Power Levels and Packet Lengths in Random Multiple Access With Multiple-Packet Reception Capability

Jie Luo, *Member, IEEE,* and Anthony Ephremides, *Fellow, IEEE*

***Abstract*—This paper extends our earlier results. We assume that the receiver has the capability of capturing multiple packets so long as the signal-to-interference-plus-noise ratio (SINR) of each packet is above a designed threshold T throughout its transmission period. We prove that, compared with a multiple-power-level system, the single-power-level system in which all nodes transmit at the maximum allowable power level achieves optimal throughput, under a condition that T exceeds the value 3.33. Given a minimum throughput requirement, under the same condition on T, the single-power-level system also achieves the maximum average packet capture probability as well as the optimum energy usage efficiency. If the multiple-power-level systems are constrained such that higher power levels always have shorter packet lengths, then the above results hold for T greater than 2.**

***Index Terms*—ALOHA, energy efficiency, power capture, random multiple access.**

I. Introduction

In distributed random multiple access, nodes transmit packets in an uncoordinated fashion. Error-controlled packet reception, being one of the key properties of the medium access control layer, is usually implemented to transform the noisy wireless channel into an error-free logical link to the upper layers. When the signal-to-interference-plus-noise ratio (SINR) of a packet is above a designed threshold T throughout its transmission period, the packet is received successfully with a low probability of reception error. If the SINR of a packet goes below T during its reception, the packet is considered unreliable and hence is dropped by the receiver. For a reasonable value of T ($T \geq 2$, for example), when multiple packets of similar powers overlap at the receiver, the SINR requirement of all packets will be violated. If two packets arrive at the receiver with different powers, however, it is possible that the SINR requirement of the high-power packet is still satisfied, and hence the packet is received successfully. Such a phenomenon is called power capture. In wireless networks, in order to take advantage of power capture, it is proposed in [2], [3] that packets can be transmitted at multiple discrete power levels, and the packet received at the highest power may be captured.

Manuscript received January 22, 2004; revised March 5, 2005. This work was supported by the National Aeronautics and Space Administration Award NCC8-235 and the Collaborative Technology Alliance for Communication and Networks sponsored by the U.S. Army Laboratory under Cooperative Agreement DAAD19-01-2-0011. Part of the material in this paper was presented at the Conference on Information Science and Systems, Princeton, NJ, March 2004. Any opinions findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Aeronautics and Space Administration or the Army Research Laboratory of the U.S. Government.

The authors are with the Electrical and Computer Engineering Department, University of Maryland, College Park, MD 20742 USA (e-mail: rockey@eng.umd.edu; tony@eng.umd.edu).

Communicated by G. Sasaki, Associate Editor for Communication Networks.
Digital Object Identifier 10.1109/TIT.2005.862076

Various power capture models have been studied in the literature. In the perfect capture model, a packet is successfully received if and only if the ratio between the power of the packet to that of any interfering packet is higher than a fixed value [3], [4]. Although the model is sometimes overly optimistic, its use [5], [6] often leads to simpler analytic derivations. A more accurate model is based on the assumption that a packet is captured if and only if its SINR is greater than a decodability threshold [3], [4], [7], [8]. This is called the SINR capture model. In all of the above works, packet lengths at different power levels are identical. When the packet lengths (or symbol durations) of different power levels are different, high SINR at the receiver matched filter output can be achieved either via a high transmission power or via a long symbol duration. Therefore, it is possible that multiple overlapping packets can satisfy the SINR requirement simultaneously. The capture model in such a situation depends on the receiver design. If the receiver has multiple matched filters with each corresponding to one of the overlapping packets, all the packets satisfying the SINR requirement can be received successfully. If the receiver contains only one matched filter and hence cannot receive multiple packets simultaneously, it was suggested that the receiver can lock onto the packet with the highest power [1], [10]. Consequently, in addition to the SINR requirement, a packet must maintain the highest reception power among all the overlapping packets throughout its transmission period in order to be received successfully. We call this the Highest Power SINR (HP-SINR) capture model.

The performance of a random multiple-access system is often measured by the system throughput, which is defined as the average number of successfully received packets per second. When all the packets are of the same length, it was shown that the use of multiple power levels increases the throughput of the system [1]. However, when both power levels and packet lengths are system parameters, transmitting packets with multiple power levels is not always beneficiary. The effect of multiple power levels and packet lengths on the system throughput and energy usage efficiency was studied in [1] under the HP-SINR capture model. It was shown in [1] that the single-power-level system in which all nodes transmit at the maximum allowable power level achieves both optimal throughput and energy usage efficiency under a condition on the decodability threshold value.

In this paper, we consider the problem as in [1] but with the general SINR capture model. In other words, we drop the assumption of the simple receiver design, and assume that meeting the predetermined SINR threshold T is the only requirement for a successful packet reception. We prove that, when $T \geq \frac{2e^2}{2e-1} \approx 3.33$, the single-power-level system achieves the optimal throughput. Given a throughput requirement, the single-power-level system also achieves the maximum average packet capture probability as well as the optimum energy usage efficiency, when $T \geq \frac{2e^2}{2e-1}$. If we only consider systems where higher power levels have shorter packet lengths, then the above results on the optimality of the single-power-level system hold for $T \geq 2$. Furthermore, for the same problem considered in [1] with the HP-SINR capture model, we show that the results of this paper hold for $T \geq \frac{2e(e-1)}{2e-1} \approx 2.11$.

The rest of the paper is organized as follows. The system model is described in Section II. The main results of the paper are presented in Section III. The detailed proofs are provided in the appendices. The throughput upper bound used in the appendices is derived based on an optimistic model, which is presented in Section IV. The paper concludes in Section V.

II. System Model

Suppose there is an infinite number of bufferless nodes in the system [1], [11]. A global clock is available to all the nodes such that slotted transmission can be achieved. There are M discrete power levels in the system. The values of the power levels and packet lengths are determined in the system deployment phase and fixed once the system is in use. For each packet, a node randomly picks a power level from the M power levels and transmits the packet at that power level. Power level of each packet is chosen independently. Time is slotted at each power level with the slot duration equaling the packet length of the corresponding power level. At each power level, packets transmissions start only at the slot edges. We do not assume slot synchronization between different power levels. If a packet is lost due to collision, it is retransmitted at a later time.

Definitions:

1. The *system throughput* is measured by the average number of successfully received packets per second.
2. The average packet capture probability is measured by

$$\frac{\text{system throughput}}{\text{offered traffic rate in packets per second}}.$$

3. The energy usage efficiency is measured by

$$\frac{\text{system throughput}}{\text{system average power consumption}}.$$

As in [1], we make the following assumptions.

Assumptions:

1. Each packet contains W symbols.
2. A packet is received correctly if and only if during the entire transmission period, the SINR of each symbol is *always* larger than a designed threshold T. The SINR is defined as symbol energy to interference plus noise ratio, that is,

$$\text{SINR} = \frac{PT_s}{I + N_0} \geq T \tag{1}$$

where P, T_s are the transmission power and symbol duration of the packet, I and N_0 are the interference energy and spectral density of the background noise in the output of the symbol matched filter.
3. There are M discrete power levels $P_1 > P_2 > \cdots > P_M$. The power levels and packet lengths are designed such that when a packet of level P_i overlaps with a packet of level P_{i+1} at the receiver, there is a positive probability that the SINR requirement of the packet at power level P_i can be satisfied.
4. Binary phase-shift keying (BPSK) modulation scheme is used with symbol waveform being the rectangular waveform.[1]
5. The transmission power of each packet is constant during the transmission period.
6. The probability of a packet transmitted at power level P_i is q_i with $\sum_{i=1}^{M} q_i = 1$. We assume these probabilities are determined at the system deployment phase and fixed once the system is in use.
7. The distance between the transmitters and the receiver are equal and the transmission medium is isotropic.
8. The maximum power that a packet can use is $P_{\max}$. We assume $P_{\max}, T$ and the bandwidth of the channel satisfy the requirement that a symbol waveform of length $\frac{N_0 T}{P_{\max}}$ can pass the channel with negligible distortion.
9. The total offered traffic on the system, including new arrivals and retransmissions, is Poisson [11].

Further explanations on the assumptions can be found in [1].

III. Optimality of the Single-Power-Level System

Based on the definitions and assumptions presented in Section II, the main result of the paper is given in the following proposition.

Proposition 1: Assume that the decodability threshold T satisfies

$$T \geq \frac{2e^2}{2e-1} \approx 3.33.$$

1. The single-power-level system where all nodes transmit at $P_{\max}$ with packet length $L = \frac{WN_0T}{P_{\max}}$ achieves the maximum possible throughput, which is

$$S_{\max} = \frac{P_{\max}\exp(-1)}{WN_0T}. \tag{2}$$

2. Given a minimum throughput requirement $S \geq S_{\min}$ (assume that the throughput requirement is achievable), the single-power-level system where all nodes transmit at $P_{\max}$ with packet length $L = \frac{WN_0T}{P_{\max}}$ achieves the maximum average packet capture probability, which is

$$p_{\max}(\text{capture}) = \frac{S_{\min}}{G} \tag{3}$$

where G is the offered traffic on the system that satisfies

$$\frac{WN_0T}{P_{\max}}G \leq 1 \quad \text{and} \quad G\exp\left(-\frac{WN_0T}{P_{\max}}G\right) = S_{\min}.$$

[1]Similar to [1], the results in this paper can be extended to other modulations and symbol waveforms with little essential modification.

3. Given a minimum throughput requirement $S \geq S_{\min}$, the single-power-level system also achieves the maximum power usage efficiency, which is

$$\text{efficiency}_{\max} = \frac{S_{\min}}{GWN_0T}. \tag{4}$$

The proof of Proposition 1 is provided in Appendix A.

In addition to Proposition 1, stronger results can be obtained for the following two special cases.

Proposition 2: Under the assumptions in Section II with an additional assumption that the packet lengths in any M-power-level system satisfy $L_1 \leq L_2 \leq \cdots \leq L_M$, then the results of Proposition 1 hold for $T \geq 2$.

Proof: When $L_1 \leq L_2 \leq \cdots \leq L_M$ is satisfied, according to the group assignment criterion presented in Section IV, the system contains M groups and each group in the M-power-level system contains only one power level. Hence, the results of Proposition 1 can be shown by following the proof in Appendix A but starting directly from Section A.2. The proof only requires $T \geq 2$. □

Proposition 3: Under the assumptions in Section II and assuming the HP-SINR capture model (the same problem considered in [1]), the results in Proposition 1 hold for

$$T \geq \frac{2e(e-1)}{2e-1} \approx 2.11.$$

The proof of Proposition 3 is presented in Appendix B and improves the results in [1].

Considering Assumption 8 in Section II, the results in this section should be interpreted as improving the bandwidth efficiency has a higher priority than exploiting the power capture effect in the system design. We want to emphasize that the results do not mitigate the value of a multiple-power-level design under the condition that the symbol duration already reaches the minimum value corresponding to the channel bandwidth.

IV. The Optimistic Model

In this section, we describe an optimistic power capture model. The model is used to provide an upper bound to the throughput of a multiple-power-level system; such an upper bound plays a key role in deriving the results in Section III. Since the model is revised from the one in [1], we focus only on the aspects that differ from [1].

Suppose two packets 1 and 2 with symbol durations T_{s1} and T_{s2}, respectively, overlap at the receiver. When the number of symbols per packet, W, is relatively large, without essential loss of generality, we assume the packets overlap over multiple symbols of each packet.[2] Therefore, at least one of the symbols of packet 1 completely overlaps with symbols of packet 2 throughout its symbol duration [1]. The offset of the symbols corresponding to the two packets is illustrated in Fig. 1, where $k = 0, 1, 2, \ldots$ is a nonnegative integer and $0 \leq \alpha, \beta < 1$. The packet lengths of packet 1 and 2 are $L_1 = WT_{s1}$ and $L_2 = WT_{s2}$, respectively. Define $N_W = WN_0$. Under the assumptions of BPSK modulation and rectangular symbol waveform, the SINR of the corresponding symbol matched filter output of packet 1 is given by[3]

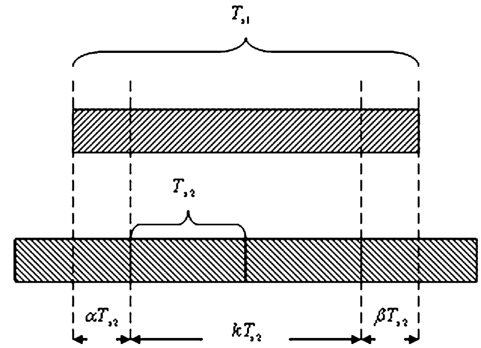

Fig. 1. Illustration of two interfering packets. $T_{s1} = (k+\alpha+\beta)T_{s2}$, $k = 0, 1, 2, \ldots$ is a nonnegative integer, $0 \geq \alpha, \beta < 1$.

$$\text{SINR}_1 = \frac{P_1L_1}{\frac{k+\alpha^2+\beta^2}{k+\alpha+\beta}P_2L_2 + N_W}. \tag{5}$$

When $L_1 < L_2$, we have [1]

$$\text{SINR}_1 \leq \frac{P_1L_1}{\frac{1}{2}P_2L_1 + N_W}. \tag{6}$$

When $L_1 \geq L_2$, we have

$$\begin{aligned}\text{SINR}_1 &= \frac{P_1L_1}{\left(1 - \frac{\alpha(1-\alpha)+\beta(1-\beta)}{k+\alpha+\beta}\right)P_2L_2 + N_W} \\ &\leq \frac{P_1L_1}{\left(1 - \frac{L_2}{2L_1}\right)P_2L_2 + N_W}.\end{aligned} \tag{7}$$

In the optimistic model, we assume that interferences from multiple overlapping packets do not have additive effects to the packet of interest. A packet P_i is successfully received if the following two requirements are satisfied.

1. The power P_i and packet length L_i satisfy

$$\frac{P_iL_i}{N_W} \geq T. \tag{8}$$

2. For all the packets P_j that overlap with packet P_i, we have

$$\begin{cases} \frac{P_iL_i}{\frac{1}{2}P_jL_i+N_W} \geq T, & \text{if } L_i < L_j \\ \frac{P_iL_i}{\left(1-\frac{L_j}{2L_i}\right)P_jL_j+N_W} \geq T, & \text{if } L_i \geq L_j. \end{cases} \tag{9}$$

If power level P_i and power level P_j satisfy requirements 1 and 2, we say "P_i survives P_j."

Next, we divide the power levels into several groups. Suppose the system has M power levels. We assign power levels to groups via the following procedure.

[2]Such an assumption might not hold when the symbol length of a packet is larger than the whole packet length of the other packet. However, in this paper, we do not consider such an extreme case in the multiple power level system design.

[3]The detailed derivation of the SINR can be found in [1, Sec. V].

Group Assignment:

1. Initialize power level index $i = 1$, group index $k = 1$. Assign power level P_1 to group g_1.
2. Let $i = i + 1$, if $i > M$, stop the group assignment.
3. If power levels P_{i-1}, P_i satisfy

$$\frac{P_{i-1}L_{i-1}}{\frac{1}{2}P_iL_{i-1} + N_W} \geq T \tag{10}$$

increase the group index by $k = k + 1$, and assign power level P_i to group g_k.

4. If power levels P_{i-1}, P_i do not satisfy inequality (10), assign power level P_i to group g_k.
5. Goto Step 2.

By Assumption 3 in Section II, P_i survives P_{i+1}. If P_i and P_{i+1} belong to the same group, we must have $P_i > P_{i+1}$, $L_i \geq L_{i+1}$. Consequently, for any $j > i$, if P_i, P_j belong to the same group, we also have $P_i > P_j$, $L_i \geq L_j$. Suppose two packets of power levels P_i, P_j $(i < j)$ overlap at the receiver, no matter if they belong to the same group or not, according to the optimistic system model, it is easily seen that P_i will be received successfully. If P_i and P_j belong to the same group, as long as $T \geq 2$, the packet at power level P_j cannot satisfy the SINR requirement. However, if the two power levels do not belong to the same group, it is possible that P_j also survives P_i. To obtain an upper bound to the system throughput, we assume that as long as P_i, P_j do not belong to the same group, they can always be captured simultaneously. Note that we are free to make such an assumption since we are constructing an optimistic system for upper-bounding purposes.

Since the offered traffic on the system is Poisson with arrival rate G (in packets per second), the offered traffic at any power level P_i is also Poisson with arrival rate $G_i = q_iG$[1]. We denote the offered traffic vector $(G_1, G_2, \ldots, G_M)$ by $\boldsymbol{G}$. Denote the group that contains P_i by $g^{(i)}$, the throughput of packets at power level P_i is upper-bounded by

$$\begin{aligned} S_i(\boldsymbol{G}) &\leq G_i \exp(-G_iL_i) \prod_{\substack{P_j \in g^{(i)} \\ j<i}} \exp(-G_jL_j) \\ &= \hat{S}_i(\boldsymbol{G}). \end{aligned} \tag{11}$$

Consequently, the overall system throughput is upper-bounded by

$$S(\boldsymbol{G}) \leq \sum_{i=1}^{M} \hat{S}_i(\boldsymbol{G}) = \hat{S}(\boldsymbol{G}). \tag{12}$$

It is important to note that, although $\hat{S}(\boldsymbol{G})$ is an upper bound to the system throughput of a multiple-power-level system due the optimistic assumptions made in this section, it becomes the *actual* throughput if the system has only a single power level.

V. Conclusion

The results in [1] are extended in this paper. We showed that, even when the receiver has multiple-packet-reception capability and implements a pure SINR power capture, the single-power-level system still achieves optimal system throughput under a condition on the decodability threshold. Given a minimum throughput requirement, the single-power-level system also achieves the maximum packet capture probability and the optimal energy usage efficiency, under the same condition on the decodability threshold. Considering the channel bandwidth assumption 8 in Section II, the results in this paper support the understanding that an efficient exploitation of the channel bandwidth takes higher priority than exploiting the advantage of the power capture.

Appendix A
Proof of Proposition 1

A. Proof

The proof contains four sections. Sections A.1 and A.2 prove result 1 of Proposition 1. Sections A.3 and A.4 prove results 2 and 3 of the proposition, respectively.

Here we describe the main structure of the proof of result 1. The idea is similar to that of [1]. Based on the optimistic system model presented in Section IV, which is a revised version of the one given in [1], the power levels are divided into several groups. We assume that the packets from different groups can be received simultaneously. Based on these optimistic assumptions, an upper bound to the system throughput of a multiple-power-level system is given in Section IV. Suppose an M-power-level system, system Ω, contains $m < M$ groups. There must be at least one group that contains more than one power levels. We pick one such group, say g_k in Ω, and construct an $(M - 1)$-power-level system, called Θ, by combining the last two power levels in group g_k of Ω into one power level in Θ. We show that the maximum value on the throughput bound of system Θ is no less than that of system Ω. The construction is performed iteratively, until we get an m-power-level system $\tilde{\Omega}$, which contains m groups and every group contains only one power level. Next, we show that the maximum throughput of the single-power-level system described in Proposition 1 is higher than or equal to the throughput upper bound of $\tilde{\Omega}$. Result 1 in the proposition then follows.

1) Proof of Result 1—Part 1: Consider an M-power-level system Ω. Assume that, according to the group assignment procedure, the M power levels in Ω are divided into $m < M$ groups. Suppose the throughput bound $\hat{S}^{\{\Omega\}}$ is maximized by traffic vector $\boldsymbol{G}^{\{\Omega\}}$, i.e.,

$$\boldsymbol{G}^{\{\Omega\}} = \arg\max_{\boldsymbol{G}} \hat{S}^{\{\Omega\}}(\boldsymbol{G}). \tag{13}$$

Since $m < M$, we can find a group g_k that contains at least two power levels. Assume that the last two power levels in g_k are P_i, P_{i+1}. In the rest of the proof, we will construct an $(M - 1)$-power-level system Θ by combining power levels P_i, P_{i+1} into one. We will also construct a traffic vector $\boldsymbol{G}^{\{\Theta\}}$ for system Θ, and show that

$$\max_{\boldsymbol{G}} \hat{S}^{\{\Theta\}}(\boldsymbol{G}) \geq \hat{S}^{\{\Theta\}}(\boldsymbol{G}^{\{\Theta\}}) \geq \max_{\boldsymbol{G}} \hat{S}^{\{\Omega\}}(\boldsymbol{G}). \tag{14}$$

To simplify the notation, we omit the superscripts on the parameters of Ω.

The $(M - 1)$-power-level system Θ, and the system traffic $\boldsymbol{G}^{\{\Theta\}}$ are constructed as follows. The parameters

$(G_j^{\{\Theta\}}, P_j^{\{\Theta\}}, L_j^{\{\Theta\}})$ for $j < i$ in system Θ are the same as (G_j, P_j, L_j) in system Ω. Construct the parameters of power level $P_i^{\{\Theta\}}$ as

$$\begin{cases} P_i^{\{\Theta\}} = P_i \\ L_i^{\{\Theta\}} = \frac{TN_W}{P_i - \frac{T}{2}P_{i+2}} \\ G_i^{\{\Theta\}} = \frac{P_i - \frac{T}{2}P_{i+2}}{TN_W}. \end{cases} \quad (15)$$

The parameters $(G_j^{\{\Theta\}}, P_j^{\{\Theta\}}, L_j^{\{\Theta\}})$ for $j > i$ in system Θ are the same as $(G_{j-1}, P_{j-1}, L_{j-1})$ in system Ω.[4] It can be seen that Θ still contains m groups, where

$$\begin{cases} P_j^{\{\Theta\}} \in g_k^{\{\Theta\}}, & \text{if } j \le i \text{ and } P_j \in g_k \text{ in } \Omega \\ P_j^{\{\Theta\}} \in g_k^{\{\Theta\}}, & \text{if } j > i \text{ and } P_{j+1} \in g_k \text{ in } \Omega. \end{cases} \quad (16)$$

From the definition of the throughput bound in (11) and (12), to have $\hat{S}^{\{\Theta\}}(\boldsymbol{G}^{\{\Theta\}}) \ge \hat{S}(\boldsymbol{G})$, it suffices to show

$$G_i^{\{\Theta\}} \exp\left(-G_i^{\{\Theta\}} L_i^{\{\Theta\}}\right) \ge \max_{G_i, G_{i+1}} \exp(-G_i L_i)(G_i + G_{i+1}\exp(-G_{i+1}L_{i+1})). \quad (17)$$

We first consider the situation when $\frac{L_i}{L_{i+1}} \ge e$. In such a case, inequality (17) becomes

$$\frac{\exp(-1)}{L_i^{\{\Theta\}}} \ge \frac{\exp(-1)}{L_{i+1}}. \quad (18)$$

Since P_{i+1} and P_{i+2} belong to different groups, according to the group assignment criterion, we have

$$P_i L_{i+1} \ge P_{i+1} L_{i+1} \ge \frac{T}{2} P_{i+2} L_{i+1} + TN_W. \quad (19)$$

Hence,

$$\frac{\exp(-1)}{L_i^{\{\Theta\}}} = \frac{\exp(-1)\left(P_i - \frac{T}{2}P_{i+2}\right)}{TN_W} \ge \frac{\exp(-1)}{L_{i+1}}. \quad (20)$$

When $1 \le \frac{L_i}{L_{i+1}} < e$, inequality (17) becomes

$$\frac{\exp(-1)}{L_i^{\{\Theta\}}} \ge \frac{\exp(-1)\exp\left(\frac{L_i}{L_{i+1}}\exp(-1)\right)}{L_i} \quad (21)$$

which is equivalent to

$$\frac{L_i P_i \left(1 - \frac{T}{2}\frac{P_{i+2}}{P_i}\right)}{TN_W} \ge \exp\left(\frac{L_i}{L_{i+1}}\exp(-1)\right). \quad (22)$$

Since $L_i \ge L_{i+1}$ and P_i survives P_{i+1} we have

$$P_i L_i \ge T P_{i+1} L_{i+1}\left(1 - \frac{L_{i+1}}{2L_i}\right) + TN_W. \quad (23)$$

From (19), we obtain

$$\frac{P_{i+2}}{P_i} \le \frac{P_{i+2}}{P_{i+1}} \le \frac{2}{T}\left(1 - \frac{TN_W}{P_{i+1}L_{i+1}}\right). \quad (24)$$

[4]Here we assume $i < M - 1$. When $i = M - 1$, we can simply assume $P_{i+2} = 0$ in system Ω and follow the same proof.

Therefore, from (23) and (24), (22) will hold if the following inequality holds:

$$T\left(1 - \frac{L_{i+1}}{2L_i}\right) + \frac{TN_W}{P_{i+1}L_{i+1}} \ge \exp\left(\frac{L_i}{L_{i+1}}\exp(-1)\right). \quad (25)$$

Define $x = \frac{L_i}{L_{i+1}} \in [1, e)$. To satisfy (25), it suffices to have

$$T \ge \frac{\exp(x\exp(-1))}{1 - \frac{1}{2x}}. \quad (26)$$

Since the right-hand side of (26) is maximized at $x = e$, (26) holds when $T \ge \frac{2e^2}{2e-1}$.

In the preceding discussion, we showed that

$$\max_{\boldsymbol{G}} \hat{S}^{\{\Theta\}}(\boldsymbol{G}) \ge \max_{\boldsymbol{G}} \hat{S}^{\{\Omega\}}(\boldsymbol{G}).$$

Since we can continue to perform the construction iteratively, there exists an m-power-level system, system $\tilde{\Omega}$, where each group in $\tilde{\Omega}$ has only one power level, and

$$\max_{\boldsymbol{G}} \hat{S}^{\{\tilde{\Omega}\}}(\boldsymbol{G}) \ge \max_{\boldsymbol{G}} \hat{S}^{\{\Omega\}}(\boldsymbol{G}). \quad (27)$$

2) Proof of Result 1—Part 2: Now consider an m-power-level system $\tilde{\Omega}$. Assume that, according to the group assignment procedure, the system has exactly m groups and each group contains only one power level. According to the optimistic assumption in Section IV, any packets from different groups can be captured simultaneously. We have

$$\max_{\boldsymbol{G}} \hat{S}(\boldsymbol{G}) = \sum_{i=1}^{m} \frac{\exp(-1)}{L_i}. \quad (28)$$

Now, denote the single-power system where all nodes transmit with $P_{\max}$ and packet length $\frac{TN_W}{P_{\max}}$ by Θ. The maximum throughput of Θ is

$$\max_{G^{\{\Theta\}}} S^{\{\Theta\}}(G^{\{\Theta\}}) = \frac{P_{\max}\exp(-1)}{TN_W}. \quad (29)$$

According to the group assignment criterion, since each group in system $\tilde{\Omega}$ has only one power level, we have

$$\begin{cases} P_i L_i \ge \frac{T}{2} P_{i+1} L_i + TN_W, & i < m \\ P_i L_i \ge TN_W, & i = m \end{cases} \quad (30)$$

where we omitted the superscripts on the parameters of $\tilde{\Omega}$. When $T \ge 2$, we have

$$\begin{aligned} \frac{P_{\max}}{TN_W} &\ge \frac{P_1}{TN_W} \\ &= \sum_{i=1}^{m-1}\left(\frac{T}{2}\right)^{i-1}\frac{P_i - \frac{T}{2}P_{i+1}}{TN_W} + \left(\frac{T}{2}\right)^{m-1}\frac{P_m}{TN_W} \\ &\ge \sum_{i=1}^{m}\frac{1}{L_i} \end{aligned} \quad (31)$$

which gives

$$\max_{G^{\{\Theta\}}} S^{\{\Theta\}}(G^{\{\Theta\}}) \ge \max_{\boldsymbol{G}} \hat{S}(\boldsymbol{G}). \quad (32)$$

3) Proof of Result 2: Suppose an M-power-level system Ω achieves the maximum average packet capture probability with a traffic vector $\boldsymbol{G}$. We again denote the single-power system where all nodes transmit with $P_{\max}$ and packet length $\frac{TN_W}{P_{\max}}$ by Θ. If

$$\sum_{i=1}^{M} G_i \geq \frac{P_{\max}}{TN_W}$$

from result 1, we have

$$\begin{aligned} p_{\max}^{\{\Omega\}}(capture) &\leq \frac{\hat{S}(\boldsymbol{G})}{\sum_{i=1}^{M} G_i} \\ &\leq \frac{P_{\max}\exp(-1)}{TN_W \sum_{i=1}^{M} G_i} \\ &\leq \exp(-1) \\ &\leq p_{\max}^{\{\Theta\}}\ \text{(capture)}. \end{aligned} \tag{33}$$

If $\sum_{i=1}^{M} G_i < \frac{P_{\max}}{TN_W}$, define

$$G^{\{\Theta\}} = \sum_{i=1}^{M} G_i$$

and define $p_i = \frac{G_i}{G^{\{\Theta\}}}$. Construct a traffic vector $\tilde{\boldsymbol{G}}$ such that

$$\tilde{G}_i = p_i \frac{P_{\max}}{TN_W} = \frac{p_i}{L^{\{\Theta\}}}. \tag{34}$$

According to result 1, we have

$$\begin{aligned} \frac{\exp(-1)}{L_i^{\{\Theta\}}} &\geq \max_{\boldsymbol{G}} \hat{S}(\boldsymbol{G}) \geq \hat{S}(\tilde{\boldsymbol{G}}) \\ &= \sum_{i=1}^{M} \tilde{G}_i \exp(-\tilde{G}_i L_i) \prod_{\substack{P_j \in g^{(i)} \\ j<i}} \exp(-\tilde{G}_j L_j) \end{aligned} \tag{35}$$

where $g^{(i)}$ is the group that contains power level P_i in system Ω. Substitute (34) into (35). We obtain

$$\exp(-1) \geq \sum_{i=1}^{M} p_i \exp\left(-\frac{p_i L_i}{L^{\{\Theta\}}}\right) \prod_{\substack{P_j \in g^{(i)} \\ j<i}} \exp\left(-\frac{p_j L_j}{L^{\{\Theta\}}}\right). \tag{36}$$

Define $y = G^{\{\Theta\}} L^{\{\Theta\}}$. Since $y \leq 1$, the function $f(x) = x^y$ is concave for $x \geq 0$, (36) yields

$$\begin{aligned} &[\exp(-1)]^y \\ &\quad \geq \sum_{i=1}^{M} p_i \exp\left(-\frac{p_i L_i y}{L^{\{\Theta\}}}\right) \prod_{\substack{P_j \in g^{(i)} \\ j<i}} \exp\left(-\frac{p_j L_j y}{L^{\{\Theta\}}}\right). \end{aligned} \tag{37}$$

This indicates that $S^{\{\Theta\}}(G^{\{\Theta\}}) \geq \hat{S}(\boldsymbol{G})$. Therefore, the single-power-level system with system traffic $G^{\{\Theta\}}$ satisfies the minimum throughput requirement and

$$p^{\{\Theta\}}\ \text{(capture)} = \frac{S^{\{\Theta\}}(G^{\{\Theta\}})}{G^{\{\Theta\}}} \geq p_{\max}^{\{\Omega\}}\ \text{(capture)}. \tag{38}$$

Hence result 2 of the proposition holds.

4) Proof of Result 3: Suppose there is a multiple-power-level system that maximizes the energy usage efficiency. There are $M > 1$ power levels; the offered traffic at power level P_i is G_i and we denote the offered traffic vector $(G_1, G_2, \ldots, G_M)$ by $\boldsymbol{G}$. The throughput of the system is $S_M(\boldsymbol{G}) \geq S_{\min}$. According to result 1 of Proposition 1, we can find a single-power-level system with transmission power $P_{\max}$ and packet length $\frac{WN_0T}{P_{\max}}$ that achieves the same system throughput S_M with offered traffic G_s, where

$$\frac{WN_0T}{P_{\max}} G_s \leq 1 \quad \text{and} \quad G_s \exp\left(-\frac{WN_0T}{P_{\max}} G_s\right) = S_M.$$

According to result 2 of the proposition, we have

$$\exp\left(-\frac{WN_0T}{P_{\max}} G_s\right) \geq \frac{S_M(\boldsymbol{G})}{\sum_{i=1}^{M} G_i}. \tag{39}$$

Therefore,

$$\frac{\exp\left(-\frac{WN_0T}{P_{\max}} G_s\right)}{WN_0T} \geq \frac{S_M(\boldsymbol{G})}{\sum_{i=1}^{M} G_i WN_0T} > \frac{S_M(\boldsymbol{G})}{\sum_{i=1}^{M} G_i P_i L_i}. \tag{40}$$

This shows that the energy usage efficiency of the single-power-level system is no less than that of the multiple-power-level system. □

Appendix B
Proof of Proposition 3

A. Proof

Since the HP-SINR capture model is considered here, if two packets of power levels P_i and P_j $(i < j)$ overlap at the receiver, the packet at power level P_j will always be lost. Therefore, we redefine the upper bound (11) on the throughput of packet at level P_i by

$$S_i(\boldsymbol{G}) \leq G_i \exp(-G_i L_i) \prod_{j<i} \exp(-G_j L_j) = \hat{S}_i(\boldsymbol{G}). \tag{41}$$

The overall system throughput is upper-bounded by

$$S(\boldsymbol{G}) \leq \sum_{i=1}^{M} \hat{S}_i(\boldsymbol{G}) = \hat{S}(\boldsymbol{G}). \tag{42}$$

We first show that result {1} of Proposition 1 holds for $T \geq \frac{2e(e-1)}{2e-1}$.

Consider an M-power-level system, Ω. Assume that the throughput bound $\hat{S}^{\{\Omega\}}$ is maximized by traffic vector $\boldsymbol{G}^{\{\Omega\}}$, i.e.,

$$\boldsymbol{G}^{\{\Omega\}} = \arg\max_{\boldsymbol{G}} \hat{S}^{\{\Omega\}}(\boldsymbol{G}).$$

We construct an $(M-1)$-power-level system Θ by combining the last two power levels, P_{M-1}, P_M of system Ω into one. The $(M-1)$-power-level system Θ, and the system traffic $\boldsymbol{G}^{\{\Theta\}}$ are constructed as follows. The parameters $(G_j^{\{\Theta\}},\ P_j^{\{\Theta\}},\ L_j^{\{\Theta\}})$ for $j < M-1$ in system Θ are the same as $(G_j,\ P_j,\ L_j)$ in system Ω. Construct the parameters of power level $P_{M-1}^{\{\Theta\}}$ as

$$\begin{cases} P_{M-1}^{\{\Theta\}} = P_{M-1} \\ L_{M-1}^{\{\Theta\}} = \frac{TN_W}{P_{M-1}} \\ G_{M-1}^{\{\Theta\}} = \frac{P_{M-1}}{TN_W} \end{cases} \tag{43}$$

where we omit the superscripts on the parameters of system Ω.

From the definition of the throughput bound in (41)and (42), to have $\hat{S}^{\{\Theta\}}(\boldsymbol{G}^{\{\Theta\}}) \geq \hat{S}(\boldsymbol{G})$, it suffices to show, as in [1], that

$$\begin{aligned} &G_{M-1}^{\{\Theta\}} \exp\left(-G_{M-1}^{\{\Theta\}} L_{M-1}^{\{\Theta\}}\right) \\ &\quad \geq \max_{G_{M-1}, G_M} e^{-G_{M-1}L_{M-1}}(G_{M-1} + G_M e^{-G_M L_M}). \end{aligned} \tag{44}$$

Consider the situation when P_{M-1} and P_M in system Ω belong to the same group. It is easy to show that inequality (44) holds if $\frac{L_{M-1}}{L_M} \geq e$.

In the situation that $1 \leq \frac{L_{M-1}}{L_M} < e$, inequality (44) is equivalent to

$$\frac{P_{M-1}L_{M-1}}{TN_W} \geq \exp\left(\frac{L_{M-1}}{L_M}\exp(-1)\right). \tag{45}$$

Since P_{M-1} and P_M in system Ω belong to the same group, we have

$$\frac{P_{M-1}L_{M-1}}{TN_W} \geq T\frac{P_M L_M}{TN_W}\left(1-\frac{L_M}{L_{M-1}}\right)+1. \tag{46}$$

Therefore, define $x = \frac{L_{M-1}}{L_M} \in [1, e)$; to satisfy (45), it suffices to have

$$T \geq \frac{\exp\left(x\exp(-1)\right)-1}{1-\frac{1}{2x}} \geq \frac{2e(e-1)}{2e-1}. \tag{47}$$

In the situation when P_{M-1} and P_M in system Ω belong to the different groups, we have

$$P_{M-1}L_{M-1} \geq \frac{T}{2}P_M L_{M-1} + TN_W. \tag{48}$$

Therefore, when $T \geq 2$

$$\begin{aligned} & G_{M-1}^{\{\Theta\}}\exp\left(-G_{M-1}^{\{\Theta\}}L_{M-1}^{\{\Theta\}}\right) \\ &\quad = \frac{P_{M-1}\exp(-1)}{TN_W} \\ &\quad = \frac{\left(P_{M-1}-\frac{T}{2}P_M\right)\exp(-1)}{TN_W} + \frac{\frac{T}{2}P_M\exp(-1)}{TN_W} \\ &\quad \geq \frac{\exp(-1)}{L_{M-1}} + \frac{\exp(-1)}{L_M} \\ &\quad \geq \max_{G_{M-1},G_M} e^{-G_{M-1}L_{M-1}}(G_{M-1}+G_M e^{-G_M L_M}). \end{aligned} \tag{49}$$

The above analysis shows that

$$\max_{\boldsymbol{G}} \hat{S}^{\{\Theta\}}(\boldsymbol{G}) \geq \hat{S}^{\{\Theta\}}(\boldsymbol{G}^{\{\Theta\}}) \geq \max_{\boldsymbol{G}} \hat{S}^{\{\Omega\}}(\boldsymbol{G}). \tag{50}$$

It is then easily seen that result 1 in Proposition 1 holds for $T \geq \frac{2e(e-1)}{2e-1}$.

As long as result 1 has been proven, results 2 and 3 of Proposition 1 can be shown by following the proofs in Sections A.3 and A.4, respectively, with only minor revisions. □

References

[1] W. Luo and A. Ephremides, "Power levels and packet lengths in random multiple access," *IEEE Trans. Inf. Theory*, vol. 48, no. 1, pp. 46–58, Jan. 2002.

[2] J. Metzner, "On improving utilization in ALOHA networks," *IEEE Trans. Commun.*, vol. COM-24, no. 4, pp. 447–448, Apr. 1976.

[3] C. Lau and C. Leung, "Capture models for mobile packet radio networks," *IEEE Trans. Commun.*, vol. 40, no. 3, pp. 917–925, May 1992.

[4] R. LaMaire, A. Krishna, and M. Zorzi, "On the use of transmitter power variations to increase throughput in multiple access radio systems," *Wireless Netw.*, vol. 4, pp. 263–277, Jun. 1998.

[5] L. Roberts, "ALOHA packet system with and without slots and capture," *Comput. Commun. Rev.*, pp. 28–42, Apr. 1975.

[6] N. Abramson, "The throughput of packet broadcasting channels," *IEEE Trans. Commun.*, vol. COM-25, no. 1, pp. 117–128, Jan. 1977.

[7] C. Lee, "Random signal levels for channel access in packet broadcast networks," *IEEE J. Sel. Areas Commun.*, vol. 5, no. 6, pp. 1026–1034, Jul. 1987.

[8] R. LaMaire, A. Krishna, and M. Zorzi, "Optimization of capture in multiple access radio systems with Rayleigh fading and random power levels," in *Multiaccess, Mobility and Teletraffic for Personal Communications*, B. Jabbari, P. Godlewski, and X. Lagrange, Eds. Norwell, MA: Kluwer, 1996, pp. 321–336.

[9] Y. Leung, "Mean power consumption of artificial power capture in wireless networks," *IEEE Trans. Commun.*, vol. 45, no. 8, pp. 957–964, Aug. 1997.

[10] C. Ware, J. Chicharo, and T. Wysocki, "Simulation of capture behavior in IEEE 802.11 radio modems," in *Proc. IEEE Vehicula Technology Conf 2001*, vol. 3, Rhodes, Greece, May 2001, pp. 1393–1397.

[11] D. Bertsekas and R. Gallager, *Data Networks*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1992.