Further Developments on Power Levels and Packet Lengths in Random Multiple Access

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***Abstract*— This paper extends the results in [1]. We assume that the receiver has the capability of capturing multiple packets as long as the signal-to-interference-plus-noise ratio (SINR) of each packet is above a designed threshold T throughout its transmission period. We prove that, compared with a multiple-power-level system, the single-power-level system in which all nodes transmit at the maximum allowable power level achieves optimal throughput; given a minimum throughput requirement, the single-power-level system also achieves the maximum average packet capture probability as well as optimal energy usage efficiency; both under a condition that T exceeds the value 3.44. In a special case when the power levels and packet lengths of a multiple-power-level system are constrained such that the higher power level always has shorter packet length, then all the results hold for T greater than 2.**

***Index Terms*— ALOHA, capture, random multiple access, energy efficiency.**

I. INTRODUCTION

IN distributed random access, nodes transmit packets in an uncoordinated fashion. For a successful packet reception, the signal-to-interference-plus-noise ratio (SINR) of the packet should be above a designed threshold T throughout its transmission period. Usually, for a reasonable T ($T \geq 2$ for example), when two packets of the same power overlap at the receiver, the SINR requirement of both packets will be violated. In such a situation, both of the packets get lost. If two packets arrive at the receiver with different powers, however, it is possible that the SINR requirement of the high power packet is still satisfied, and hence the packet is received successfully. Such a phenomenon is called power capture. In wireless networks, in order to take the advantage of power capture, it is proposed in [2][3] that packets can be transmitted at multiple discrete power levels, and the packet received at the highest power may be captured. In a multiple-power-level system, there are M discrete power levels. Each packet is transmitted at a power level that is independently and randomly chosen from the M power levels.

Various power capture models have been studied in the literature. In the perfect capture model, a packet is successfully received if and only if the ratio of the power of the packet to that of any interferencing packet is higher than a fixed value [3][4]. Although the model is sometime overly optimistic, its use [5][6][7] often leads to simpler analytic derivations. A more accurate model is based on the assumption that a packet is captured if and only if its SINR is greater than a certain decodability threshold [3][4][8][9][10]. This is called the SINR capture model. In all the above works, packet lengths at different power levels are identical. Throughput of the system is measured by the average number of successfully received packets per slot. It was shown that the use of multiple power levels increases the throughput of the system [1].

When the packet lengths of different power levels are different, it is possible that multiple packets can satisfy the SINR requirement simultaneously. The capture model in such a situation depends on the receiver design. If the receiver is not designed to provide multiple-packet-reception capability, it is suggested that the receiver can lock to the packet with the highest power [1]. Such a design can be viewed as an approximation to the physical operation of an IEEE 802.11 radio modem [12]. We call this the Highest-Power-SINR (HP-SINR) capture model. It is observed in [1] that for a packet transmitted at a higher power level, the length of the packet can be reduced since the required SINR can be met through transmission at correspondingly increased rate. The effect of the use of multiple transmission power levels and of the corresponding packet lengths on the system throughput and energy usage efficiency was studied in [1]. The throughput is measured by the average number of successfully received packets per second. The energy usage efficiency is measured by the system throughput divided by the system average power consumption, as defined in [11]. It is proved in [1] that under the HP-SINR capture model, the single-power-level system in which all nodes transmit at the maximum allowable power level achieves both optimal throughput and energy usage efficiency under a condition on the decodability threshold value.

Although the HP-SINR capture model is derived from the operation of current radio modems, with an advanced hardware design, it is possible that future radio modems will be able to capture multiple packets simultaneously. In this paper, we consider the SINR capture model where

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the only requirement for a successful packet capture is that the SINR of the packet should be higher than or equal to a designed threshold T. We prove that, when $T \geq \frac{2e^2}{2e-1} \approx 3.44$, the single-power-level system achieves the optimal throughput. Given a throughput requirement, the single-power-level system also achieves the maximum average packet capture probability as well as optimal energy usage efficiency, when $T \geq \frac{2e^2}{2e-1}$. In the special case when the power levels and packet lengths in a multiple-power-level system are constrained such that the higher power level always has shorter packet length, then all the results hold for $T \geq 2$. Furthermore, when the receiver does not have multiple-packet-capture capability and the HP-SINR capture model is implemented, the results of this paper hold for $T \geq \frac{2e(e-1)}{2e-1} \approx 2.11$.

II. System Model

Suppose there is an infinite number of bufferless nodes in the system [1][14]. A global clock is available to all the nodes such that slotted transmission can be achieved. There are M discrete power levels in the system. The values of the power levels and packet lengths are determined in the system deployment and fixed once the system is in use. For each packet, a node randomly pick a power level from the M power levels and transmit the packet at that power level. Nodes chose power levels randomly and independently for each packet. Time is slotted at each power level, and the slot duration equals to the packet length of the corresponding power level. At each power level, packets transmissions start only at the slot edges. However, since the packet lengths of different power levels may be different, the slot durations at different power levels may also be different. Therefore, we do not have slot synchronization between different power levels. We assume that, if a packet is lost due to collision, it is retransmitted at a later time. The overall packet arrival pattern including the new arrivals and retransmission is Possion [14].

The system throughput, average packet capture probability and energy usage efficiency used in this paper are defined as follows.

Definitions:

1) The *system throughput* is measured by the average number of successfully received packets per second.
2) The *average packet capture probability* is measured by $\frac{\text{system throughput}}{\text{offered traffic rate in packets per second}}$.
3) The *energy usage efficiency* is measured by $\frac{\text{system throughput}}{\text{system average power consumption}}$.

As in [1], we make the following assumptions.

Assumptions:

1) Each packet contains W symbols.
2) A packet is received correctly if and only if during the whole transmission period, the packet's SINR is *always* larger than a designed threshold T. The SINR is defined as symbol energy to interference plus noise ratio,
$$\text{SINR} = \frac{PT_s}{I + N_0} \geq T \qquad (1)$$
where P, T_s are the transmission power and symbol duration of the packet, I and N_0 are the interference energy and spectral density of the background noise in the output of the symbol matched filter. There is no coding and each symbol is detected individually.
3) There are M discrete power levels $P_1 > P_2 > \ldots > P_M$. The power levels and packet lengths are designed such that when a packet of level P_i overlaps with a packet of level P_{i+1} at the receiver, there is a positive probability that the SINR requirement of the packet at power level P_i can be satisfied.
4) BPSK modulation scheme is used.
5) The transmission power of each packet is constant during the transmission period.
6) The probability of a packet transmitted at power level P_i is q_i with $\sum_{i=1}^{M} q_i = 1$. We assume these probabilities are determined at the system deployment and fixed once the system is in use.
7) The distance between the transmitters and the receiver are equal and the transmission medium is isotropic.
8) The maximum power that a packet can use is P_{max}.
9) The offered traffic on the system, including new arrivals and retransmissions, is Poisson.

Further explanations on the assumptions can be found in [1].

III. Optimality of Single-Power-Level System

Based on the definitions and assumptions presented in section II, we have

Proposition 1: Given a peak power constraint P_{max}, if the decodability threshold T satisfies $T \geq \frac{2e^2}{2e-1} \approx 3.44$, the single-power-level system where all nodes transmit at P_{max} with packet length $L = \frac{WN_0T}{P_{max}}$ achieves maximum possible throughput, which is

$$S_{max} = \frac{P_{max}\exp(-1)}{WN_0T} \qquad (2)$$

The detailed proof can be found in [15]. Here we describe the main structure of the proof. The idea is similar to that of [1]. We first present an optimistic power capture model, which is a revised version of that in [1]. The power levels are then divided into several groups. We assume that the packets from different groups can be received simultaneously. Based on this assumption, an upper bound on the system throughput of a multiple-power-level system is given. Suppose a M-power-level system, system Ω, contains $m < M$ groups. There must be at least one group that contains more than one power levels. We pick one such group, say g_k in Ω, and construct a $(M-1)$-power-level system, called Θ, by combining the last two power levels in group g_k of Ω into one power level in Θ. We show that the maximum value on the throughput bound of system Θ is no less than that of system Ω. The construction is performed iteratively, till we get a m-power-level system $\tilde{\Omega}$. System $\tilde{\Omega}$ contains m groups and every group contains only one power level. Next, we show that the maximum throughput of the single-power-level system described in

Proposition 1 is higher than or equal to the throughput upper bound of $\tilde{\Omega}$. The result in the Proposition then follows.

Proposition 2: Under the same assumptions as in Proposition 1, given a minimum throughput requirement $S \geq S_{min}$ (assume that the throughput requirement is achievable), the single-power-level system where all nodes transmit at P_{max} with packet length $L = \frac{WN_0T}{P_{max}}$ achieves maximum average packet capture probability, which is

$$p_{max}(\text{capture}) = \frac{S_{min}}{G} \tag{3}$$

where G is the offered traffic on the system that satisfies $\frac{WN_0T}{P_{max}}G \leq 1$ and $G\exp(-\frac{WN_0T}{P_{max}}G) = S_{min}$.

The detailed proof can be found in [15].

Proposition 3: Under the same assumptions as in Proposition 1, given a minimum throughput requirement $S \geq S_{min}$ (assume that the throughput requirement is achievable), the single-power-level system where all nodes transmit at P_{max} with packet length $L = \frac{WN_0T}{P_{max}}$ achieves maximum power usage efficiency, which is

$$\text{efficiency}_{max} = \frac{S_{min}}{GWN_0T} \tag{4}$$

where $\frac{WN_0T}{P_{max}}G \leq 1$ and $G\exp(-\frac{WN_0T}{P_{max}}G) = S_{min}$.

Proof: Suppose there is a multiple-power-level system that maximizes the power usage efficiency. There are $M > 1$ power levels, the offered traffic at power level P_i is G_i and we denote the offered traffic vector $(G_1, G_2, \ldots, G_M)$ by $\boldsymbol{G}$. The throughput of the system is $S_M(\boldsymbol{G}) \geq S_{min}$. According to Proposition 1, we can find a single-power-level system with transmission power P_{max} and packet length $\frac{WN_0T}{P_{max}}$ that achieves the same system throughput S_M with offered traffic G_s, where $\frac{WN_0T}{P_{max}}G_s \leq 1$ and $G_s\exp(-\frac{WN_0T}{P_{max}}G_s) = S_M$. According to Proposition 2, we have

$$\exp\left(-\frac{WN_0T}{P_{max}}G_s\right) \geq \frac{S_M(\boldsymbol{G})}{\sum_{i=1}^{M} G_i} \tag{5}$$

Therefore,

$$\frac{\exp\left(-\frac{WN_0T}{P_{max}}G_s\right)}{WN_0T} \geq \frac{S_M(\boldsymbol{G})}{\sum_{i=1}^{M} G_i WN_0T} > \frac{S_M(\boldsymbol{G})}{\sum_{i=1}^{M} G_i P_i L_i} \tag{6}$$

This shows that the energy usage efficiency of the single-power-level system is no less than that of the multiple-power-level system. ♢

Furthermore, in the special case when the M-power-level system satisfy $L_1 \leq L_2 \leq \ldots \leq L_M$, we also have

Proposition 4: Under the assumption that the M-power-level system satisfies $L_1 \leq L_2 \leq \ldots \leq L_M$, given a peak power constraint P_{max}, the results in Propositions 1, 2, 3 hold for $T \geq 2$.

The detailed proof can be found in [15].

In the above propositions, we considered the SINR capture model where multiple-packet-capture is possible at the receiver. As a side product of the analysis, if the receiver cannot capture multiple packets simultaneously and uses the HP-SINR capture model, we have

Proposition 5: Under the assumptions in section II and assume that the HP-SINR capture model (see [1] for details) is implemented at the receiver, the results in Propositions 1, 2 and 3 hold for $T \geq \frac{2e(e-1)}{2e-1} \approx 2.11$.

The proof is presented in [15] and improves the results in [1].

IV. CONCLUSIONS

When the receiver has multi-packet reception capability and implements a pure SINR power capture, the single-power-level system in which all nodes transmit at the maximum allowable power level achieves optimal system throughput under a condition on the decodability threshold. Given a minimum throughput requirement, the single-power-level system also achieves the maximum packet capture probability and optimal energy usage efficiency, under the same condition on the decodability threshold.

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