boldsymbol{R}$. We say $\boldsymbol{R}$ is asymptotically achievable if there exists a sequence of decoding algorithms under which the following two conditions are satisfied. First, for all message sequences $\{\boldsymbol{W}^{(N)}\}$ with $\boldsymbol{r}(\boldsymbol{W}^{(N)}) \in \boldsymbol{R}$ for all N and $\lim_{N\to\infty} \boldsymbol{r}(\boldsymbol{W}^{(N)}) \in \boldsymbol{R}$, we have $\lim_{N\to\infty} P_e(\boldsymbol{W}^{(N)}) = 0$. Second, for all message sequences $\{\boldsymbol{W}^{(N)}\}$ with $\boldsymbol{r}(\boldsymbol{W}^{(N)}) \notin \text{closure}(\boldsymbol{R}_c)$ for all N and $\lim_{N\to\infty} \boldsymbol{r}(\boldsymbol{W}^{(N)}) \notin \text{closure}(\boldsymbol{R}_c)$, we have $\lim_{N\to\infty} P_c(\boldsymbol{W}^{(N)}) = 1$. ■

The following theorem shows that the maximum asymptotically achievable rate region of a random coding scheme can be characterized by a set of mutual information inequalities.

Theorem 2: Consider a discrete-time memoryless multiple access channel $P_{Y|X_1,\ldots,X_K}$ and an asymptotic conditional input distributions $\boldsymbol{P_{X|r}}$. Assume for any user i, that $P_{X_i|r_i}$ is only discontinuous in r_i at a finite number of points. For any sequence of random coding schemes $\{(\boldsymbol{\mathcal{L}}^{(N)}, \boldsymbol{\gamma}^{(N)})\}$ that follows the asymptotic conditional input distribution $\boldsymbol{P_{X|r}}$, the following standard communication rate region $\boldsymbol{R}$ is asymptotically achievable.

$$\boldsymbol{R} = \left\{ \hat{\boldsymbol{r}} \;\middle|\; \begin{array}{l} \forall S \subseteq \{1,\ldots,K\}, \text{either } \hat{\boldsymbol{r}}_{i\in S} = \boldsymbol{0}, \\ \text{or } \sum_{i\in S} \hat{r}_i < I_{\hat{\boldsymbol{r}}}(\boldsymbol{X}_{i\in S}; Y|\boldsymbol{X}_{i\notin S}) \end{array} \right\}, \tag{6}$$

where the mutual information $I_{\hat{\boldsymbol{r}}}(\boldsymbol{X}_{i\in S}; Y|\boldsymbol{X}_{i\notin S})$ is computed based on the distribution $\boldsymbol{P_{X|\hat{r}}}$.

Furthermore, for any sequence of random coding schemes following the asymptotic conditional input distribution $\boldsymbol{P_{X|r}}$, assume $\tilde{\boldsymbol{R}}$ is an asymptotically achievable rate region. Let $\hat{\boldsymbol{r}}$ be an arbitrary rate vector inside $\tilde{\boldsymbol{R}}$ in the sense that we can find $\delta > 0$ with $\boldsymbol{r} \in \tilde{\boldsymbol{R}}$ for all $\hat{\boldsymbol{r}} \le \boldsymbol{r} \le \hat{\boldsymbol{r}} + \delta \times \boldsymbol{1}$, where $\boldsymbol{1}$ is a vector of all 1's. Let $S \subseteq \{1,\ldots,K\}$ be an arbitrary user subset. If the asymptotic conditional input distribution $\boldsymbol{P_{X|r}}$ is continuous in $\boldsymbol{r}_{i\in S}$ at $\hat{\boldsymbol{r}}$, then we must have

$$\sum_{i\in S} \hat{r}_i \le I_{\hat{\boldsymbol{r}}}(\boldsymbol{X}_{i\in S}; Y|\boldsymbol{X}_{i\notin S}). \tag{7}$$

■

When the asymptotic conditional input distribution $\boldsymbol{P_{X|r}}$ is not a function of $\boldsymbol{r}$, i.e. codewords of each user are generated according to the same input distribution, the achievable rate region $\boldsymbol{R}$ given in (6) becomes

$$\boldsymbol{R} = \left\{ \hat{\boldsymbol{r}} \;\middle|\; \forall S \subseteq \{1,\ldots,K\}, \sum_{i\in S} \hat{r}_i < I(\boldsymbol{X}_{i\in S}; Y|\boldsymbol{X}_{i\notin S}) \right\}. \tag{8}$$

[1]Because, as we will see, the value of $R_{\max}$ is not important, there is no loss of generality by assuming all users have the same maximum rate.

This is identical to the Shannon information rate region without a convex hull operation for the multiple access channel under a given input distribution [1].

Note that random access communication does not assume joint rate determination among users [8]. Hence whether the actual rate vector is within the achievable rate region is unknown to any particular user. This is similar to the random channel case we discussed in Section II. In our coding approach, codebooks of the users contain large numbers of codewords. However, the codewords are indexed by their standard rate parameters and the receiver only searches for appropriate codewords within the achievable rate region. The receiver reports a collision if an appropriate codeword cannot be found.

IV. Simple Examples

In this section, we illustrate the asymptotically achievable rate region results in two simple examples of random multiple access systems.

Example 1: Consider a K-user, random multiple access communication system over a memoryless Gaussian channel modeled by $Y = \sum_{i=1}^{K} X_i + V$, where V is the additive white Gaussian noise with zero mean and variance N_0.

Assume that the input distribution of user k, for all k, is Gaussian with zero mean and variance P_k, irrespective of the rate parameter. According to Theorem 2, the maximum achievable rate region is given by

$$\boldsymbol{R} = \left\{ \boldsymbol{r} \left| \forall S \subseteq \{1, \dots, K\}, \sum_{i \in S} r_i < \frac{1}{2} \log \left(1 + \frac{\sum_{i \in S} P_i}{N_0} \right) \right. \right\}. \tag{9}$$

Note that the achievable rate region $\boldsymbol{R}$ is identical to the Shannon channel capacity region [1].

If for all k, the input distribution is Gaussian with zero mean and variance P_k for any non-zero rate, and user k idles at rate zero, the achievable rate region is still given by (9).

Example 2: Consider a K-user, random multiple access communication system over a memoryless collision channel. We define an nth order collision channel as follows. The channel input alphabet of any user is given by $\mathcal{X} = \{0, 1, \dots, 2^n\}$, where 0 represents an idle symbol. The channel output alphabet is given by $\mathcal{Y} = \{0, 1, \dots, 2^n, c\}$, where c represents a collision symbol. If all users idle, the receiver receives an idle symbol, $Y = 0$; if only one user, say user k, transmits a non-zero symbol X_k, the receiver receives $Y = X_k$; if multiple users transmit non-zero symbols, the receiver receives $Y = c$, i.e., a collision symbol. We assume in all input distributions the non-zero symbols always take equal probabilities. Consequently, an input distribution $P_{X_k|r_k}$ can be characterized through a single parameter $p(r_k)$, which is the probability that any particular symbol in the transmitted codeword takes a non-zero value.

Proposition 1: Assume the conditional input distribution of user k, for all k, is given by

$$P_{X_k|r_k} = \begin{cases} 1 - \sqrt{r_k/n} & \text{for } X_k = 0 \\ \frac{1}{2^n}\sqrt{r_k/n} & \text{for } X_k = j \in \{1, \dots, 2^n\} \end{cases}. \tag{10}$$

In other words, $p(r_k) = \sqrt{r_k/n}$. The following rate region $\boldsymbol{R}$ is asymptotically achievable.

$$\boldsymbol{R} = \left\{ \boldsymbol{r} \left| \sum_{i=1}^{K} \sqrt{r_k/n} < 1 \right. \right\}. \tag{11}$$

■

V. Conclusion

We proposed a new channel coding approach for coding within each packet in random access communication with bursty sources. Extensions of the coding approaches, such as the receiver reporting a collision for each individual user, are presented in [10]. Error performance analysis of the coding schemes is given in [11]. Further discussions about coding in random access communication can also be found in [10].

There are numerous questions left open that would further tighten the connection between random access networking and Information Theory. We believe that our approach contributes toward that connection by relaxing the joint rate determination assumption among users, and by distinguishing the issues of reliable communication and reliable collision detection in a rigorous manner.

References

[1] T. Cover and J. Thomas, *Elements of Information Theory,* 2nd Edition, Wiley Interscience, 2005.

[2] R. Gallager, *A Simple Derivation of The Coding Theorem and Some Applications,* IEEE Transactions on Information Theory, Vol. 11, pp. 3-18, January 1965.

[3] Y. Polyanskiy, V. Poor, S. Verdú, *Channel Coding Rate in The Finite Blocklength Regime,* submitted to IEEE Transactions on Information Theory.

[4] I. Telatar and R. Gallager, *Combining Queueing Theory with Information Theory for Multiaccess,* IEEE Journal on Selected Areas in Communications, Vol. 13, pp. 963-969, August 1995.

[5] R. Berry and R. Gallager, *Communication over Fading Channels with Delay Constraints,* IEEE Transactions on Information Theory, Vol. 48, pp. 11351149, May 2002.

[6] A. Ephremides and B. Hajek, *Information Theory and Communication Networks: An Unconsummated Union,* IEEE Transactions on Information Theory, Vol. 44, pp. 2416-2434, October 1998.

[7] D. Bertsekas and R. Gallager, *Data Network,* 2nd Edition, Prentice Hall, 1992.

[8] J. Luo and A. Ephremides, *On the Throughput, Capacity, and Stability Regions of Random Multiple Access,* IEEE Transactions on Information Theory, Vol. 52, pp. 2593-2607, June 2006.

[9] S. Shamai, I. Teletar, and S. Verdú, *Fountain Capacity,* IEEE Transactions on Information Theory, Vol. 53, pp. 4372-4376, November 2007.

[10] J. Luo and A. Ephremides, *A New Channel Coding Approach for Random Access with Bursty Traffic,* submitted to IEEE Transactions on Information Theory, http://www.engr.colostate.edu/∼rockey/Papers/RandomAccessCodingJournal.pdf.

[11] Z. Wamg and J. Luo, *Achievable Error Exponent of Channel Coding in Random Access Communication,* submitted to IEEE ISIT, Austin, TX, June 2007.