IMPROVED MULTIUSER DETECTION IN CODE-DIVISION MULTIPLE ACCESS COMMUNICATIONS

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In Code Division Multiple Access (CDMA) communications, Multiuser Detection (MUD), that reduces the multi-access interference (MAI) and that solves the near-far problem, has been widely studied for over 15 years. Since optimal multiuser detection is generally NP hard, many sub-optimal algorithms that provide reliable performance and ensure polynomial complexity have been proposed. However, there is still a large gap between the performance of the sub-optimal detectors and that of the optimal detector. Due to the advances in the hardware computational speeds, advanced MUD algorithms that achieve near-optimal performance, while maintaining high computational efficiency, are of special interest to both researchers and industry.

The main objective of this research is to improve the existing multiuser detectors and propose new advanced near-optimal and optimal detectors.

In our research, we improve the performance of the Group Decision Feedback (GDF) detector by finding the optimal user partitioning and ordering. We solve the time labeling issue in asynchronous CDMA and improve the performance of the DF detector by finding the optimal time labeling and user ordering.

Based on the user ordering and Branch-and-Bound search, we improve the computational efficiency of the optimal detector by proposing a fast optimal algorithm that significantly reduces the average computational cost.

In addition, we also develop a new multiuser detection algorithm based on the idea of Probabilistic Data Association (PDA) from target tracking. The PDA detector achieves near-optimal performance in both synchronous and asynchronous systems with $O(K^3)$ complexity where K is the number of users. The situation of overloaded system in both synchronous and asynchronous cases are also studied. The soft-output feature of the PDA method makes it extremely flexible and easy to extend to multiuser detection problems in a wide variety of communication settings.

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Chapter 1

Introduction

1.1 Background

The demand for personal communication services (PCS), especially those related to wireless communications, has been increasing dramatically during the past two decades. With the goals of improving service quality and increasing system capacity, wireless communication systems tend to utilize advanced channel access technologies. The commonly considered channel access schemes include Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA), and Code Division Multiple Access (CDMA). Unlike FDMA, which transmits signals over carriers with separated frequencies, and TDMA, which transmits signals to the receiver through separated time slots, CDMA identifies channels by the user-specific signature waveforms (signature sequences). When the correlations between user signatures are zero, CDMA can be treated as a generalized orthogonal channel access scheme. However, theoretical analysis showed that CDMA with non-orthogonal signature design gives a higher channel capacity than that of TDMA and FDMA over the same communication bandwidth [Gh91]. In mobile communications, due to the voice activity gain, the antenna sectorization gain and the frequency reuse gain, the capacity of CDMA can be four times that of TDMA and twenty times that of FDMA [Gh91]. Aside from the theoretical advantage in capacity, CDMA also has several other desirable features in wireless communications [Vt94] [Wx96].

Nevertheless, with a non-orthogonal signature design, interference between user signals is introduced due to the correlation among signatures. This is called the Multi-Access Interference (MAI). Generally, the interference from one user to another is proportional to the signatures correlation as well as the powers of the signals. Especially when user signal powers differ significantly from one another, the MAI from strong users to weak users can be very serious. This is known as the "near-far" problem. In mobile channels, reflectors in wireless channel generate "multipath", and the movement of a mobile results in a "Doppler frequency shift"; these together are termed "channel fading" and they further aggravate the "near-far" problem in CDMA.

Multiuser Detection (MUD) algorithms that reduce the interference and combat the near-far problem have been hotly discussed from mid 1980's [Vd85] [Vd83] [Vd86]. Due to the variety of communication environments, multiuser detection problems assume different forms in different applications. In cellular communications, the up-link communications (from mobile to base station) are often characterized as asynchronous CDMA since the mobiles usually send their signals to the base station with their bit intervals unaligned. The reverse-link communications (from base station to mobile), however, are characterized as synchronous CDMA since it is feasible for the base station to send user signals synchronously. In both situations, when the communication rate is high and the mobile is moving fast, channel fading will occur due to environmental changes and the effect of Doppler shift. Although the multiuser detection problem for synchronous CDMA without channel fading is relatively simple to formulate, optimal detection has been shown to be generally NP hard, meaning that it cannot be solved with a computational effort that scales polynomially with the number of users [LV89]. It is thus unlikely to be implemented in practice. For this reason, prior research has focused on developing reliable sub-optimal methods with polynomial complexity.

One of the most popular classes of sub-optimal multiuser detector is the linear detector, which generally applies a linear mapping to the output of the matched filter bank. The decorrelating linear detector, called the decorrelator, is proposed in [Kh83] and extensively analyzed in [LV89]. The decorrelator eliminates the MAI at the expense of increasing background noise power. One of its good features is that the algorithm does not require the powers of the interferers.

Another well known linear detector is the Minimum Mean Square Error (MMSE) based detector, originally proposed in [X90]. The performance of the

MMSE detector approaches that of the decorrelator if the environment is interference limited, and approaches the performance of the conventional detector if the environment is background noise limited.

Linear detectors improve the performance of the conventional matched filter significantly in most of the cases, while their computational complexities are $O(K^2)$ where K is the number of users. The linear structure make these detectors easy to analyze as well as to extend to various CDMA communication environments such as the long-code CDMA [GRL99] where signatures are changing for each time frame and blind CDMA multiuser detection [HMV95] [UY98a] where no information on interferers is available.

Decision driven multiuser detectors form another popular class of suboptimal detectors in CDMA. The multistage detector was originally proposed in [Va90] and extended in [Va91]. This detector makes temporary decisions in the first several stages. The final decision is obtained by canceling the MAI, which is approximated based on the temporary decisions from the previous stages. There are several variations on the MAI cancellation, including the parallel MAI cancellation, the sequential MAI cancellation and the partial MAI cancellation. The sequential multistage detection algorithm can also be interpreted as a first order coordinate descent search [LLPW00].

Decision Feedback (DF) detector was originally proposed in [Dh93], extended in [Dh95] and extensively analyzed in [Va99]. The DF detector makes decisions sequentially on one user at a time with the weak users taking the advantage of the decisions on strong users to cancel out the MAI. Although, usually, the performance of the DF detector is significantly better than the linear detectors, its complexity remains $O(K^2)$ and can be even lower than that of the linear decorrelator [LPWL00]. However, the performance strongly relies on the detection order. If the decisions on weak users are made first, the probability of error will be high because of the strong interference from other users. In addition, canceling the MAI based on wrong decisions on the previously-decided user signals exacerbates the noise in the later-decided signals. Although it is intuitive to order users according to their received power, the optimal user ordering involves not only the received power, but also the correlations among signature waveforms of different users. Optimal user ordering for the DF detector in synchronous CDMA is found in [Va99] and is in fact an extension to the user ordering from [WFGV98].

Group detection was first proposed in [Va95]. Unlike the DF detector, a group detector partitions users into several groups. Users with highly correlated signature waveforms are assigned to the same group and the signature correlations between the groups are relatively low. While the decisions on different groups are made sequentially, the decisions on user signals within a group are made simultaneously by exploring all the possible combinations within the group. Therefore, the sequential group detection, termed the Group Decision Feedback (GDF), can be viewed as a generalization of the DF detector. The complexity of the GDF detector remains $O(K^2)$, but it is exponential in the maximum group size, which is usually small. Similarly to the DF detector, user ordering as well as user partitioning are critical to the performance of the GDF detector.

Among the decision-driven multiuser detectors, the DF detector is computationally most efficient. The probability of error of a DF detector can be 2 orders of magnitude lower than the linear detectors, while the computational cost can be also lower than the decorrelator. However, the performance of the decision driven detector strongly depends on the structure of the correlation matrix and is hard to control in practical situations. Furthermore, there is usually still a large gap between the probability of the DF detector and that of the ML optimal detector.

Due to the increasing demand on bandwidth efficiency in modern communication systems, advanced multiuser detection algorithms that fill the performance gap between the decision-driven multiuser detectors and the ML detector become more attractive. Since the multiuser detection problem is generally a quadratic optimization problem with binary or integer constraints, many advanced methods in binary quadratic programming have been applied to multiuser detection in the past several years. Among them are the coordinate descent methods [LLPW00], the quadratic programming methods with various constraints [HLPW02], semi-definite relaxation [MDWLC00] [TR01], Tabu search [TR02], Boltzmann machine [HLPW02] and genetic algorithm [EH00].

Coordinate descent is a local search method that ensures a local minimum of the final solution [LLPW00]. A good initial point is critical for the performance of the coordinate descent search. Although the first order coordinate descent is a good patch to improve other multiuser detectors, searching more than one coordinate at a time will increase the computational complexity substantially.

Quadratic programming and semidefinite programming methods are relaxation methods that solve a relaxed problem first and then find the final decisions by projecting the result onto the binary set. Quadratic programming methods [HLPW02] relax the binary constrains on user signals to box constraints (they assume signals to take values in $[-1, 1]$) and then solve a quadratic programming problem using a primal-dual method [Bk99]. Similarly, the semidefinite-relaxation method [MDWLC00] [TR01] relaxes the original problem to a semidefinite programming problem and solves it using interior-point methods [VB96]. Although the quadratic programming method is simple, its performance is also moderate. The semidefinite-relaxation method, however, is reported to have close to optimum performance with a complexity of $O(K^{3.5})$. Nevertheless, when compared with other detectors, the computational cost of the semidefinite relaxation method is relatively high even for small-sized problems [LPWF01a].

Tabu search [Gv86], Boltzmann machine [Gd96] and genetic algorithm [Hl75] are global search strategies for solving the binary programming problems. While the main purpose of the optimization is to decrease the cost function, these methods allow search directions, with a relatively small probability, to increase the cost of the objective function. Such strategies help these algorithms to avoid local minima. Tabu search starts from an initial point and searches the binary signal set by changing one signal value at a time. If a signal value is changed in one step, it will be forced to remain the same value in the following several steps. Although the algorithm tries to flip the signal that decreases the cost value, it also forces flips even when no improvement in the cost can be found. Unlike the Tabu search, the Boltzmann machine changes user signal values in a random fashion. The change that decreases the cost is accepted with a large probability while the change increasing the cost is accepted with a relatively small probability. These probabilities are determined by checking the difference between the resulting costs with binary values of a user signal at its two extremes. The probability of accepting a decision that increases cost is also reduced in subsequent iterations. The genetic algorithm, introduced in [EH00], starts with a solution pool. The solutions with low cost values are called "good", while the solutions with high cost values are termed "bad". At each step, the bad solutions are replaced by solutions that are randomly generated from the good solutions by following certain evolution rules. In addition, a one-step coordinate descent search is performed at the end on all the solutions to generate solutions with possibly lower costs.

Although, in general, the optimization algorithms for binary quadratic programming problems can be applied to multiuser detection, multiuser detection problems also differ from general optimization problems in the following ways:

- The parameters in the cost function are not generated arbitrarily. The observation vector yields a statistical model defined by the CDMA system.

- Since the noise in the CDMA system is generally small, in most of the cases, simple multiuser detectors can provide solutions close to optimal.

- Communication systems evaluate the detection results using the probability of error. Therefore, as long as an algorithm can achieve the same probability of error as the optimal detector, it is equivalent to the optimal detector from a communication point of view. In other words, we do not care how close the solution is to the optimal one; what we really are interested is the probability that the solution equals the true signal.

Due to the above differences and in view of the fact that the MUD problem is also a maximum likelihood (ML) estimation problem, several methods have also been proposed from a statistical point of view. The Space Alternating Generalized EM (SAGE) algorithm, proposed in [NP96], is based on the Expectation-Maximization (EM) algorithm [DLR77]. By considering other user signals as missing data, the initial estimation of the user signals can be obtained as a soft-valued decorrelator. Coordinate descent search is added in the following stages to improve its performance. The key idea of soft initialization makes the probability of error of the SAGE algorithm much lower than that of the multistage detector. Another method based on the Gibbs sampling idea [GS90] was proposed in [WC00]. Gibbs sampling is a general method for ML estimation when a closed form of the marginal density of the variables of interest is not available. The advantage of sample-based methods is that they support complicated statistical models since no marginal densities need be computed in the algorithm. However, a major disadvantage of the sample-based methods is their high computational complexity.

Although MUD problem differs from a general optimization problem, it also differs from the general ML estimation problem. The MUD problem must be solved online and very quickly. The issue is not to find the global optimal solution, but to find the best solution given a computational limit. In addition, the resulting probability of error must be very small in order to satisfy the quality of service requirements of practical communication systems. Therefore, combining the optimization methods with statistical ones is desirable and is a major goal of the research in this thesis.

In synchronous CDMA, the outstanding performance of the DF detector relies strongly on intelligent user ordering, which is based on the statistical information from the system model. Motivated by the DF user ordering [Va99], we derive the optimal user partitioning and user ordering for the GDF detector, and significantly enhance its performance. During the research, we also found that the DF solution is actually a first order approximation to the optimal solution. By combining the user ordering idea with an optimal branch-and-bound algorithm for binary quadratic programming, we developed a fast optimal algorithm for multiuser detection in synchronous CDMA. Although the worst case computational cost is still exponential in the number of users, good user ordering helps reduce the average computational cost dramatically. While it is hard for a conventional branch-and-bound method to solve a 30-user problem, the fast optimal algorithm makes it possible to simulate a 60-user problem with $500,000$ Monte-Carlo runs in a reasonable time. In asynchronous CDMA, in addition to the user ordering, we find that the performance of the DF detector is also affected by time labeling. Although mentioned originally in [Dh95], time labeling was not studied extensively in the literature. However, different time labelings do affect the detection sequence of the signals which, in turn, affect the overall performance of the DF detector. In our research we derive the optimal time labeling and user ordering for the ideal DF detector that assumes no error propagation. Our simulations also show that the optimal time labeling and user ordering helps the actual DF detector to achieve its performance lower bound.

In addition to improving the DF detector, the GDF detector and the optimal detector, we also propose a new detection algorithm based on the Probabilistic Data Association (PDA) [BT75] [BL95], an idea that was originally proposed in target tracking. PDA models user signals as binary random variables and approximates the MAI, together with the channel noise, by a single Gaussian with matched mean and covariance. The ML estimation is obtained by iteratively updating the associated probabilities. Although theoretical analysis for the convergence as well as the performance is not available, simulation results show that the probability of error of the PDA detector is very close to, and sometime even indistinguishable from, that of the optimal detector. The computational complexity of the PDA method is $O(K^3)$ and is much lower than other methods with comparable performance. The soft-output feature of the PDA method also makes it extremely flexible and easily extended to more realistic communication settings. In our research, we extend the PDA method to asynchronous CDMA and overloaded systems where the number of users is larger than the signature length. An interesting result is that the performance of the *a*synchronous PDA detector does not degrade significantly when the number of users exceeds the signature length. This is in contrast to synchronous CDMA, where the performance of even the optimal detector degrades dramatically when the system becomes overloaded and when the signature sequences are not very carefully designed.

1.2 Outline of the Thesis

The goal of our research is to improve the performance of the existing multiuser detection methods, and develop a new multiuser detection methods that give high performance with a relatively low computational complexity. Throughout the thesis, we have made the following assumptions:

- The channel is perfectly estimated, i.e., the powers of user signals are known to the receiver.

- The signatures of all active users are known to the receiver.

- The noise before the matched-filter at the receiver is white Gaussian with zero mean. The power spectral density of this white Gaussian noise is known.

- In asynchronous CDMA we assume that the delays of the user signals are known.

Although practical systems may involve noncoherent reception, coding, fading, multipath, intersymbol interference, etc., they are beyond the scope of this research and are not addressed here. However, ideas for dealing with some of these problems are discussed in Chapter 5.

In Chapter 2 we introduce synchronous and asynchronous CDMA system models. Several common multiuser detectors are described and studied. Closed form analyses for the probability of error of the related detectors are given.

In Chapter 3 we study the user ordering of DF detector in synchronous CDMA. As an extension to the DF user ordering, given the maximum group size, the optimum user partitioning and user ordering algorithm for the GDF detector is derived. The time labeling issue for the DF detector in asynchronous CDMA is studied in the last section. The optimal time labeling and user ordering for the ideal DF detector is derived.

In Chapter 4, based on the user ordering and the Branch-and-Bound search, a fast optimum multiuser detection algorithm for synchronous CDMA is introduced and analyzed. As a byproduct, a suboptimal “any-time” algorithms is also proposed.

In Chapter 5 we propose the PDA detector for synchronous CDMA. The algorithm is then modified and extended to the synchronous overloaded system. PDA detection for asynchronous CDMA is studied in the final section of the chapter. Several simulations of the PDA detector in heavily overloaded asynchronous systems are shown. Since there is no theoretical difference in the multiuser detection of asynchronous non-overloaded and overloaded system, asynchronous transmission with designed delay pattern is recommended as an alternative to the synchronous CDMA system for a heavily overloaded system.

The thesis is summarized in Chapter 6 with a brief introduction to some future research directions.

1.3 Major Contributions

1. Derived the optimal user partitioning and ordering for Group Decision Feedback detector in synchronous CDMA given the maximum group size.

2. Proposed a fast optimal algorithm for synchronous CDMA multiuser detection. The average computational cost is significantly reduced in comparison to the method introduced in [PR90]. The fast optimal algorithm makes it

possible to simulate a 60-user detection problem with $500,000$ Monte-Carlo runs in a reasonable time.

3. Obtained the optimal time labeling and user ordering for ideal Decision Feedback detector in asynchronous CDMA.

4. Proposed the Probabilistic Data Association (PDA) detector for synchronous CDMA. Achieved near optimal performance with $O(K^3)$ complexity where K is the number of users.

5. Extended the PDA detector to synchronous overloaded system as well as the asynchronous CDMA multiuser detection. Close to optimal performance is achieved.

1.4 Publications (since 1998)

Related journal papers:

[1] J. Luo, K. Pattipati, P. Willett, F. Hasegawa, "Near-Optimal Multiuser Detection in Synchronous CDMA using Probabilistic Data Association", IEEE Communications Letters, Vol. 5, pp. 361-363, Sept. 2001.

[2] J. Luo, K. Pattipati, P. Willett, "Optimal Grouping Algorithm for a Group Decision Feedback Detector in Synchronous CDMA Communications" accepted for publication in IEEE Trans. on Communications.

[3] J. Luo, G. Levchuk, K. Pattipati, P. Willett, "Fast Optimal and Sub-optimal Any-Time Algorithms for CWMA Multiuser Detection based on Branch and Bound", submitted to IEEE Trans. on Communications, Sept. 2000.

[4] J. Luo, K. Pattipati, P. Willett, F. Hasegawa, "Optimal User Ordering and Time Labeling for Decision Feedback Detection in Asynchronous CDMA", submitted to IEEE Trans. Comm., November 2001.

[5] F. Hasegawa, J. Luo, K. Pattipati, P. Willett, "Speed and Accuracy Comparison of Techniques for Multi-user Detection in Synchronous CDMA", submitted to IEEE Trans. Comm., March 2002.

Related conference papers:

[1] J. Luo, K. Pattipati, P. Willett, F. Hasegawa, "Multiuser Detection in Asynchronous CDMA Using PDA", IEEE CISS 2002, Princeton, NJ, Mar. 2002.

[2] J. Luo, K. Pattipati, P. Willett, F. Hasegawa, "Optimal User Ordering and Time Labeling for Decision Feedback Detection in Asynchronous CDMA", IEEE ICASSP 2002, Orlando, FL, May 2002.

[3] F. Hasegawa, J. Luo, K. Pattipati, P. Willett, "Speed and Accuracy Comparison of Techniques to Solve a Binary Quadratic Programming Problem

with Applications to Synchronous CDMA", IEEE CDC 2001, Orlando, FL, Dec. 2001.

[4] J. Luo, K. Pattipati, P. Willett, "Optimal Grouping and User Ordering for Sequential Group Detection in Synchronous CDMA", IEEE GlobeCom 2001, San Antonio, TX, Nov. 2001.

[5] J. Luo, K. Pattipati, P. Willett, "A Sub-optimal Soft Decision PDA Method for Binary Quadratic Programming", IEEE SMC 2001, Tucson, AZ, Oct. 2001.

[6] F. Hasegawa, J. Luo, K. Pattipati, P. Willett, "Performance of Various Methods for the Solution of Binary Quadratic Programming Problems", IEEE SMC 2001, Tucson, AZ, Oct. 2001.

[7] J. Luo, K. Pattipati, P. Willett, G. Levchuk, "Fast Optimal and Suboptimal Any-time Algorithms for CWMA Multiuser Detection", IEEE ISIT 2001, Wanshington D.C., June 2001.

[8] J. Luo, G. Levchuk, K. Pattipati, P. Willet, "A Class of Coordinate Descent Algorithm for Multiuser Detection", IEEE ICASSP 2000, Istanbul, Turkey, June 2000.

[9] J. Luo, G. Levchuk, K. Pattipati, P. Willett, "A Fast Optimal Algorithm for CWMA Multiuser Detection", IEEE CISS 2000, Princeton, NJ, March 2000.

Other journal papers:

[1] G. Levchuk, Y. Levchuk, J. Luo, K. Pattipati, D. Kleinman, “Normative Design of Organizations - Part I: Mission Planning”, accepted for publication in IEEE Trans. on SMC: Part A - Systems and Humans.

[2] G. Levchuk, Y. Levchuk, J. Luo, K. Pattipati, D. Kleinman, “Normative Design of Organizations - Part II: Organizational Structure”, accepted for publication in IEEE Trans. on SMC: Part A - Systems and Humans.

Other conference papers:

[1] G. Levchuk, Y. Levchuk, J. Luo, F. Tu, K. Pattipati, “Optimization Algorithms for Organizational Design”, IEEE SMC 2000, Nashville, TN, Oct. 2000.

[2] Y. Shlapak, J. Luo, G. Levchuk, F. Tu, K. Pattipati, “A Software Environment for the Design of Organizational Structures”, 2000 Command and Control Symposium, Monterey, CA, June 2000.

[3] G. Levchuk, Y. Levchuck, J. Luo, F. Tu, K. Pattipati, “A Library of Optimization Algorithms for Organizational Design”, 2000 Command and Control Symposium, Monterey, CA, June 2000.

[4] V. Rajan, K. Pattipati, J. Luo, “Fault Diagnosis in Mixed-Signal Circuits via Neural Network based Classification Algorithms”, IEEE IMSTW 2000, Montpelier, France, June 2000.

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Chapter 2

System Models and Multiuser Detectors

In this chapter, we introduce the system models of synchronous and asynchronous CDMA systems. We investigate the structures of several common multiuser detectors related to our research. The performance is evaluated by the average probability of group detection error. [1]

The chapter is organized as follows. Section 2.1 introduces the mathematical model of the synchronous CDMA system. The related multiuser detectors are presented in section 2.2. Asynchronous CDMA is introduced in section 2.3 and the related detectors are given in section 2.4

2.1 Synchronous CDMA System

Consider a synchronous CDMA system illustrated in Figure 1.

[1]The probability that all user signals in the same time frame are detected correctly.

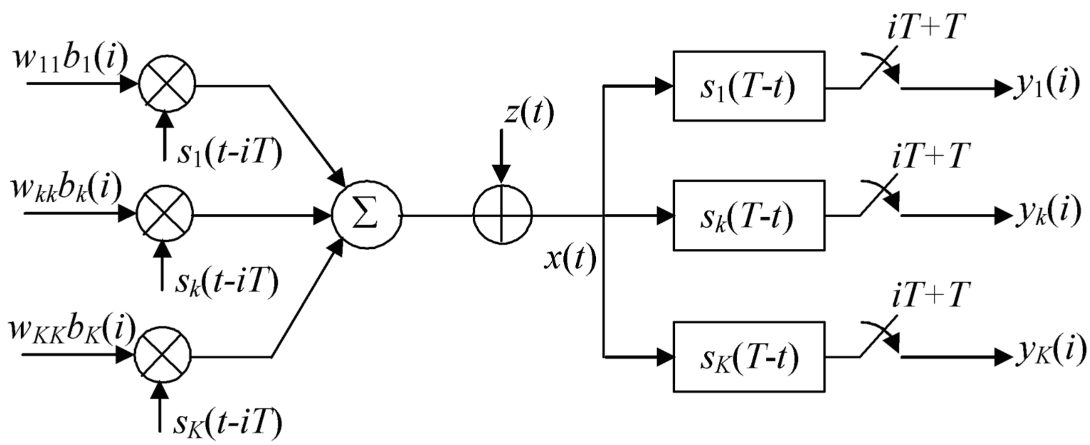

Figure 1: Synchronous CDMA system

The received baseband signal of the i-th symbol period can be represented by

$$x(t) = \mathbf{b}(i)^T \boldsymbol{W}\mathbf{s}(t - iT) + z(t) \qquad t \in [iT, iT + T] \tag{1}$$

where $\boldsymbol{b}(i) \in \{-1, +1\}^K$ denotes the K-length vector of bits transmitted by the K active users during the ith symbol period. $\boldsymbol{W}$ is a diagonal matrix whose kth diagonal element, w_{kk}, is the square root of the received signal energy per bit of the k-th user. $\mathbf{s}(t)$ is a column vector whose kth element $s_k(t)$ is the designed signature waveform for the kth user. $z(t)$ is a zero mean white Gaussian random process whose variance is σ^2. Without loss of generality, we assume that the signature waveform is normalized in the sense that

$$\int_{\mathbf{0}}^{T} s_k^2(\tau)d\tau = 1 \tag{2}$$

At the receiver side, the matched filter output of the ith symbol period $\boldsymbol{y}(i)$ is obtained by passing $x(t)$ through the K symbol matched filters and sampling at time $(i+1)T$, which can be represented by

$$\boldsymbol{y}(i) = \boldsymbol{RWb} + \boldsymbol{n}(i) \tag{3}$$

Here $\boldsymbol{R}$ is the normalized correlation matrix whose component located on the kth row and jth column, r_{kj}, is

$$r_{kj} = \int_{\mathbf{0}}^{T} s_k(\tau)s_j(\tau)d\tau \tag{4}$$

Here $\boldsymbol{n}(i)$ is a real-valued zero-mean Gaussian random vector with a covariance matrix $\sigma^2\boldsymbol{R}$.

In Direct Sequence (DS) CDMA, user signature waveforms $s_k(t)$ can be further represented by

$$s_k(t) = [\phi(t), \phi(t-T_c), \ldots, \phi(t-(N-1)T_c)]\boldsymbol{s}_k \tag{5}$$

where $\boldsymbol{s}_k$ is a $N \times 1$ vector, also known as the signature sequence for user k. N is called the spreading factor [Vd98]. $\phi(t)$, $t \in [0, T_c]$ is the chip waveform and T_c is the chip period. Assume $\boldsymbol{s}_k$ and $\phi(t)$ are also normalized,

$$\begin{aligned} \boldsymbol{s}_k^T\boldsymbol{s}_k &= 1 \\ \int_{\mathbf{0}}^{T_c} \phi_j^2(\tau)d\tau &= 1 \end{aligned} \tag{6}$$

Another equivalent implementation, also known as the chip-matched-filter for synchronous DS-CDMA, is shown in 2

The chip-matched-filter output of the ith time frame is

$$\boldsymbol{x}(i) = \boldsymbol{SWb}(i) + \boldsymbol{z}(i) \tag{7}$$

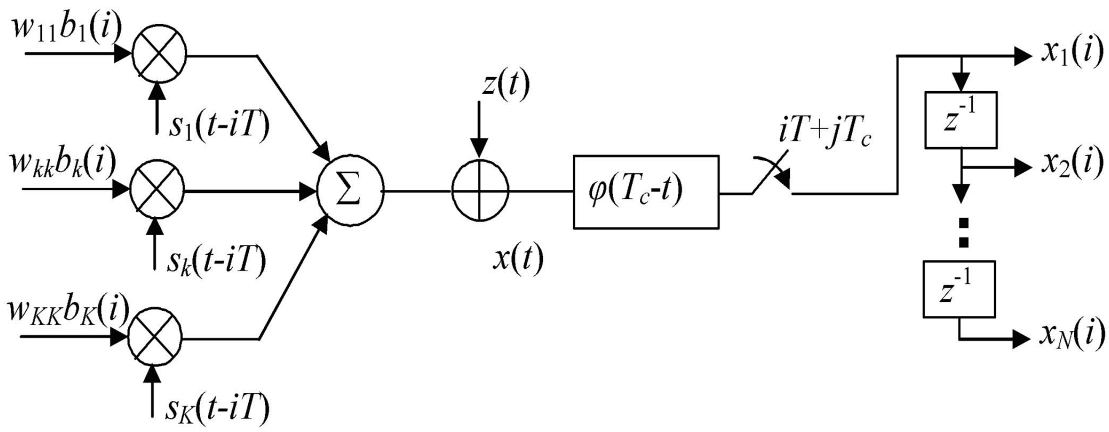

Figure 2: Chip Matched Filter Synchronous DS-CDMA system

where $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_K]$ is the signature matrix and $\boldsymbol{z}(i)$ is a white Gaussian noise with zero mean and covariance matrix $\sigma^2\boldsymbol{I}$.

The symbol matched filter output $\boldsymbol{y}(i)$ satisfies,

$$\boldsymbol{y}(i) = \boldsymbol{S}^T\mathbf{x}(i) = \boldsymbol{RWb}(i) + \boldsymbol{n}(i) \tag{8}$$

which is the same as (3).

In synchronous CDMA, since the detection of user signals in each time frame is independent of those from other time frames, for the convenience of notation, we usually ignore the time index and write the system model as

$$\boldsymbol{y} = \boldsymbol{RWb} + \boldsymbol{n} \tag{9}$$

Denote $\boldsymbol{L}^T\boldsymbol{L} = \boldsymbol{R}$ as the Cholesky decomposition of $\boldsymbol{R}$. Then (9) can be equivalently written as the white noise model,

$$\tilde{\boldsymbol{y}} = \boldsymbol{L}^{-T}\boldsymbol{y} = \boldsymbol{LWb} + \tilde{\boldsymbol{n}} \tag{10}$$

where $\boldsymbol{L}^{-T}$ denotes the inverse of $\boldsymbol{L}^T$, and $\tilde{\boldsymbol{n}} = \boldsymbol{L}^{-T}\boldsymbol{n}$ is a zero mean white Gaussian noise with covariance matrix $\sigma^2\boldsymbol{I}$.

2.2 Common Multiuser Detectors in Synchronous CDMA

2.2.1 Detection Algorithms

Assume all the user signals are equally probable. If user signature sequences are orthogonal to each other, i.e., $\forall k \neq j$, $\boldsymbol{s}_k^T \boldsymbol{s}_j = 0$, $\boldsymbol{R}$ becomes an identity matrix. Then, the optimal decision on $\boldsymbol{b}$ is given by the conventional matched filter,

$$\Phi_{MF} : \hat{\boldsymbol{b}} = sign(\boldsymbol{y}) \tag{11}$$

However, there are theoretical advantages to use non-orthogonal signature sequences. In this case, the optimal solution of (9) is the output of a Maximum Likelihood (ML) detector [LV89]

$$\Phi_{ML} : \hat{\boldsymbol{b}} = \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left(\boldsymbol{b}^T \boldsymbol{WRWb} - 2\boldsymbol{y}^T \boldsymbol{Wb} \right) \tag{12}$$

The ML detector has the property that it minimizes, among all detectors, the probability that not all users' decisions are correct. Usually, Φ_{ML} is considered NP-hard and exponentially complex to implement [LV89] unless some special structure of the correlation matrix can be found [SE98] [UY98]. Due to this, the focus is then on developing easily implementable and effective multiuser detectors.

The linear detector, including the conventional matched filter (11), the decorrelator and the MMSE detector, is one of the most popular classes of suboptimal detectors. The decision of the decorrelator [LV89]

$$\Phi_D : \hat{\boldsymbol{b}} = sign(\boldsymbol{R}^{-1}\boldsymbol{y}) \tag{13}$$

is found in two steps. First, the unconstrained solution $\tilde{\boldsymbol{b}} = \boldsymbol{R}^{-1}\boldsymbol{y}$ is computed. This is then projected onto the constraint set via: $\hat{b}_k = \text{sign}\left(\tilde{b}_k\right)$.

Similar to the decorrelator, the MMSE detector [X90] make decisions via

$$\Phi_{MMSE} : \hat{\boldsymbol{b}} = sign(\boldsymbol{W}(\boldsymbol{WRW} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{Wy}) \tag{14}$$

The reason why it is called MMSE is that $\boldsymbol{C} = \boldsymbol{W}(\boldsymbol{WRW} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{W}$ actually minimizes the mean square error $E\left(\|\boldsymbol{Wb} - \boldsymbol{Cy}\|_2^2\right)$.

Since we assume $\boldsymbol{W}$, $\boldsymbol{R}$ and σ are known to the receiver, the computational complexities of the decorrelator and the MMSE detector are evidently $O(K^2)$. The complexity per user is also linear in the number of users.

Another popular class of the suboptimal detectors is the decision-driven detector, including the multistage detector, the DF detector and the GDF detector.

The multistage detector [Va90] initializes the decision of the first stage using the decorrelator.

$$\hat{\boldsymbol{b}}^{(0)} = sign(\boldsymbol{R}^{-1}\boldsymbol{y}) \tag{15}$$

The kth decision of the $(m+1)$st stage is then refined by canceling MAI based on decisions of the mth stage

$$\Phi_{MS} : \hat{\boldsymbol{b}}_k^{(m)} = sign(y_k - \sum_{j \neq k} w_{jj}\mathbf{r}_j b_j^{(m-1)}) \tag{16}$$

where $\boldsymbol{r}_j$ denotes the jth column of $\boldsymbol{R}$. The complexity of the multistage detector is also $O(K^2)$ and is linear in the number of stages.

The decorrelator-based DF detector [Dh93] makes decisions on user signals sequentially via

$$\Phi_{DF} : \hat{b}_k = sign\left([\boldsymbol{L}^{-T}\boldsymbol{y}]_k - \sum_{j=1}^{k-1} l_{kj} w_{jj} \hat{b}_j\right) \tag{17}$$

Since the higher-indexed users cancel MAI based on the decisions of lower-indexed user signals, the performance of the DF detector is strongly affected by the detection order. The optimal user ordering for the decorrelating-based DF detector is given in Theorem 1 of [Va99], and a proof of the optimality can be dated back to [WFGV98]. Although finding the optimal user order requires $O(K^3)$ computations, it is considered as an offline computational load since it only requires $\boldsymbol{W}$ and $\boldsymbol{R}$, which are known to the receiver. Therefore, the online computational cost of the DF detector remains $O(K^2)$.

The sequential group detection, which can be viewed as an extension of the DF detector, was first introduced in [Va95]. Suppose users are partitioned into an ordered set of P groups, $G_{\mathbf{0}}, \ldots, G_{P-1}$. The number of users in group G_j is denoted by $|G_j|$, and naturally $\sum_{j=\mathbf{0}}^{P-1} |G_j| = K$. The decision on group $\{G_{\mathbf{0}}\}$ is made by

$$\hat{\boldsymbol{b}}_{G_0} = \arg \min_{\boldsymbol{b}_{G_0} \in \{-1,+1\}^{|G_0|}} \left[\min_{\boldsymbol{b}_{\bar{G}_0}} \left(\boldsymbol{b}^T \boldsymbol{WRWb} - 2\boldsymbol{y}^T \boldsymbol{Wb}\right)\right] \tag{18}$$

where $\boldsymbol{b}_{G_0}$ denotes the part of vector $\boldsymbol{b}$ that corresponds to users in group $G_{\mathbf{0}}$, and $\bar{G}_{\mathbf{0}}$ denotes the complement of $G_{\mathbf{0}}$, i.e., the union of $G_1, \ldots, G_{P-1}$. The decisions of (18) are then used to subtract the multiple-access interference due to users in $G_{\mathbf{0}}$ from the remaining decision statistics $\boldsymbol{y}_{\bar{G}_0}$. The detector for the next group G_1 is designed under the assumption that the multiple-access interference cancelation is perfect. This process of interference cancelation and group detection is carried out sequentially for users in groups $G_{\mathbf{2}}, \ldots, G_{P-1}$, with the group detector for group G_j taking advantage of the decisions made by group detectors for $G_{\mathbf{0}}, \ldots, G_{j-1}$. Denote the channel model for the user expurgated channel that only has users in groups $G_j, \ldots, G_{P-1}$ by

$$\boldsymbol{y}^{(j)} = \boldsymbol{R}^{(j)} \boldsymbol{W}^{(j)} \boldsymbol{b}^{(j)} + \boldsymbol{n}^{(j)} \tag{19}$$

Define $\bar{G}_j^{(j)}$ as the complement of G_j in the user-expurgated channel, i.e., the union of $G_{j+1}, \ldots, G_{P-1}$. The decisions on group G_j can be represented as

$$\hat{\boldsymbol{b}}_{G_j} = \arg \min_{\boldsymbol{b}^{(j)}_{G_j} \in \{-1,+1\}^{|G_j|}} \left[\min_{\boldsymbol{b}^{(j)}_{\bar{G}_j^{(j)}}} \left(\boldsymbol{b}^{(j)^T} \boldsymbol{W}^{(j)} \boldsymbol{R}^{(j)} \boldsymbol{W}^{(j)} \boldsymbol{b}^{(j)} - 2 \boldsymbol{y}^{(j)^T} \boldsymbol{W}^{(j)} \boldsymbol{b}^{(j)} \right) \right] \tag{20}$$

Define $|G|_{max} = \max(|G_{\mathbf{0}}|, \ldots, |G_{P-1}|)$. The complexity of the GDF detector can be expressed as $O(e^{|G|_{max}} K^{\mathbf{2}})$ since (66) is generally NP hard. Similar to the DF detector, the user partitioning and ordering is critical to the performance of the GDF detector. However, only ordering is given in [Va95] (without proof of optimality), and that is also based on the assumption that the user partitioning is given.

In addition to the linear detector and the decision driven detector, many advanced detectors have been proposed in the last several years. Here we will only introduce the semi-definite relaxation method. A detailed discussion and comparison of advanced detection algorithms can be found in [HLPW02].

Define vector $\boldsymbol{u}^T = [\boldsymbol{b}^T, u_{K+1}]$, where u_{K+1} is a dummy variable which is forced to be 1. The ML detector can be represented by

$$\begin{aligned}
\Phi_{ML} : \hat{\boldsymbol{b}} &= \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left(\boldsymbol{u}^T \begin{bmatrix} \boldsymbol{WRW} & -\boldsymbol{Wy} \\ -\boldsymbol{Wy} & \boldsymbol{0} \end{bmatrix} \boldsymbol{u} \right) \\
&= \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} trace \left(\begin{bmatrix} \boldsymbol{WRW} & -\boldsymbol{Wy} \\ -\boldsymbol{Wy} & \boldsymbol{0} \end{bmatrix} \boldsymbol{u}\boldsymbol{u}^T \right) \\
\boldsymbol{u}^T &= [\boldsymbol{b}^T, 1] \qquad (21)
\end{aligned}$$

Based on (21), the semi-definite relaxation method can be described as a two step procedure. In the first step, a semidefinite programming problem is solved.

$$\begin{aligned}
\hat{\boldsymbol{X}} &= \arg \min_{\boldsymbol{X}} trace \left(\begin{bmatrix} \boldsymbol{WRW} & -\boldsymbol{Wy} \\ -\boldsymbol{Wy} & \boldsymbol{0} \end{bmatrix} \boldsymbol{X} \right) \\
s.t. \quad & \boldsymbol{X} \geq 0 \qquad \forall k, x_{kk} = 1 \qquad (22)
\end{aligned}$$

In the second step, the decisions on user signals are made by taking the first K elements of vector $\hat{\boldsymbol{u}}$, which is obtained by

$$\hat{\boldsymbol{u}} = sign(\xi_{max}[\xi_{max}]_{K+1}) \qquad (23)$$

where ξ_{max} is the eigenvector corresponding to the maximum eigenvalue of $\hat{\boldsymbol{X}}$.

The computational complexity of solving (22) is $O(K^{3.5})$ [VB96].

2.2.2 Performance Evaluation

In multi-user detection, the Asymptotic Effective Energy (AEE) and the Symmetric Energy (SE) are two important performance measures [Va99]. Define $P_k(e)$ to be the probability of error for user k, and define $P(e)$ to be the probability that not all users are detected correctly. The AEE of user k is define by

$$E_k = sup\left(E \geq 0; \lim_{\sigma\to 0} \frac{P_k(e)}{Q(\sqrt{E}/\sigma)} < \infty\right) \tag{24}$$

where $Q(.)$ is defined by $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$. The SE of the detector Φ is given by

$$E(\Phi) = sup\left(E \geq 0; \lim_{\sigma\to 0} \frac{P(e)}{Q(\sqrt{E}/\sigma)} < \infty\right) \tag{25}$$

It is easy to see that the SE of the ML detector is

$$E(\Phi_{ML}) = \min_{\boldsymbol{e}\in\{-1,0,1\}^K, \boldsymbol{e}\neq\boldsymbol{0}} \{\mathbf{e}^T \boldsymbol{WRWe}\} \tag{26}$$

Usually,

$$d_{min} = \sqrt{\min_{\boldsymbol{e}\in\{-1,0,1\}^K, \boldsymbol{e}\neq\boldsymbol{0}} \{\mathbf{e}^T \boldsymbol{He}\}} \tag{27}$$

is termed the minimum distance of matrix $\boldsymbol{H}$.

For the decorrelator, the decision on user 1 can be represented by

$$\hat{b}_1 = sign([\boldsymbol{L}^{-1}\boldsymbol{L}^{-T}\boldsymbol{y}]_1) \tag{28}$$

From the white noise model (10), we have

$$\begin{aligned}\hat{b}_1 = &\quad sign([\boldsymbol{L}^{-1}]_{11}\tilde{y}_1) \\ = &\quad sign(\frac{1}{l_{11}}\tilde{y}_1) \\ = &\quad sign(w_{11}b_1 + \frac{1}{l_{11}}\tilde{n}_1)\end{aligned} \tag{29}$$

Since $\tilde{n}_1$ is zero mean white Gaussian with variance σ^2, we have

$$P_1(e) = Q(\frac{w_{11}l_{11}}{\sigma}) = Q(\frac{w_{11}}{\sigma\sqrt{[\boldsymbol{R}^{-1}]_{11}}}) \tag{30}$$

Due to the symmetry of (13), the AEE of user k is

$$E_k = \frac{w_{kk}}{\sqrt{[\boldsymbol{R}^{-1}]_{kk}}} \tag{31}$$

Since when $\sigma \to 0$

$$P(e) \approx \sum_{k=1}^{K} P_k(e) \tag{32}$$

We get the SE of the decorrelator as

$$E(\Phi_D) = \min_k \left\{ \frac{w_{kk}}{\sqrt{[\boldsymbol{R}^{-1}]_{kk}}} \right\} \tag{33}$$

For the DF detector, assume that the decision on user signal j is correct for $j < k$. Then, the AEE of user k can be obtained from (17) as [Va99]

$$E_k = w_{kk}l_{kk} \tag{34}$$

Consequently, the SE of the DF detector can be represented by

$$E(\Phi_D) = \min_k \{w_{kk}l_{kk}\} \tag{35}$$

For GDF detector, we first define the Asymptotic Effective Group Energy (AEGE) for each user group. Define the probability that not all users in group $\{G_j\}$ are detected correctly in a GDF detector Φ_{GDF} as $P_{G_i}(e)$. Then, the AEGE for group G_j is given by

$$E_{G_j} = sup\left\{E \geq 0; \lim_{\sigma \to \mathbf{0}} \frac{P_{G_j}(e)}{Q\left(\frac{\sqrt{E}}{\sigma}\right)} < \infty\right\} \tag{36}$$

Similar to the DF detector, assume that the decisions on user signals in group $i < j$ are correct. The AEGE for group G_j can be expressed by

$$E_{G_j} = \sqrt{\min_{\boldsymbol{e} \in \{-1,0,1\}^{|G_j|}, \boldsymbol{e} \neq \mathbf{0}} \{\mathbf{e}^T \boldsymbol{W}_{G_jG_j} \boldsymbol{L}_{G_jG_j}^T \boldsymbol{L}_{G_jG_j} \boldsymbol{W}_{G_jG_j} \mathbf{e}\}} \tag{37}$$

Evidently, the SE of the GDF detector can be obtained via

$$E(\Phi_{GDF}) = \min(E_{G_0}, \ldots, E_{G_{P-1}}) \tag{38}$$

Unfortunately, the SE of most other advanced multiuser detectors are not available in the literature and thus can only be obtained via computer simulations.

2.3 Asynchronous CDMA

2.3.1 System Model

The asynchronous CDMA system is illustrated in Figure 3.

The received baseband signal can be represented by

$$x(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{K} b_k(i) w_{kk} s(t - iT - \tau_k) + z(t) \tag{39}$$

where τ_k is the delay corresponding to user signal k. Assume that the delays are known to the receiver, and are not changing during the whole transmission. Also

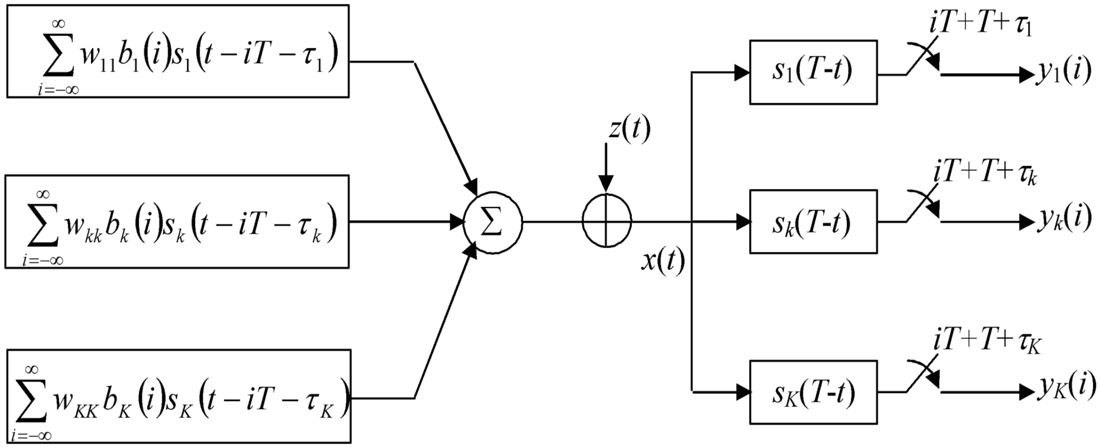

Figure 3: Asynchronous CDMA system

assume that the kth matched filter is synchronized with the signal of the kth user. The kth matched filter output in its ith time frame can be represented by

$$
\begin{aligned}
y_k(i) = \; & \sum_{j=-\infty}^{\infty} \int_{iT+\tau_k}^{iT+T+\tau_k} \sum_{m=1}^{K} w_{mm} b_m(j) s_m(t-jT-\tau_m) s_k(t-iT-\tau_k) dt \\
& + \int_{iT+\tau_k}^{iT+T+\tau_k} z(t) s_k(t-iT-\tau_k) dt
\end{aligned} \tag{40}
$$

Suppose users are ordered so that $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_K$. The integral in (40) can be further separated and written as

$$
\begin{aligned}
y_k(i) = \; & \int_{iT+\tau_k}^{iT+T+\tau_k} \sum_{m=k+1}^{K} w_{mm} b_m(j) s_m(t-jT+T-\tau_m) s_k(t-iT-\tau_k) dt \\
& + \int_{iT+\tau_k}^{iT+T+\tau_k} \sum_{m=1}^{K} w_{mm} b_m(j) s_m(t-jT-\tau_m) s_k(t-iT-\tau_k) dt \\
& + \int_{iT+\tau_k}^{iT+T+\tau_k} \sum_{m=1}^{k-1} w_{mm} b_m(j) s_m(t-jT-T-\tau_m) s_k(t-iT-\tau_k) dt \\
& + \int_{iT+\tau_k}^{iT+T+\tau_k} z(t) s_k(t-iT-\tau_k) dt
\end{aligned} \tag{41}
$$

Define two matrices $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$ such that

$$
r[0]_{km} = \int_{max(\tau_k,\tau_m)}^{T+min(\tau_k,\tau_m)} s_k(t-\tau_k) s_m(t-\tau_m) dt = r[0]_{mk}
$$

$$r[1]_{km} = \begin{cases} \int_{\tau_k}^{\tau_m} s_m(t+T-\tau_m)s_k(t-\tau_k)dt & \tau_m > \tau_k \\ 0 & \tau_m \leq \tau_k \end{cases} \tag{42}$$

Evidently, $\boldsymbol{R}[0]$ is a symmetric positive definite matrix with unity diagonal elements, and $\boldsymbol{R}[1]$ is a singular matrix.

The matched filter output vector $\boldsymbol{y}(i)$ can then be represented by [Vd98]

$$\boldsymbol{y}(i) = \boldsymbol{R}[1]^T \boldsymbol{W}\boldsymbol{b}(i+1) + \boldsymbol{R}[0]\boldsymbol{W}\boldsymbol{b}(i) + \boldsymbol{R}[1]\boldsymbol{W}\boldsymbol{b}(i-1) + \boldsymbol{n}(i) \tag{43}$$

Applying the Z-transform to (43), we obtain,

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{W}\boldsymbol{b}(z) + \boldsymbol{n}(z) \tag{44}$$

where $\boldsymbol{R}(z) = \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1}$ is the correlation matrix. Here, $\boldsymbol{n}(z)$ is a zero mean colored Gaussian noise with correlation matrix $\sigma^2\boldsymbol{R}(z)$ [Vd98].

2.3.2 White Noise Model and Related Computations

Similar to the Cholesky decomposition in the synchronous case, $\boldsymbol{R}(z)$ can be factored as [Dh95]

$$\begin{aligned} \boldsymbol{R}(z) &= \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1} \\ &= (\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^T(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}) \end{aligned} \tag{45}$$

$\boldsymbol{F}[0]$ and $\boldsymbol{F}[1]$ can be obtained by the following iterative procedure [AR98],

(1) Set $j = 0$ and initialize $\boldsymbol{F}[0]^{(0)}$ by

$$[\boldsymbol{F}[0]^{(0)}]^T[\boldsymbol{F}[0]^{(0)}] = \boldsymbol{R}[0] \tag{46}$$

(2) Initialize $\boldsymbol{F}[1]^{(\mathbf{0})}$ by

$$[\boldsymbol{F}[1]^{(\mathbf{0})}] = [\boldsymbol{F}[0]^{(\mathbf{0})}]^{-T}\boldsymbol{R}[1] \tag{47}$$

(3) Let $j = j + 1$, and compute

$$\begin{aligned} &[\boldsymbol{F}[0]^{(j)}]^T[\boldsymbol{F}[0]^{(j)}] = \boldsymbol{R}[0] - [\boldsymbol{F}[1]^{(j)}]^T[\boldsymbol{F}[1]^{(j)}] \\ &[\boldsymbol{F}[1]^{(j)}] = [\boldsymbol{F}[0]^{(j)}]^{-T}\boldsymbol{R}[1] \end{aligned} \tag{48}$$

(4) Goto step 3 till convergence.

With the factorization, (44) can be equivalently written as the white noise model

$$\begin{aligned} \tilde{\boldsymbol{y}}(z) &= (\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T}\boldsymbol{y}(z) \\ &= (\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1})\boldsymbol{W}\boldsymbol{b}(z) + \tilde{\boldsymbol{n}}(z) \end{aligned} \tag{49}$$

where $\tilde{\boldsymbol{n}}(z) = (\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T}\boldsymbol{n}(z)$ is a zero mean white Gaussian noise with covariance matrix $\sigma^2\boldsymbol{I}$.

There are two ways to implement the anti-causal filter $(\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T}$ and get $\tilde{\boldsymbol{y}}(i)$ in time domain.

The first way is to convert $(\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T}$ to a truncated anti-causal Moving Average (MA) filter

$$(\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T} \approx \sum_{j=\mathbf{0}}^{J}(-1)^j\boldsymbol{F}[0]^{-T}(\boldsymbol{F}[1]^T\boldsymbol{F}[0]^{-T}z)^j \tag{50}$$

and approximate $\tilde{\boldsymbol{y}}(i)$ by

$$\tilde{\boldsymbol{y}}(i) \approx \sum_{j=\mathbf{0}}^{J}(-1)^j\boldsymbol{F}[0]^{-T}(\boldsymbol{F}[1]^T\boldsymbol{F}[0]^{-T})^j\boldsymbol{y}(i+j) \tag{51}$$

The MA implementation requires minimum delay, and the computational cost of getting $\tilde{\boldsymbol{y}}(i)$ is $O(JK^2)$ where J is the order of the MA filter.

The other way is to implement the anti-causal AR filter directly and obtain $\tilde{\boldsymbol{y}}(i)$ by

$$\tilde{\boldsymbol{y}}(i) = \boldsymbol{F}[0]^{-T}(\boldsymbol{y}(i) - \boldsymbol{F}[1]^T \tilde{\boldsymbol{y}}(i+1)) \tag{52}$$

Since J in the MA implementation is usually much larger than 2, the computational cost of the AR implementation is evidently lower than the MA model. However, the AR implementation requires the receipt of the entire data, which will certainly cause a significant delay on the detection output.

In a practical situation, one can combine the MA and AR implementation together to balance complexity and delay.

2.4 Common Multiuser Detectors in Asynchronous CDMA

Suppose there are overall M time frames in the transmission. Stack the observations of all time frames together and define

$$\boldsymbol{y}^T = [\boldsymbol{y}(0)^T, \boldsymbol{y}(1)^T, \ldots, \boldsymbol{y}(M-1)^T] \tag{53}$$

Similarly, treat each bit as if it was transmitted by a different (fictitious) user. Define

$$\boldsymbol{b}^T = [\boldsymbol{b}(0)^T, \boldsymbol{b}(1)^T, \ldots, \boldsymbol{b}(M-1)^T] \tag{54}$$

The K-user M-frame asynchronous CDMA system can be viewed as a KM user synchronous system [Vd98]

$$\boldsymbol{y} = \boldsymbol{R}\Xi\boldsymbol{b} + \boldsymbol{n} \tag{55}$$

where

$$\begin{aligned}
\boldsymbol{R} &= \begin{bmatrix} \boldsymbol{R}[0] & \boldsymbol{R}[1]^T & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{R}[1] & \boldsymbol{R}[0] & \ddots & \ddots & \vdots \\ \boldsymbol{0} & \ddots & \ddots & \ddots & \boldsymbol{0} \\ \vdots & \ddots & \ddots & \boldsymbol{R}[0] & \boldsymbol{R}[1]^T \\ \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{R}[1] & \boldsymbol{R}[0] \end{bmatrix} \\
\Xi &= \begin{bmatrix} \boldsymbol{W} & \boldsymbol{0} & \dots & \dots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{W} & \boldsymbol{0} & \ddots & \vdots \\ \ddots & \ddots & \ddots & \ddots & \ddots \\ \vdots & \ddots & \boldsymbol{0} & \boldsymbol{W} & \boldsymbol{0} \\ \boldsymbol{0} & \dots & \dots & \boldsymbol{0} & \boldsymbol{W} \end{bmatrix} \\
\boldsymbol{n}^T &= [\boldsymbol{n}(0)^T, \boldsymbol{n}(1)^T, \dots, \boldsymbol{n}(M-1)^T]
\end{aligned} \tag{56}$$

Define $\boldsymbol{L}^T\boldsymbol{L} = \boldsymbol{R}$ to be the Cholesky decomposition matrix of $\boldsymbol{R}$. From the iterative procedure of computing $\boldsymbol{F}[0]$ and $\boldsymbol{F}[1]$, it can be easily verified [AR98]

$$\boldsymbol{L} = \begin{bmatrix} \ddots & \boldsymbol{0} & \dots & \dots & \boldsymbol{0} \\ \boldsymbol{F}[1] & \boldsymbol{F}[0] & \boldsymbol{0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \boldsymbol{F}[1]^{(1)} & \boldsymbol{F}[0]^{(1)} & \boldsymbol{0} \\ \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{F}[1]^{(0)} & \boldsymbol{F}[0]^{(0)} \end{bmatrix} \tag{57}$$

Theoretically, any multiuser detection algorithm for synchronous CDMA can be directly extended to asynchronous CDMA by considering the above equivalent synchronous system. However, since $MK >> K$, the computational complexity of a direct extension of the advanced detection algorithms can be very high.

In (55), the optimal decision is again given by the ML detector [2]

$$\Phi_{ML} : \hat{\boldsymbol{b}} = \arg \min_{\boldsymbol{b} \in \{-1,+1\}^{KM}} \left(\boldsymbol{b}^T \Xi \boldsymbol{R} \Xi \boldsymbol{b} - 2\boldsymbol{y}^T \Xi \boldsymbol{b} \right) \tag{58}$$

ML detection is generally NP hard. Although the overall computational cost can be significantly lower than the worst case one [Vd98], it is still too high to be implemented in practice. In computer simulations, it is hard to do large numbers of Monte-Carlo runs with ML detection for even moderate-sized problem.

Similar to the synchronous situation, the conventional matched filter makes decisions by directly taking the signs of the observations

$$\Phi_{MF} : \hat{\boldsymbol{b}}(i) = sign(\boldsymbol{y}(i)) \tag{59}$$

The DF detector [Dh95] works on the white noise model (44), which can also be represented in time domain by

$$\tilde{\boldsymbol{y}}(i) - \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(i-1) = \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(i) + \tilde{\boldsymbol{n}}(i) \tag{60}$$

DF detector makes decisions sequentially and utilizes past decisions in addition to channel outputs, where $\hat{b}_k(i)$ is given by

$$\Phi_{DF} : \hat{b}_k(i) = \text{sign}\left(\tilde{y}_k(i) - \sum_{j=1}^{k-1} F[0]_{kj} \hat{b}_j(i) - \sum_{j=1}^{K} F[1]_{kj} \hat{b}_j(i-1) \right) \tag{61}$$

[2]ML detector for the equivalent synchronous system corresponds to the ML sequence detector for asynchronous CDMA

The DF detector assumes that the decisions for user bits $b_k(i-1)$ are made prior to the decision of $b_j(i)$, $\forall k, j$.

The Decorrelator multiplies $\boldsymbol{R}(z)^{-1}$ from left on both sides of (44) first to obtain

$$\tilde{\tilde{\boldsymbol{y}}}(z) = \boldsymbol{R}(z)^{-1}\boldsymbol{y}(z) \tag{62}$$

and then makes decisions by

$$\Phi_D : \hat{\boldsymbol{b}}(i) = sign(\tilde{\tilde{\boldsymbol{y}}}(i)) \tag{63}$$

Since

$$\boldsymbol{R}(z)^{-1} = (\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1})^{-1}(\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T} \tag{64}$$

We can compute and implement $\boldsymbol{R}(z)^{-1}$ in a way similar to the implementation of $(\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^{-T}$

Unfortunately, the performance analysis for most multiuser detectors in asynchronous CDMA is not available in the literature. The reason is that due to the overlap of user signals, a decision error in one time frame may cause a detection error in the next time frame. Such error propagation makes the actual performance of a multiuser detector hard to analyze except via simulation.

Chapter 3

Improving Decision-Feedback-based Detectors

Decision Feedback detector is one of the most efficient methods in both synchronous and asynchronous CDMA due to its simplicity and its outstanding performance. However, there is usually still a large gap between the performance of the DF detector and that of the optimal detector. The original motivation of our research was to improve the performance of the DF detector while maintaining its high computational efficiency.

The performance of the DF detector depends critically on the detection order. Although the DF idea is rather a simple idea in optimization, its outstanding performance comes from the use of statistical information of the system model in its user ordering algorithm. By studying the user ordering proposed in [Va99] for the DF detector, we found the optimal user partitioning and ordering for the GDF detector, which can be generally considered as an extension to the DF detector. In asynchronous CDMA, we found that the performance of the DF detector is affected not only by user ordering, but also by the time labeling of the system. An optimal time labeling and user ordering algorithm for the ideal DF detector is then derived.

The chapter is organized as follows. The optimal user ordering for the DF detector is introduced in section 3.1. Based on the idea of optimal user ordering, given the maximum group size, the optimal user partitioning and ordering for the GDF detector is derived. The DF detector in asynchronous CDMA is studied and the optimal user ordering and time labeling is presented in section 3.2.

3.1 Optimal User Partitioning and Ordering for A Group DF Detector in Synchronous CDMA

3.1.1 Group Decision Feedback

In the GDF detector, suppose users are partitioned into an ordered set of P groups, $G_{\mathbf{0}}, \ldots, G_{P-1}$. Denote the channel model for the user expurgated channel that only has users in groups $G_j, \ldots, G_{P-1}$ by

$$\boldsymbol{y}^{(j)} = \boldsymbol{R}^{(j)}\boldsymbol{W}^{(j)}\boldsymbol{b}^{(j)} + \boldsymbol{n}^{(j)} \tag{65}$$

The decisions on group G_j can be represented by

$$\hat{\boldsymbol{b}}_{G_j} = \arg \min_{\boldsymbol{b}^{(j)}_{G_j} \in \{-1,+1\}^{|G_j|}} \left[\min_{\boldsymbol{b}^{(j)}_{\bar{G}^{(j)}_j}} \left({\boldsymbol{b}^{(j)}}^T \boldsymbol{W}^{(j)}\boldsymbol{R}^{(j)}\boldsymbol{W}^{(j)}\boldsymbol{b}^{(j)} - 2{\boldsymbol{y}^{(j)}}^T \boldsymbol{W}^{(j)}\boldsymbol{b}^{(j)} \right) \right] \tag{66}$$

The complexity of the GDF detector can be expressed as $O(e^{|G|_{max}}K^2)$.

Define $\boldsymbol{J}^{(j)} = \left[\boldsymbol{W}^{(j)}\boldsymbol{R}^{(j)}\boldsymbol{W}^{(j)}\right]^{-1}$, and denote $\boldsymbol{J}^{(j)}_{G_jG_j}$ to be the sub-matrix of $\boldsymbol{J}^{(j)}$ that only contains the columns and rows corresponding to users in G_j. Define

d_{G_j} to be the minimum distance of matrix $\left(\boldsymbol{J}^{(j)}_{G_jG_j}\right)^{-1}$, i.e.,

$$d^2_{G_j} = \min_{\boldsymbol{e}\in\{-1,0,1\}^{|G_j|}\setminus\{\boldsymbol{0}\}^{|G_j|}} \boldsymbol{e}^T \left(\boldsymbol{J}^{(j)}_{G_jG_j}\right)^{-1} \boldsymbol{e} \tag{67}$$

Assuming that the decisions on user signals in group $i < j$ are correct, the AEGE for group G_j can be expressed by

$$E_{G_j} = d^2_{G_j} \tag{68}$$

Consequently, the SE of the GDF detector is,

$$E(\Phi_{GDF}) = \min(d^2_{G_0}, \ldots, d^2_{G_{P-1}}) \tag{69}$$

3.1.2 Optimal Grouping and Detection Order

Since the overall computation of GDF detector is exponential in $|G|_{max}$, which is the maximum group size, in this section, we develop a grouping and ordering algorithm that maximizes the SE of the GDF detector given $|G|_{max}$ as a design parameter.

Define

$$\begin{aligned} \boldsymbol{H} &= \boldsymbol{W}\boldsymbol{R}\boldsymbol{W} \\ \tilde{\boldsymbol{L}} &= \boldsymbol{L}\boldsymbol{W} \end{aligned} \tag{70}$$

The white noise model (10) can be written as

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{L}}\boldsymbol{b} + \tilde{\boldsymbol{n}} \tag{71}$$

Define $\bar{G}_{\mathbf{0}}$ to be the complement of $G_{\mathbf{0}}$, i.e., the union of $G_1, \ldots, G_{P-1}$. Partition the matrices and vectors according to $G_{\mathbf{0}}$ and $\bar{G}_{\mathbf{0}}$ to obtain

$$\begin{bmatrix} \tilde{\boldsymbol{y}}_{G_0} \\ \tilde{\boldsymbol{y}}_{\bar{G}_0} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{L}}_{G_0G_0} & 0 \\ \tilde{\boldsymbol{L}}_{\bar{G}_0G_0} & \tilde{\boldsymbol{L}}_{\bar{G}_0\bar{G}_0} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}_{G_0} \\ \boldsymbol{b}_{\bar{G}_0} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{n}}_{G_0} \\ \tilde{\boldsymbol{n}}_{\bar{G}_0} \end{bmatrix} \tag{72}$$

Since $\boldsymbol{H} = \tilde{\boldsymbol{L}}^T \tilde{\boldsymbol{L}}$, we have

$$\tilde{\boldsymbol{L}}^T_{G_0G_0} \tilde{\boldsymbol{L}}_{G_0G_0} = \left[(\boldsymbol{H}^{-1})_{G_0G_0} \right]^{-1} = \left[\boldsymbol{J}^{(\mathbf{0})}_{G_0G_0} \right]^{-1} \tag{73}$$

A similar result can be obtained for group G_j. In the user expurgated system with parameter $\boldsymbol{H}^{(j)} = \boldsymbol{W}^{(j)} \boldsymbol{R}^{(j)} \boldsymbol{W}^{(j)}$, if we let $\boldsymbol{H}^{(j)} = \left[\tilde{\boldsymbol{L}}^{(j)} \right]^T \left[\tilde{\boldsymbol{L}}^{(j)} \right]$, then $\left[\tilde{\boldsymbol{L}}^{(j)}_{G_jG_j} \right]^T \left[\tilde{\boldsymbol{L}}^{(j)}_{G_jG_j} \right] = \left(\boldsymbol{J}^{(j)}_{G_jG_j} \right)^{-1}$. Since $\boldsymbol{H}^{(j)}$ is the south-east sub-diagonal matrix of $\boldsymbol{H}$, it is easy to see that $\tilde{\boldsymbol{L}}^{(j)}$ is the south-east sub-diagonal matrix of $\tilde{\boldsymbol{L}}$ and $\tilde{\boldsymbol{L}}^{(j)}_{G_j} = \tilde{\boldsymbol{L}}_{G_j}$. Hence,

$$\tilde{\boldsymbol{L}}^T_{G_jG_j} \tilde{\boldsymbol{L}}_{G_jG_j} = \left(\boldsymbol{J}^{(j)}_{G_jG_j} \right)^{-1} \tag{74}$$

The above result shows that d_{G_j} is determined by the diagonal block-matrix $\tilde{\boldsymbol{L}}_{G_j}$ of $\tilde{\boldsymbol{L}}$. Now, since the SE of GDF detector is given by

$$E(\phi_{GDF}) = \min(d^2_{G_0}, ..., d^2_{G_{P-1}}) \tag{75}$$

and $|G|_{max}$ is given as a design parameter, the problem is then to find an optimal partition and detection order that maximizes $\min(d^2_{G_0}, ..., d^2_{G_{P-1}})$. Notice that different GDF detectors may have the same $|G|_{max}$ but different numbers of groups since P is not a design parameter.

Grouping and Ordering Algorithm : Find the optimal grouping and detection order via the following steps.

Step 1: Partition the K users into two groups $\{G_{\mathbf{0}}\}$ and $\{\bar{G}_{\mathbf{0}}\}$ with $|G_{\mathbf{0}}| \leq |G|_{max}$. Among these partitions ($\{G_{\mathbf{0}}\}$ and $|G_{\mathbf{0}}|$ are not fixed), select the one that maximizes $d_{G_0,min}$ (which is the minimum distance of matrix $\left[\boldsymbol{J}^{(\mathbf{0})}_{G_0G_0}\right]^{-1}$).

Step 2: Partition the remaining $K - |G_{\mathbf{0}}|$ users into two groups G_1 and $\bar{G}_1$ with $|G_1| \leq |G|_{max}$. Among these partitions, select the one that maximizes $d_{G_1,min}$.

Step 3: Continue this process until all the users are assigned to groups.

Example 1 : The algorithm is illustrated by the following 4-user example. Suppose the H matrix is given by

$$\boldsymbol{H} = \begin{bmatrix} 4.30 & 1.00 & 0.60 & 0.30 \\ 1.00 & 3.00 & 1.70 & 0.50 \\ 0.60 & 1.70 & 2.20 & 0.70 \\ 0.30 & 0.50 & 0.70 & 1.90 \end{bmatrix} \tag{76}$$

Assume that the desired maximum group size is $|G|_{max} = 2$. In step 1 of the algorithm, the possible choices for group $G_{\mathbf{0}}$ and the resulting $d^2_{G_0}$ are shown in Table 1.

User(s)	0	1	2	3	0,1
$d^2_{G_0}$	3.96	1.62	1.14	1.67	1.69
User(s)	0,2	0,3	1,2	1,3	2,3
$d^2_{G_0}$	1.14	1.68	1.74	1.62	1.24

Table 1: Choices of group $G_{\mathbf{0}}$ and the corresponding $d^2_{G_0,min}$

The best choice for group G_0 is {user 0}. Then, for the user expurgated channel, we have

$$\boldsymbol{H}^{(1)} = \begin{bmatrix} 3.00 & 1.70 & 0.50 \\ 1.70 & 2.20 & 0.70 \\ 0.50 & 0.70 & 1.90 \end{bmatrix} \tag{77}$$

The possible choices for group G_1 and the resulting $d_{G_1,min}$ are shown in Table 2. We can see that the best choice for group G_1 is {user 1, user 2}. Naturally {user

User(s)	1	2	3	1,2	1,3	2,3
$d^2_{G_1}$	1.69	1.14	1.68	1.78	1.68	1.24

Table 2: Choices of group G_1 and the corresponding $d_{G_1,min}$

3} will be the last group. The resulting SE for this partitioning and ordering is $E = 1.78$.

Note that the above example has 4 users and $|G|_{max} = 2$. One may think that partitioning users into 2 groups with 2 users in each group is a good choice. However, since user 0 is a strong user, this user has to be detected first. And since user 1 and user 2 are strongly correlated, they have to be assigned to the same group. If, for example, we assign two groups as $\{user0, user3\}$ and $\{user1, user2\}$, then, as a penalty for detecting the weak user (user 3) first, we obtain $E = 1.68 < 1.78$.

Before giving the proof of optimality to the GDF ordering, we present the following three key lemmas.

Lemma 1: Suppose $\boldsymbol{H} = \tilde{\boldsymbol{L}}^T \tilde{\boldsymbol{L}}$ is partitioned on an arbitrary diagonal element as

$$\begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{21}^T \\ \boldsymbol{H}_{21} & \boldsymbol{H}_{22} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{L}}_{11} & 0 \\ \tilde{\boldsymbol{L}}_{21} & \tilde{\boldsymbol{L}}_{22} \end{bmatrix}^T \begin{bmatrix} \tilde{\boldsymbol{L}}_{11} & 0 \\ \tilde{\boldsymbol{L}}_{21} & \tilde{\boldsymbol{L}}_{22} \end{bmatrix} \tag{78}$$

For any permutation matrix $\boldsymbol{P}$ of the same size as $\boldsymbol{H}_{22}$, if

$$\begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{H}_{21}^T\boldsymbol{P} \\ \boldsymbol{P}\boldsymbol{H}_{21} & \boldsymbol{P}\boldsymbol{H}_{22}\boldsymbol{P} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}_{11} & 0 \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{C}_{11} & 0 \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix} \tag{79}$$

The following results hold.

$$\boldsymbol{C}_{11} = \tilde{\boldsymbol{L}}_{11} \quad , \quad \boldsymbol{C}_{22}^T\boldsymbol{C}_{22} = \boldsymbol{P}\tilde{\boldsymbol{L}}_{22}^T\tilde{\boldsymbol{L}}_{22}\boldsymbol{P} \tag{80}$$

The proof is straightforward and is therefore omitted. □

Lemma 2: Suppose $\boldsymbol{H}$ is a $m \times m$ symmetric and positive definite matrix. Suppose $\boldsymbol{H} = \tilde{\boldsymbol{L}}^T \tilde{\boldsymbol{L}}$ is the Cholesky decomposition. Partition $\boldsymbol{H}$ and $\tilde{\boldsymbol{L}}$ on the last (south-east) diagonal component as

$$\begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{h}_{21}^T \\ \boldsymbol{h}_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{L}}_{11} & 0 \\ \tilde{\boldsymbol{l}}_{21} & \tilde{l}_{22} \end{bmatrix}^T \begin{bmatrix} \tilde{\boldsymbol{L}}_{11} & 0 \\ \tilde{\boldsymbol{l}}_{21} & \tilde{l}_{22} \end{bmatrix} \tag{81}$$

Now "move up" the last "user" to the first, denote the action and the new Cholesky decomposition matrix by

$$\begin{bmatrix} h_{22} & \boldsymbol{h}_{21} \\ \boldsymbol{h}_{21}^T & \boldsymbol{H}_{11} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ \boldsymbol{c}_{21} & \boldsymbol{C}_{22} \end{bmatrix}^T \begin{bmatrix} c_{11} & 0 \\ \boldsymbol{c}_{21} & \boldsymbol{C}_{22} \end{bmatrix} \tag{82}$$

Then matrix $\boldsymbol{C}_{22}^T\boldsymbol{C}_{22} - \tilde{\boldsymbol{L}}_{11}^T\tilde{\boldsymbol{L}}_{11}$ is non-negative definite.

Proof : Substituting (81) into (82) yields

$$\boldsymbol{C}_{22}^T \boldsymbol{L}_{22} - \tilde{\boldsymbol{L}}_{11}^T \tilde{\boldsymbol{L}}_{11} = \tilde{\boldsymbol{l}}_{21}^T \tilde{\boldsymbol{l}}_{21} \geq 0 \tag{83}$$

□

Lemma 3: Suppose $\boldsymbol{L}$ and $\tilde{\boldsymbol{L}}$ are two lower triangular matrices of size $m \times m$, and assume that $\boldsymbol{L}^T \boldsymbol{L} - \tilde{\boldsymbol{L}}^T \tilde{\boldsymbol{L}} \geq 0$. Partition $\boldsymbol{L}$ on an arbitrary diagonal component, and partition $\tilde{\boldsymbol{L}}$ accordingly as

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{L}_{11} & 0 \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix} \quad , \quad \tilde{\boldsymbol{L}} = \begin{bmatrix} \tilde{\boldsymbol{L}}_{11} & 0 \\ \tilde{\boldsymbol{L}}_{21} & \tilde{\boldsymbol{L}}_{22} \end{bmatrix} \tag{84}$$

We have

$$\boldsymbol{L}_{11}^T \boldsymbol{L}_{11} - \tilde{\boldsymbol{L}}_{11}^T \tilde{\boldsymbol{L}}_{11} \geq 0 \quad , \quad \boldsymbol{L}_{22}^T \boldsymbol{L}_{22} - \tilde{\boldsymbol{L}}_{22}^T \tilde{\boldsymbol{L}}_{22} \geq 0 \tag{85}$$

Proof : Since $\boldsymbol{L}^T \boldsymbol{L} - \tilde{\boldsymbol{L}}^T \tilde{\boldsymbol{L}} \geq 0$, we can find a lower triangular matrix $\boldsymbol{C}$ which satisfies

$$\boldsymbol{L}^T \boldsymbol{L} = \tilde{\boldsymbol{L}}^T \left(\boldsymbol{I} + \boldsymbol{C}^T \boldsymbol{C} \right) \tilde{\boldsymbol{L}} \tag{86}$$

According to (84), partition $\boldsymbol{C}$ as

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_{11} & 0 \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix} \tag{87}$$

Substitute (84) and (87) into (86) to obtain

$$\begin{aligned} \boldsymbol{L}_{22}^T \boldsymbol{L}_{22} &= \tilde{\boldsymbol{L}}_{22}^T \left(\boldsymbol{I} + \boldsymbol{C}_{22}^T \boldsymbol{C}_{22} \right) \tilde{\boldsymbol{L}}_{22} \\ \boldsymbol{L}_{11}^T \boldsymbol{L}_{11} &= \tilde{\boldsymbol{L}}_{11}^T \left(\boldsymbol{I} + \boldsymbol{C}_{11}^T \boldsymbol{C}_{11} \right) \tilde{\boldsymbol{L}}_{11} + \triangle \end{aligned} \tag{88}$$

where $\triangle$ is a symmetric non-negative definite matrix. The proof is complete. □

Note that in Lemma 3, we can continue partitioning the sub-diagonal block matrices, and apply Lemma 3 iteratively to obtain a result similar to (85) for an arbitrary partition.

With the help of the above three lemmas, we now show the following results.

Proposition 1 : The proposed grouping and ordering algorithm maximizes the ASE in (75).

Proof : Denote the optimal group and detection sequence determined by the proposed algorithm as G, which has groups $G_{\mathbf{0}}, \ldots, G_{P-1}$. Denote the GDF detector using detection sequence G by Φ_{G-GDF}. The idea of the proof can be summarized as follows. Suppose there is another group and detection sequence $G^{(i)}$, which has groups $G_{\mathbf{0}}^{(i)}, \ldots, G_{P^{(i)}-1}^{(i)}$. Without loss of generality, assume $\forall j$ $(0 \leq j < i)$ $G_j^{(i)} = G_j$ (The superscript (i) means that the first i groups in $G^{(i)}$ are identical to the first i groups in G). $\square$

Now construct a new group and detection sequence $G^{(i+1)}$. The groups of $G^{(i+1)}$ are defined by

$$\begin{cases} G_j^{(i+1)} = G_j^{(i)} = G_j & 0 \leq j < i \\ G_j^{(i+1)} = G_j & j = i \\ G_j^{(i+1)} = G_{j-1}^{(i)} \setminus G_i & j > i \end{cases} \tag{89}$$

To simplify the notation, in the above construction, if $G_j^{(i+1)} = NULL$, we still keep group $G_j^{(i+1)}$ and define $d^2_{G_j^{(i+1)}} = \infty$. Evidently, $G^{(i+1)}$ has one more group than $G^{(i)}$. The following result holds for $G^{(i+1)}$.

Proposition 2: If $G^{(i+1)}$ is constructed according to the above definition, then

(1) $\forall j$ $(0 \leq j < i)$, $d^2_{G_j^{(i+1)}} = d^2_{G_j^{(i)}}$.

(2) $d^2_{G_i^{(i+1)}} \geq d^2_{G_i^{(i)}}$.

(3) $\forall j$ $(i < j \leq P^{(i)})$, $d^2_{G_j^{(i+1)}} \geq d^2_{G_{j-1}^{(i)}}$.

Proof :

(1) For any $j < i$, the decision for group $G_j^{(i)}$ is made by treating the signal corresponding to $G_{j+1}^{(i)}$, ..., $G_{P^{(i)}-1}^{(i)}$ as noise and minimizing the probability of error in ML sense. Therefore, any swapping of users within groups of index larger than j will not affect the performance of $G_j^{(i)}$. This result can be formally proved using Lemma 1.

(2) Since $G_j^{(i)} = G_j^{(i+1)}$ $(\forall j < i)$, this result can be directly obtained from the definition of the optimal grouping and ordering algorithm.

(3) The proof for this part is relatively tricky. In fact, the construction of $G^{(i+1)}$ from $G^{(i)}$ can be divided into three stages. Define the users in group G_i as $K_{\mathbf{0}}$, ..., $K_{|G_i|-1}$. For convenience of discussion, we first consider user $K_{\mathbf{0}}$.

Stage 1 Suppose, in $G^{(i)}$, user $K_{\mathbf{0}}$ belongs to group $G_j^{(i)}$ $(j \geq i)$. Define the action "take out user $K_{\mathbf{0}}$ from group $G_j^{(i)}$", which converts $G^{(i)}$ to

$G^{(S1)}$, as,

$$\begin{cases} G_k^{(S1)} = G_k^{(i)} & k < j \\ G_k^{(S1)} = \{userK_{\mathbf{0}}\} & k = j \\ G_k^{(S1)} = G_j^{(i)} \setminus \{userK_{\mathbf{0}}\} & k = j+1 \\ G_k^{(S1)} = G_{k-1}^{(i)} & k > j+1 \end{cases} \tag{90}$$

The "take out" action is illustrated in Figure 4.

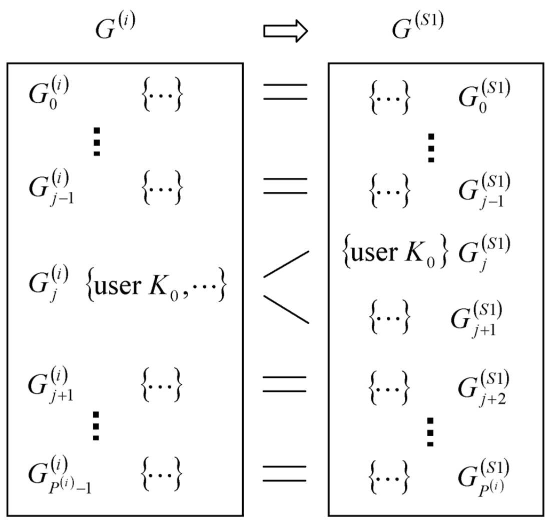

Figure 4: Illustration of "take out user $K_{\mathbf{0}}$ from group $G_j^{(i)}$"

Stage 2 Now in $G^{(S1)}$, we have $G_j^{(S1)} = \{userK_{\mathbf{0}}\}$. Define the action "move up user $K_{\mathbf{0}}$ to follow group $G_{i-1}^{(S1)}$", which converts $G^{(S1)}$ to $G^{(S2)}$, as follows,

$$\begin{cases} G_k^{(S2)} = G_k^{(S1)} & k < i \\ G_k^{(S2)} = \{userK_{\mathbf{0}}\} & k = i \\ G_k^{(S2)} = G_{k-1}^{(S1)} & i < k \leq j \\ G_k^{(S2)} = G_k^{(S1)} & k > j \end{cases} \tag{91}$$

And this is illustrated in Figure 5.

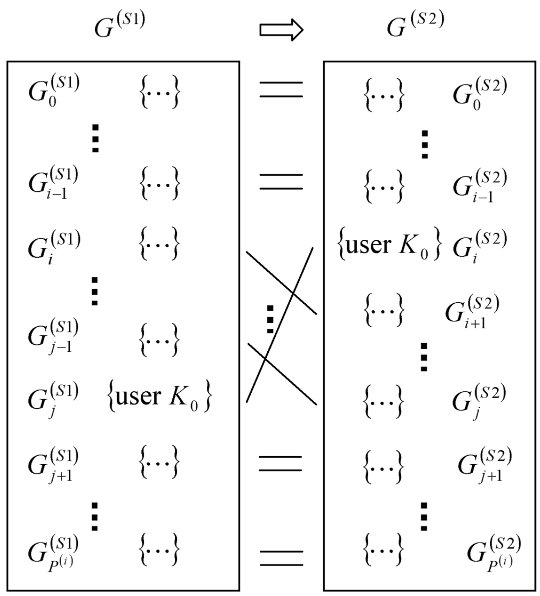

Figure 5: Illustration of "move up user $K_{\mathbf{0}}$ to follow group $G_{i-1}^{(S1)}$"

Continue performing the above two stages on all users $K_{\mathbf{0}}$, ..., $K_{|G_i|-1}$. Denote the resulting group and detection sequence as $G^{(S3)}$. Denote the number of groups in $G^{(S3)}$ by $P^{(S3)}$.

Stage 3 In $G^{(S3)}$, combine groups $\{K_{|G_i|-1}\}$, ..., $\{K_{\mathbf{0}}\}$, which converts $G^{(S3)}$ to $G^{(i+1)}$, as,

$$\begin{cases} G_k^{(i+1)} = G_k^{(S3)} & k < i \\ G_k^{(i+1)} = \{user\, K_{\mathbf{0}}, ..., K_{|G_i|-1}\} & k = i \\ G_k^{(i+1)} = G_{k-|G_i|+1}^{(S3)} & k > i \end{cases} \tag{92}$$

The "Combine" action is illustrated in Figure 6.

In the first stage, without loss of generality, suppose user $K_{\mathbf{0}}$ is the first user in group $G_j^{(i)}$. The "take out" action does not change the order of the users, thus the Cholesky decomposition matrix $\tilde{\boldsymbol{L}}$ remains unchanged. This shows

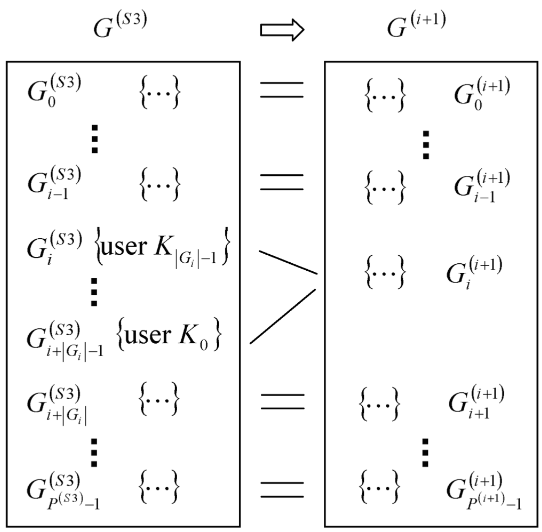

Figure 6: Illustration of "combine groups $\{K_{|G_i|-1}\}$, ..., $\{K_{\mathbf{0}}\}$"

that $\tilde{\boldsymbol{L}}_{G_{j+1}^{(S1)}G_{j+1}^{(S1)}}$ is the south-east diagonal sub-block of $\tilde{\boldsymbol{L}}_{G_j^{(i)}G_j^{(i)}}$. Therefore,

$$d^2_{G_{j+1}^{(S1)}} \geq d^2_{G_j^{(i)}} \tag{93}$$

In the second stage, since the "minimum distance" of a sub-block is the performance measure for the corresponding user group given all the user groups with smaller indices are correctly detected, putting more users into the detected user list will result in a better performance and a larger "minimum distance". In fact, from Lemma 2 and Lemma 3, for any groups $G_k^{(S2)} = G_{k-1}^{(S1)}$, $i < k \leq j$, we have,

$$\tilde{\boldsymbol{L}}^T_{G_k^{(S2)}G_k^{(S2)}}\tilde{\boldsymbol{L}}_{G_k^{(S2)}G_k^{(S2)}} - \tilde{\boldsymbol{L}}^T_{G_{k-1}^{(S1)}G_{k-1}^{(S1)}}\tilde{\boldsymbol{L}}_{G_{k-1}^{(S1)}G_{k-1}^{(S1)}} \geq 0 \tag{94}$$

Hence, in $G^{(i+1)}$, for any $j > i$, $d^2_{G_j^{(i+1)}} \geq d^2_{G_{j-1}^{(i)}}$, which proves part (3) of proposition 3. □

Proposition 2 shows that

$$E(\Phi_{G^{(i+1)}-GDF}) \geq E(\Phi_{G^{(i)}-GDF}) \tag{95}$$

By iteratively using the above construction procedure in the proof of Proposition 1, we will finally get $G^{(P)} = G$ and

$$E(\phi_{G^{(P)}-GDF}) \geq E(\phi_{G^{(i)}-GDF)} \tag{96}$$

which completes the proof. $\square$

The grouping and ordering algorithm is also optimal in the following sense.

Proposition 3 : The proposed grouping and ordering algorithm maximizes the performance lower bound in (37) for every group. In other words, suppose G is the grouping and ordering result obtained from the proposed algorithm, and G_k is one of the groups in G. Further suppose there is another group and detection sequence $\hat{G}$ with $\hat{G}_l$ being one of the groups in $\hat{G}$, and $\hat{G}_l = G_k$. The following result holds,

$$\min(d^2_{G_1}, \ldots, d^2_{G_k}) \geq \min(d^2_{\hat{G}_1}, \ldots, d^2_{\hat{G}_l}) \tag{97}$$

Proof : In the above proof for proposition 1, let $G^{(i)} = \hat{G}$. Construct $G^{(i+1)}$ using the same procedure. Note that $G^{(i)}_l = \hat{G}_l = G_k$, and $G_k \cap G_i = NULL$. Therefore, in $G^{(i+1)}$, we have $G^{(i+1)}_{l+1} = G_k$. And

$$\min(d^2_{G^{(i+1)}_0}, \ldots, d^2_{G^{(i+1)}_{l+1}}) \geq \min(d^2_{\hat{G}_0}, \ldots, d^2_{\hat{G}_l}) \tag{98}$$

By iteratively using the construction procedure, we will finally get $G^{(P)} = G$ which satisfies

$$\min(d^2_{G_0}, \ldots, d^2_{G_k}) \geq \min(d^2_{\hat{G}_0}, \ldots, d^2_{\hat{G}_l}) \tag{99}$$

Hence the proof is complete. □

In addition to the above 2 propositions, we can derive a fast computational method for the GDF detector. We suggest the following steps for the group detection.

Computational Method for GDF Detector: Suppose the GDF detector has P groups, G_0, ..., G_{P-1}.

1) Initialize $\tilde{\boldsymbol{y}}^{(1)} = (\boldsymbol{L}^{-1})^T \boldsymbol{y}$, $\tilde{\boldsymbol{L}} = \boldsymbol{L}\boldsymbol{W}$, $\tilde{\boldsymbol{L}}^{(1)} = \tilde{\boldsymbol{L}}$. Let $j = 1$;

2) Form the white noise system model for the user-expurgated channel, partition the vectors and matrices according to group G_j and its complement $\bar{G}_j$ as

$$\begin{bmatrix} \tilde{\boldsymbol{y}}^{(j)}_{G_j} \\ \tilde{\boldsymbol{y}}^{(j)}_{\bar{G}_j} \end{bmatrix} = \begin{bmatrix} \tilde{\boldsymbol{L}}^{(j)}_{G_j G_j} & 0 \\ \tilde{\boldsymbol{L}}^{(j)}_{\bar{G}_j G_j} & \tilde{\boldsymbol{L}}^{(j)}_{\bar{G}_j \bar{G}_j} \end{bmatrix} \begin{bmatrix} \boldsymbol{b}^{(j)}_{G_j} \\ \boldsymbol{b}^{(j)}_{\bar{G}_j} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{n}}^{(j)}_{G_j} \\ \tilde{\boldsymbol{n}}^{(j)}_{\bar{G}_j} \end{bmatrix} \tag{100}$$

Find the decision on group G_j by

$$\hat{\boldsymbol{b}}_{G_j} = \arg \min_{\boldsymbol{b}_{G_j} \in \{-1,+1\}^{|G_j|}} \left\| \tilde{\boldsymbol{L}}_{G_j G_j} b_{G_j} - \tilde{\boldsymbol{y}}^{(j)}_{G_j} \right\|_2^2 \tag{101}$$

3) Compute $\tilde{\boldsymbol{y}}^{(j+1)}$ by

$$\tilde{\boldsymbol{y}}^{(j+1)} = \tilde{\boldsymbol{y}}^{(j)}_{\bar{G}_j} - \tilde{\boldsymbol{L}}^{(j)}_{\bar{G}_j G_j} \hat{\boldsymbol{b}}_{G_j} \tag{102}$$

Let

$$\tilde{\boldsymbol{L}}^{(j+1)} = \tilde{\boldsymbol{L}}^{(j)}_{\bar{G}_j \bar{G}_j} \tag{103}$$

4) Let $j = j + 1$. If $j < P$, go to step 2; otherwise, stop the computation.

The computational cost for step 1 is $\frac{K(K+1)}{2}$ multiplications and $\frac{K(K-1)}{2}$ additions. Assume the computational cost for step 2 can be bounded by

$$\text{“}\times\text{”} \leq M(|G_j|) \quad , \quad \text{“}+\text{”} \leq S(|G_j|) \tag{104}$$

where “ × ” denotes the number of multiplications and “ + ” denotes the number of additions. In step 3, since $\boldsymbol{b}$ can only take known discrete values and $\tilde{\boldsymbol{L}}$ can be precomputed and stored, only $|G_j| \sum_{k=j+1}^{P-1} |G_k|$ additions are needed. Therefore, the overall computational cost is bounded by

$$\begin{aligned} \text{“}\times\text{”} &\leq \frac{K(K+1)}{2} + \sum_{k=0}^{P-1} [M(|G_k|)] \\ \text{“}+\text{”} &\leq \frac{K(K-1)}{2} + \sum_{k=0}^{P-1} \left[S(|G_k|) + |G_k| \sum_{j=k+1}^{P-1} |G_j| \right] \end{aligned} \tag{105}$$

3.1.3 Simulation Results

Example 1 - continued : In the previous 4-user example, $\eta(\phi_{GDFD}) = 1.78$. The SE for optimal decorrelating-DF detector and the ML detector can be obtained from [Va99] as $E(\phi_{D-DF}) = 1.69$ and $E(\phi_{ML}) = 1.8$. The simulation results are shown in Figure 7, which are consistent with the theoretical analysis.

Example 2 : In this 60-user example, we use 63-length Gold codes as user signature sequences. The power of the user signals are generated by $w_{kk} \sim N(4.5, 4)$ ($N(.)$ represents the Gaussian distribution) and are limited within the

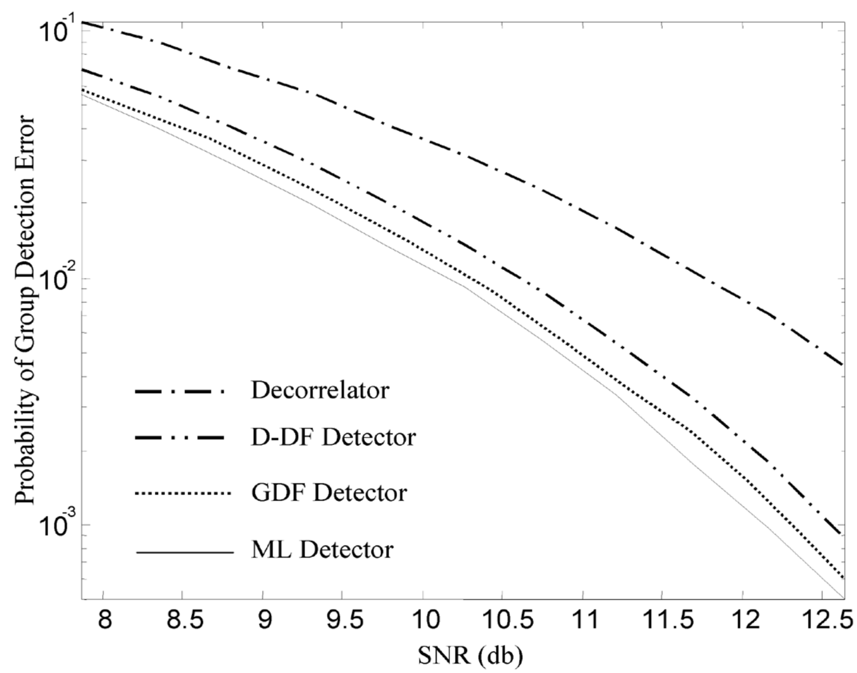

Figure 7: Performance of various methods (4 users, 100000 Monte-Carlo runs with importance sampling)

range of $[2, 7]$. The maximum group size is assumed to be 5. Figure 8 shows the simulation results of the performances of different detectors.

Example 3 : In the last example, we compare the computational loads for different multiuser detectors. We fix the SNR at 12 dB and fix the maximum group size at 5. The signature sequences are randomly generated and the ratio between the spreading factor and the number of users is fixed at 1.2. Let the number of users vary from 5 to 60. Figure 9 shows the worst case computational complexity measured in terms of the number of multiplications plus number of additions of different detectors. For GDF detector, although the computation for finding the optimal user partitioning and user ordering is $O(K^{|G|_{max}})$, it can be done offline. When $|G|_{max}$ is small, as can be seen from the figure, the increase in

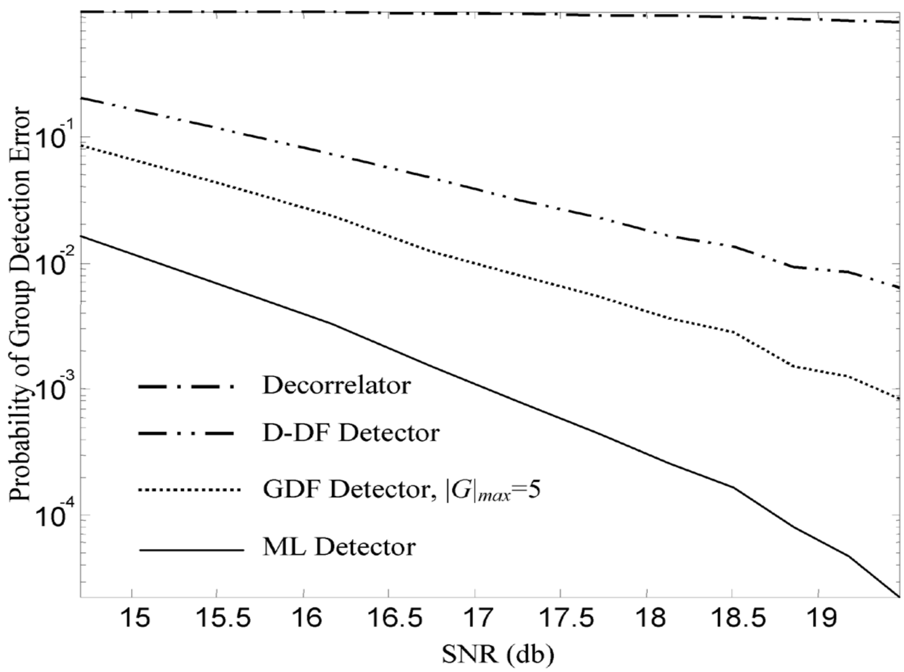

Figure 8: Performance of various methods (60 users, 63-length Gold codes as signature sequences, 100000 Monte-Carlo runs with importance sampling)

the online computational cost of the GDF detector to D-DF detector is marginal. The computational method used for the D-DF detector as well as for the ML detector in this example may be found in [LPWL00].

3.1.4 Conclusions

An optimal grouping and ordering algorithm for Group Decision Feedback Detector is proposed. Together with a fast computational method based on the idea of branch and bound, the proposed algorithm provides a systematic way of improving the Decision Feedback Detector, especially when strong correlation exists among the users. Simulation results show that GDF detector with the optimal grouping and ordering algorithm provides a considerable improvement

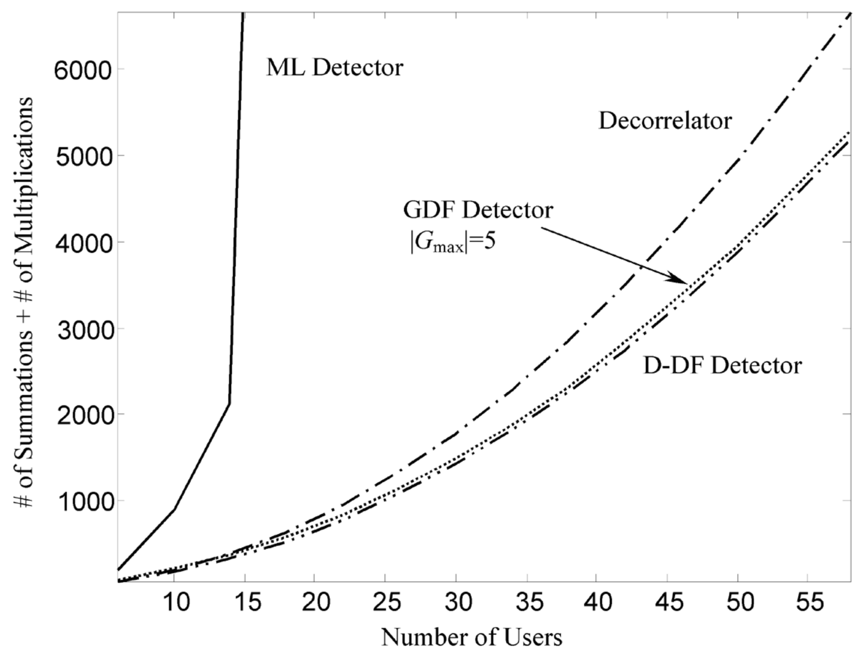

Figure 9: Comparison of worst-case computational cost (random signature sequences, 10000 Monte-Carlo runs)

over DF detector, while the increase in online computational cost is marginal. The proposed method can be easily extended to finite-alphabet signals instead of binary ones.

3.2 Optimal Time Labeling and User Ordering for Ideal DF Detector in Asynchronous CDMA

In asynchronous CDMA, it is commonly (and wrongly) accepted that users should be detected either in decreasing order of their signal powers or in chronological order of their arrival times. In synchronous CDMA, as we have shown in the previous sections, there are $K!$ different user orders, and ordering users according to decreasing signal power is not necessarily optimal when user correlations are considered; this remains so in the asynchronous case. Furthermore, although it may at first appear that ordering in terms of arrival times is at least a fixed strategy, one can actually consider any user as the first-arriving user by fixing a fictitious initial bit to be zero.

In this section, we study user ordering and time labeling for the DF detector. Although our final goal is to minimize its probability of error, even the asymptotic performance of a DF detector is, due to error propagation, hard to estimate. In [Dh95], assuming no error propagation, the theoretical asymptotic performance of the ideal DF detector is given. Building on this, we find a user ordering and time labeling that maximizes the SE of the ideal DF detector. We further show in computer simulations that, with the proposed user ordering and time labeling, the asymptotic performance of an actual DF detector is indistinguishable from the theoretical performance bound. The overall computation for the optimal user ordering and time labeling is shown to be $O(K^4)$, and is, of course, considered as offline computational load since it is required only once for a given user configuration.

3.2.1 DF Detector and The Time Labeling Issue

The asynchronous CDMA system can be described in the z domain by [Vd98]

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{W}\boldsymbol{b}(z) + \boldsymbol{n}(z) \tag{106}$$

where $\boldsymbol{R}(z) = \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1}$ is the correlation matrix.

The asynchronous CDMA system is illustrated in Figure 10.

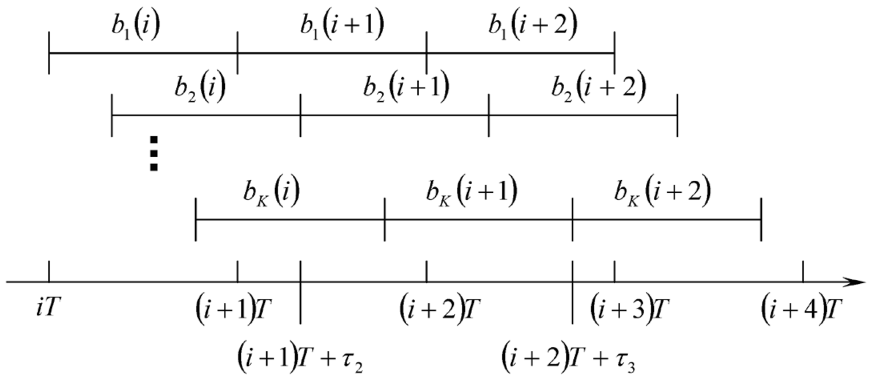

Figure 10: Bit epochs for Asynchronous CDMA (T is the symbol duration, τ_i is the time delay for user i).

In the time domain representation, we denote $\boldsymbol{b}(i)$ to be the binary signal vector for the i^{th} time frame and denote $b_k(i)$ to be the binary signal of user k in time frame i. We refer the time labeling issue as the assignment of the i^{th} or $(i+1)^{st}$ bit of each user to $\boldsymbol{b}(i)$.

It is easy to see that the time labeling of the system is not unique. In Figure 10, suppose we change the time label for user 1: an equivalent bit epoch can be obtained as in Figure 11.

Now, given a time labeling, and after ordering users according to their times of arrival, $\boldsymbol{R}[1]$ becomes an upper triangular matrix with zero diagonal components. Factorize $\boldsymbol{R}(z)$ as $\boldsymbol{R}(z) = (\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1})$. The corresponding time-domain representation of the white noise model is

$$\tilde{\boldsymbol{y}}(i) = \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(i) + \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(i-1) + \tilde{\boldsymbol{n}}(i) \tag{107}$$

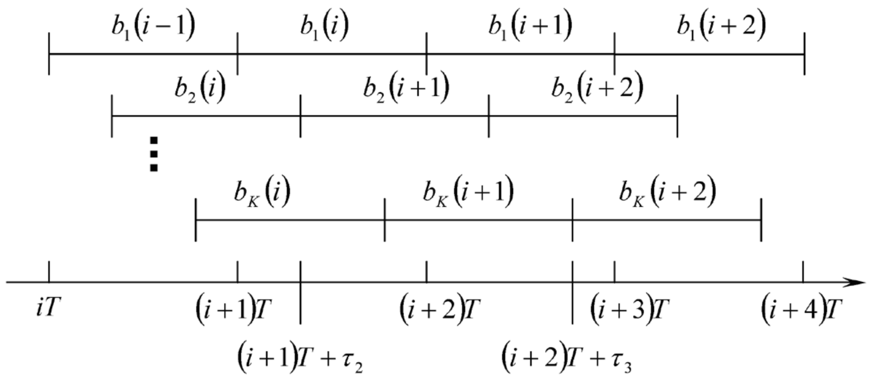

Figure 11: Bit epochs for an equivalent system by changing the time labeling of user 1 in Figure 10 ($b_1(i-1)$ in this figure is physically $b_1(i)$ in Figure 10).

The DF detector assumes that the decisions for user bit $b_k(i-1)$ are made prior to the decision of $b_j(i)$, $\forall k, j$. Hence the performance is affected by both user ordering and time labeling.

Assuming that the past decisions are correct, the SE of the ideal DF detector at time index i can be found via [Va99]

$$E(\Phi_{ideal\ DF}) = \min_{k=1,2,\ldots,K} (F[0]_{kk} w_{kk})^2 \tag{108}$$

3.2.2 Optimal Time Labeling and User Ordering

Rewrite the system model (188) as,

$$\boldsymbol{F}[0]^T \left[\tilde{\boldsymbol{y}}(i) - \boldsymbol{F}[1]\,\boldsymbol{W}\boldsymbol{b}(i-1)\right] = \boldsymbol{F}[0]^T \boldsymbol{F}[0]\,\boldsymbol{W}\boldsymbol{b}(i) + \boldsymbol{F}[0]^T \tilde{\boldsymbol{n}}(i) \tag{109}$$

Given the idealized assumption that the past decisions are correct, the above system model is equivalent to a synchronous CDMA model, in which $R_{AEC} = \boldsymbol{F}[0]^T \boldsymbol{F}[0]$, termed the asymptotic effective correlation, is the equivalent signature correlation matrix. We begin with:

Proposition 1: Given the time labeling, suppose $\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P}$ is the signature correlation matrix of the same system but with a different user order ($\boldsymbol{P}$ is an arbitrary permutation matrix). The AEC matrix of the permuted system satisfies $\tilde{\boldsymbol{R}}_{AEC} = \boldsymbol{P}^T\boldsymbol{R}_{AEC}\boldsymbol{P}$. The SE of the ideal DF detector is maximized by applying the user ordering technique in Theorem 1 of [Va99] to $\boldsymbol{W}\boldsymbol{R}_{AEC}\boldsymbol{W}$.

Proof: Define a sequence of matrices $\boldsymbol{R}^{(j)}$, where

$$\boldsymbol{R}^{(j)} = \boldsymbol{R}[0] - \boldsymbol{R}[1]^T \left[\boldsymbol{R}^{(j-1)}\right]^{-1} \boldsymbol{R}[1] \tag{110}$$

and $\boldsymbol{R}^{(\mathbf{0})} = \boldsymbol{R}[0]$.

From the iterative procedure proposed in [AR98], it follows that, in the above procedure, $\boldsymbol{R}^{(j)} \to \boldsymbol{R}_{AEC}$ as $j \to \infty$. Since $\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P}$, we have $\tilde{\boldsymbol{R}}[0] = \boldsymbol{P}^T\boldsymbol{R}[0]\boldsymbol{P}$ and $\tilde{\boldsymbol{R}}[1] = \boldsymbol{P}^T\boldsymbol{R}[1]\boldsymbol{P}$. By defining the corresponding iterative procedure for the permuted system, we can see that

$$\tilde{\boldsymbol{R}}^{(j)} = \boldsymbol{P}^T\boldsymbol{R}^{(j)}\boldsymbol{P} \tag{111}$$

Therefore, $\tilde{\boldsymbol{R}}_{AEC} = \boldsymbol{P}^T\boldsymbol{R}_{AEC}\boldsymbol{P}$. The proof follows from Theorem 1 of [Va99].

□

Via Proposition 1, we can perform user ordering for all possible time labelings and choose the one that maximizes the SE. However, the matrices $\boldsymbol{R}[0]$ and $\boldsymbol{R}[1]$ for different time labelings are different: we certainly do not want to apply the expensive iterative procedure (110) to all time labelings to find the best.

Fortunately, this is unnecessary. Given a time labeling, we first order users according to their times of arrival (see Figure 10). Apparently, the chronological user ordering vector uniquely represents the corresponding time labeling. For example, the time labeling in Figure 10 can be represented by a vector $T_1 = [1, \ldots, K]$, and the time labeling in Figure 11 is represented by a vector $T_2 = [2, \ldots, K, 1]$. To change time labeling from T_1 to T_2, we only need to change the time index definition of user 1, i.e., $\forall i$, redefine $b_1(i)$ in time labeling T_2 to be $b_1(i+1)$ in time labeling T_1 [1] . Consequently, we denote the conversion from time labeling T_1 to T_2 by

$$T_1 \xrightarrow{\{user1\}} T_2 \tag{112}$$

Note that for a valid time labeling, $b_k(i)$ must overlap with $b_j(i)$ $\forall k, j$ and for any i. We have:

Proposition 2: Suppose there is a time labeling T_G, where $T_1 \xrightarrow{G} T_G$ that converts T_1 to T_G. Then this G can be separated into two sets, G_1, G_2, where $G_1 = \{user\ 1\}$ and G_2 is the rest of the users in G, i.e., $G_2 = G \setminus G_1$. The operation $T_1 \xrightarrow{G} T_G$ can also be written as

$$T_1 \xrightarrow{G_1} T_2 \quad \text{followed by} \quad T_2 \xrightarrow{G_2} T_G \tag{113}$$

Proof: Suppose $user\ i \in G$ and $user\ 1 \notin G$. Redefine $b_k(i)$ in time label T_G to $b_k(i+1)$ in T_1. Now $b_k(i)$ and $b_1(i)$ in time label T_G do not overlap, and this is not valid. Therefore, $user\ 1 \in G$ must be true. $\square$

---

[1] An equivalent conversion can be expressed as, $\forall j \neq 1, \forall i$, redefining $b_j(i)$ in time labeling T_2 to be $b_j(i-1)$ in time labeling T_1. However, without loss of generality, we only consider a single-direction conversion in this paper, i.e., $\forall(k,i)$, the redefinition of $b_k(i)$ in T_2 to be $b_k(i-1)$ in T_1 is prohibited.

It is clear now that the only valid time labelings are circular permutations, that is, $T_1 = [1, \ldots, K]$, $T_2 = [2, \ldots, K, 1]$, …, $T_K = [K, 1, \ldots, K-1]$. Even better, we have:

Proposition 3: Partition $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$, $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ on their second diagonal components (from upper-left corner) as

$$\boldsymbol{R}[0] = \begin{bmatrix} r_{11}[0] & \boldsymbol{r}_{21}[0]^T \\ \boldsymbol{r}_{21}[0] & \boldsymbol{R}_{22}[0] \end{bmatrix}, \quad \boldsymbol{R}[1] = \begin{bmatrix} 0 & \boldsymbol{r}_{12}[1] \\ \boldsymbol{0} & \boldsymbol{R}_{22}[1] \end{bmatrix}$$

$$\boldsymbol{F}[0] = \begin{bmatrix} f_{11}[0] & \boldsymbol{0} \\ \boldsymbol{f}_{21}[0] & \boldsymbol{F}_{22}[0] \end{bmatrix}, \quad \boldsymbol{F}[1] = \begin{bmatrix} 0 & \boldsymbol{f}_{12}[1] \\ \boldsymbol{0} & \boldsymbol{F}_{22}[1] \end{bmatrix}$$

Then, the matrices corresponding to time labeling T_2 become, respectively, $\tilde{\boldsymbol{R}}[0], \tilde{\boldsymbol{R}}[1], \tilde{\boldsymbol{F}}[0]$ and $\tilde{\boldsymbol{F}}[1]$:

$$\tilde{\boldsymbol{R}}[0] = \begin{bmatrix} \boldsymbol{R}_{22}[0] & \boldsymbol{r}_{12}[1]^T \\ \boldsymbol{r}_{12}[1] & r_{11}[0] \end{bmatrix}, \quad \tilde{\boldsymbol{R}}[1] = \begin{bmatrix} \boldsymbol{R}_{22}[1] & \boldsymbol{r}_{21}[0] \\ \boldsymbol{0} & 0 \end{bmatrix}$$

$$\tilde{\boldsymbol{F}}[0] = \begin{bmatrix} \boldsymbol{F}_{22}[0] & \boldsymbol{0} \\ \boldsymbol{f}_{12}[1] & f_{11}[0] \end{bmatrix}, \quad \tilde{\boldsymbol{F}}[1] = \begin{bmatrix} \boldsymbol{F}_{22}[1] & \boldsymbol{f}_{21}[0] \\ \boldsymbol{0} & 0 \end{bmatrix}$$

Proof: Suppose the asynchronous CDMA system has M time frames. We can view the asynchronous CDMA system as a KM-user synchronous CDMA system (as introduced in [Vd98]). The overall signature correlation matrix $\boldsymbol{R}$ is illustrated in Figure 12.

By partitioning the $\boldsymbol{R}$ matrix according to the time frame definition of T_1 (shown by black solid lines in Figure 12), the diagonal block matrices are equal to $\boldsymbol{R}[0]$, while the first off-diagonal block matrices are $\boldsymbol{R}[1]$ and $\boldsymbol{R}[1]^T$ (shown by

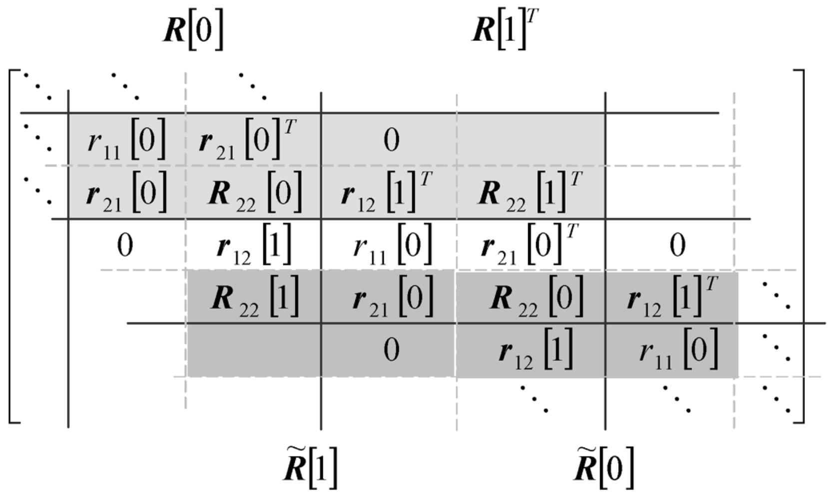

Figure 12: Relation between signature correlation matrices of different time labelings.

the light-grey blocks in Figure 12). Conversion of time labeling from T_1 to T_2 only changes the time frame definition; hence, if we partition the $\boldsymbol{R}$ matrix according to the time frame definition of T_2 (dashed grey lines in Figure 12), the resulting diagonal block matrices must equal $\tilde{\boldsymbol{R}}[0]$ and the first off-diagonal block matrices are $\tilde{\boldsymbol{R}}[1]$ and $\tilde{\boldsymbol{R}}[1]^T$ (the dark-grey blocks in Figure 12), respectively. This can be easily extended to all time labelings. □

It may also be observed from proposition 3 that the SEs of the ideal DF detectors using chronological user ordering are identical. Furthermore, for chronological user ordering, since time labeling does not change the detection order of the physical signals, apparently, the performances of the actual DF detectors with chronological user ordering are identical.

With the above results, the user ordering and time labeling algorithm proceeds as follows.

User Ordering and Time Labeling Procedure:

(1) Choose an arbitrary time labeling. Order users according to their times of arrival.

(2) Apply the iterative procedure proposed in [AR98] to obtain $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$.

(3) Via Proposition 2, obtain the other $K-1$ time labelings with the corresponding chronological user order.

(4) Via Proposition 3, obtain the corresponding $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ matrices for the other $K-1$ time labelings.

(5) Compute $\boldsymbol{R}_{AEC}$ for all K different time labelings.

(6) Apply the user ordering proposed in Theorem 1 of [Va99] to $\boldsymbol{W}\boldsymbol{R}_{AEC}\boldsymbol{W}$ for the K time labelings to obtain the optimal user order and the corresponding SE of the ideal DF detector for the K time labelings.

(7) Choose the time-labeling and user-ordering pair that maximizes the ideal SE.

The following 3-user example illustrates the user ordering and time labeling procedure.

Example 1: Suppose users are ordered according to their physical times of arrival (without adding any fictitious signal bits). Define the original time labeling as $T_1 = [1,2,3]$. The correlation matrices of the system and the $\boldsymbol{W}$ matrix are given by

$$\boldsymbol{R}_{T_1}[0] = \begin{bmatrix} 1.0 & -0.27 & -0.01 \\ -0.27 & 1.0 & 0.16 \\ -0.01 & 0.16 & 1.0 \end{bmatrix}; \; \boldsymbol{R}_{T_1}[1] = \begin{bmatrix} 0 & -0.06 & 0.55 \\ 0 & 0 & -0.49 \\ 0 & 0 & 0 \end{bmatrix};$$
$$\boldsymbol{W} = diag(4.36, 4.48, 4.1) \tag{114}$$

By applying the iterative procedure proposed in [AR98], the factorization matrices corresponding to time labeling T_1 are obtained as

$$\boldsymbol{F}_{T_1}[0] = \begin{bmatrix} 0.96 & 0 & 0 \\ -0.28 & 0.97 & 0 \\ -0.01 & 0.25 & 0.75 \end{bmatrix}; \; \boldsymbol{F}_{T_1}[1] = \begin{bmatrix} 0 & -0.06 & 0.43 \\ 0 & 0 & -0.51 \\ 0 & 0 & 0 \end{bmatrix} \tag{115}$$

According to proposition 2, the two other valid time labelings are $T_2 = [2,3,1]$:

$$\boldsymbol{R}_{T_2}[0] = \begin{bmatrix} 1.0 & 0.16 & -0.06 \\ 0.16 & 1.0 & 0.55 \\ -0.06 & 0.55 & 1.0 \end{bmatrix}; \; \boldsymbol{R}_{T_2}[1] = \begin{bmatrix} 0 & -0.49 & -0.27 \\ 0 & 0 & -0.01 \\ 0 & 0 & 0 \end{bmatrix} \tag{116}$$

and $T_3 = [3,1,2]$:

$$\boldsymbol{R}_{T_3}[0] = \begin{bmatrix} 1.0 & 0.55 & -0.49 \\ 0.55 & 1.0 & -0.27 \\ -0.49 & -0.27 & 1.0 \end{bmatrix}; \; \boldsymbol{R}_{T_3}[1] = \begin{bmatrix} 0 & -0.01 & 0.16 \\ 0 & 0 & -0.06 \\ 0 & 0 & 0 \end{bmatrix} \tag{117}$$

From proposition 3, the factorization matrices corresponding to time labeling T_2 and T_3 are,

$$\boldsymbol{F}_{T_2}[0] = \begin{bmatrix} 0.97 & 0 & 0 \\ 0.25 & 0.75 & 0 \\ -0.06 & 0.43 & 0.96 \end{bmatrix}; \quad \boldsymbol{F}_{T_2}[1] = \begin{bmatrix} 0 & -0.51 & -0.28 \\ 0 & 0 & -0.01 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{F}_{T_3}[0] = \begin{bmatrix} 0.75 & 0 & 0 \\ 0.43 & 0.96 & 0 \\ -0.51 & -0.28 & 0.97 \end{bmatrix}; \quad \boldsymbol{F}_{T_3}[1] = \begin{bmatrix} 0 & -0.01 & 0.25 \\ 0 & 0 & -0.06 \\ 0 & 0 & 0 \end{bmatrix} \tag{118}$$

In step 5, we calculate the $\boldsymbol{R}_{AEC}$ matrices for all 3 time labelings,

$$(\boldsymbol{R}_{AEC})_{T_1} = \begin{bmatrix} 1.0 & -0.27 & -0.01 \\ -0.27 & 1.0 & 0.19 \\ -0.01 & 0.19 & 0.56 \end{bmatrix}$$

$$(\boldsymbol{R}_{AEC})_{T_2} = \begin{bmatrix} 1.0 & 0.16 & -0.06 \\ 0.16 & 0.74 & 0.41 \\ -0.06 & 0.41 & 0.92 \end{bmatrix}$$

$$(\boldsymbol{R}_{AEC})_{T_3} = \begin{bmatrix} 1.0 & 0.55 & -0.49 \\ 0.55 & 1.0 & -0.27 \\ -0.49 & -0.27 & 0.93 \end{bmatrix} \tag{119}$$

By using the user ordering algorithm proposed in Theorem 1 of [Va99], the optimal SE and the corresponding optimal user order of the ideal DF detector corresponding to time labelings T_1, T_2 and T_3 are $E^{T_1} = 9.43$, for which the algorithm of [Va99] finds $\{user1, user2, user3\}$ to be the best user ordering within this time labeling; $E^{T_2} = 12.49$, for which we get $\{user2, user1, user3\}$; and $E^{T_3} = 13.26$, which yields $\{user2, user1, user3\}$; respectively. Therefore, the optimal time labeling is T_3 and the optimal user order is $\{user2, user1, user3\}$.

Since the iterative procedure (110) needs to be applied at least once in the DF detector for asynchronous CDMA, the extra computation to obtain the optimal user order and time labeling is $O(K^4)$.

3.2.3 Computer Simulations

In this section, we use computer simulations to verify the optimality of the proposed user ordering and time labeling algorithm on actual DF detectors. The following conventions are used:

(1) **Optimal DF Detector:** The DF detector that uses the optimal user order and time labeling proposed here.

(2) **Chronological User Order:** The DF detector whose order of decisions is exactly that of the time of arrival of each bit.

(3) **Decreasing Power User Order with Physical Time Labeling:** DF detector with the same time labeling of bit epochs as (2). However, within each epoch, ordering is based on user signal powers.

(4) **Decreasing Power User Order with Optimal Time Labeling:** DF detector with optimal time labeling as (1). However, ordering within each epoch is based on user signal powers.

(5) **Ideal DF Detector:** Optimal DF detector (as in (1)) for which previously-decoded bits are error-free. This can be considered as a lower bound on the probability of group detection error for a DF detector.

(6) **Truncated DF Detector:** Although the proposed algorithm maximizes the SE of the ideal DF detector, the performance of the actual DF detector is hard to estimate due to error propagation [Dh95]. Suppose the entire transmission data has M time frames. As shown in [Vd98], one can consider every bit as being transmitted by a different fictitious user and consider the K-user M-frame asynchronous CDMA system as a MK user synchronous system. Consequently, the optimal DF detector that minimizes the probability of asymptotic group detection error for the K user asynchronous CDMA is obtained by applying the optimal user ordering algorithm, introduced in Theorem 1 of [Va99], to the MK user synchronous CDMA.[2]

Unfortunately, such a DF detector is quite hard to implement. To avoid this, in (185), we only consider the data within a processing window from time frame $(i-L)$ to $(i+L)$ when detecting the user signal vector in time frame (i). When L is sufficiently large, the performance of the truncated

[2] Asymptotically, the probability of error of the K user asynchronous CDMA system is dominated by the users with minimum Asymptotic Effective Energy [Dh95], which is also the SE of the MK user synchronous system.

DF detector is practically the same as that of the optimal DF detector. In computer simulations, we also assume that the user signals outside the processing window are known.

Example 1 (continued): In Figure 27, the performances of four DF detectors are considered. The performances of the actual DF detector and the ideal DF detector are indistinguishable. Although the optimal user ordering in this example happens to be the decreasing power user order, without a good time labeling the decreasing-power user ordering strangely cannot even ensure a near-optimal performance. From the simulation results, the performance of the DF detector with decreasing power user order is the same as that of the chronological user order.

Although in most of the cases, the proposed DF detector achieves the same performance as its corresponding ideal version, one of the exceptions is shown below.

Example 2 : The correlation matrices of the system and the $\boldsymbol{W}$ matrix of this 3-user example is given by

$$\boldsymbol{R}[0] = \begin{bmatrix} 1.0 & 0.47 & -0.44 \\ 0.47 & 1.0 & -0.47 \\ -0.44 & -0.47 & 1.0 \end{bmatrix}$$

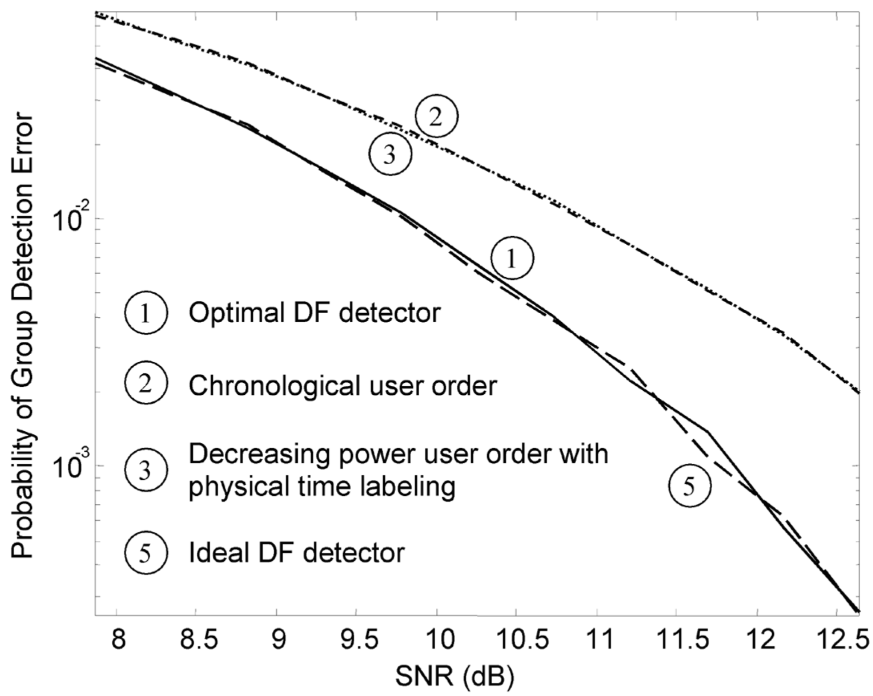

Figure 13: Performance comparison of DF detectors with different user order and time labeling. 3-Users, 100000 Monte-Carlo runs.

$$\begin{aligned} \boldsymbol{R}[1] &= \begin{bmatrix} 0 & -0.51 & 0.02 \\ 0 & 0 & 0.01 \\ 0 & 0 & 0 \end{bmatrix} \\ \boldsymbol{W} &= diag(4.45, 4.53, 4.45) \end{aligned} \tag{120}$$

The performances of several detectors are depicted in Figure 28. Due to error propagation, the performance of the actual DF detector does not match its ideal version. However, the lower bound provided by the truncated DF is still achieved; note that the ideal DF detector does not necessarily provide a tight bound and, in fact, the truncated DF detector is a reasonable bound on achievable performance. In this example, since there is no significant difference in user signal powers, the decreasing power user ordering strategy is equivalent to a random user ordering in terms of performance. Even with optimal time labeling, the DF detector with decreasing power user order shows a significantly degraded performance.

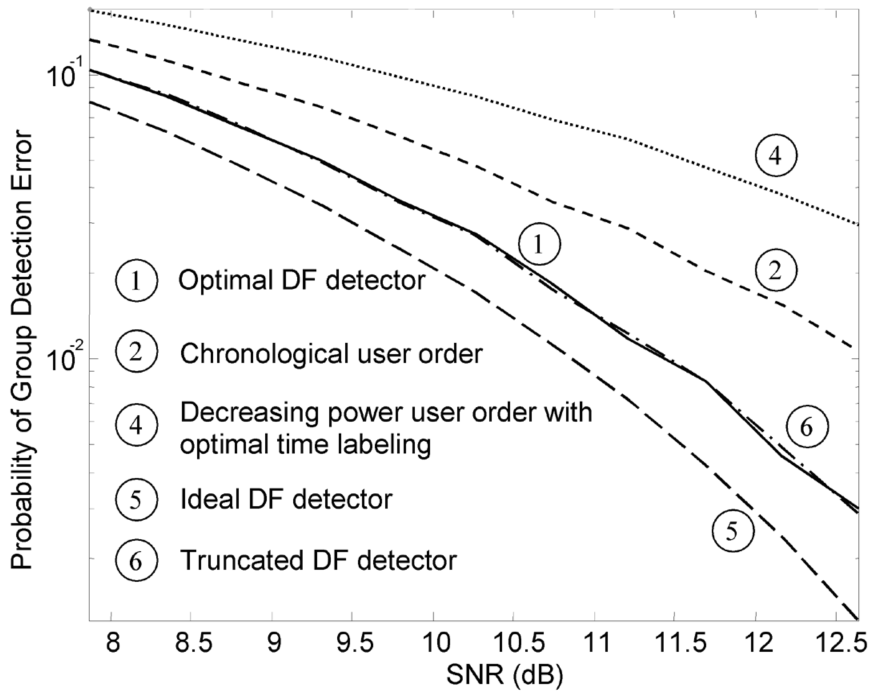

Figure 14: Performance Comparison of DF detectors with different user ordering and time labeling. 3-Users, 100000 Monte-Carlo runs. The truncated DF detector uses a window width $L = 81$, meaning that it is equivalent to synchronous CDMA with 243 “users”.

Example 3 : In the last example, we consider an overloaded system that has 70 users. The signature sequences are chosen to be 31-length Gold codes. The time delays of user signals are random and uniformly-distributed within a symbol duration and we use the system model introduced in [Ps77] to generate the signature correlation matrix. The signal to noise ratio is fixed at $16.6dB$. The square roots of user signal powers are generated randomly by $w_{kk} \sim N(4.5, 4)$ ($N(.)$ represents the Gaussian distribution) and are limited within the range of

[3, 6]. Figure 15 shows the histogram of the performances of 70 DF detectors with randomly generated user orders and time labelings. Apparently, the performance of a DF detector with random user ordering and time labeling can be far from optimal. In order to show the effect of time labeling only, in Figure 16, we show the histogram of performances of 70 DF detectors with different time labelings and the corresponding optimal user orders. We can see that, even when an optimal user order is guaranteed, the performance improvement with an optimal time labeling can still be significant.

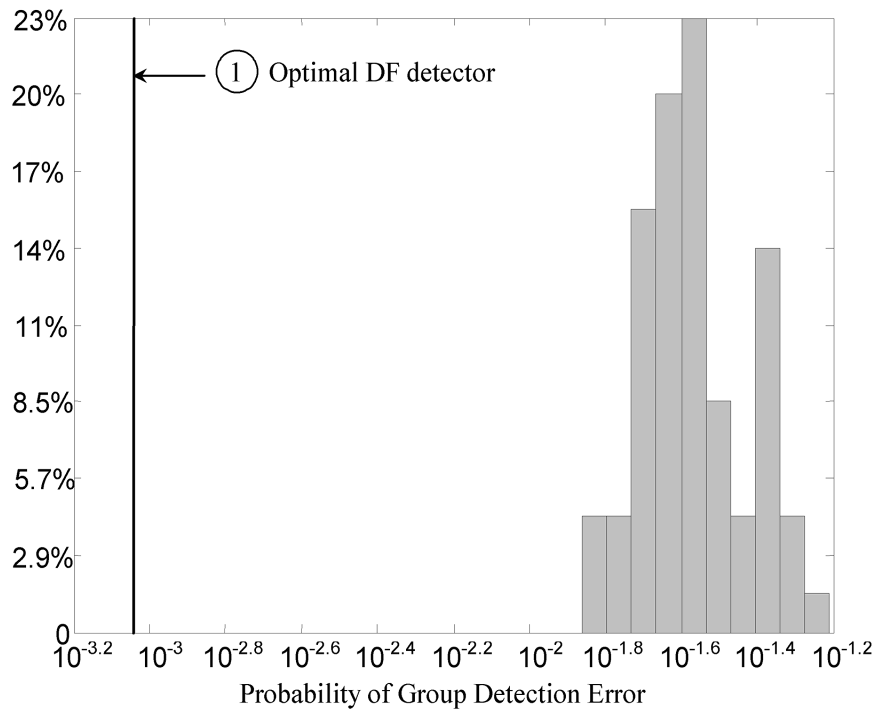

Figure 15: Performance histogram (Random time labeling with random user ordering, 70 users with 31-length Gold codes, 200000 Monte-Carlo runs)

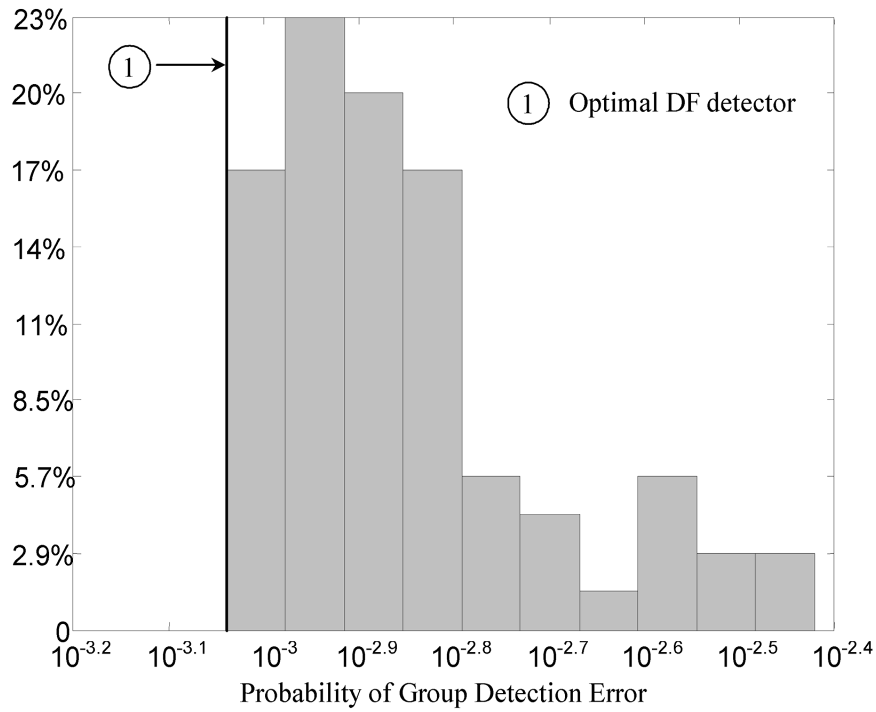

Figure 16: Performance histogram (Random time labeling with optimal user ordering, 70 users with 31-length Gold codes, 200000 Monte-Carlo runs)

3.2.4 Conclusions

The time labeling and user ordering that jointly optimize the asymptotic performance of an ideal decision feedback detector in asynchronous CDMA are given. Simulation results show that the ordering provided is not just asymptotically optimal (i.e., in terms of symmetric energy), but is also practically significant. The proposed ordering can be performed offline with a computational complexity of $O(K^4)$. The performance improvement can be substantial when compared to a chronological or a received signal power user-ordering, the natural and simplest first choices. The ideas can easily be extended to group decision feedback detection [LPW01] in asynchronous CDMA.

Chapter 4

Fast Optimal and Suboptimal "Any-time" Algorithm Based on Branch-and-Bound

Although optimal multiuser detection is generally NP hard and is therefore unlikely to be implemented in practice, it is often included in computer simulations for comparisons and research purposes. Fast optimal algorithms enable the simulation of optimal detection for large size systems and are very helpful for the research of advance suboptimal detection methods.

Define $\boldsymbol{H}$ and $\tilde{\boldsymbol{L}}$ by (70). Then, the ML optimal detector can be represented by

$$\Phi_{ML} : \hat{\boldsymbol{b}} = \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left(\boldsymbol{b}^T \boldsymbol{H} \boldsymbol{b} - 2 \boldsymbol{y}^T \boldsymbol{W} \boldsymbol{b} \right) \tag{121}$$

The idea of using a branch and bound method in solving optimization problems is already well known [PR90]. However, the tradeoff between a tight lower bound and a lower bound with less computational requirements is common to most of the problems. In multiuser detection, branch-and-bound method with breadth-first search has been used in [SW97] to find the minimum distance of $\boldsymbol{H}$. In this chapter, we propose a new optimal algorithm based on the Branch-and-Bound search. Although the worst case computational cost for any optimal algorithm is exponential in the number of users, with the help of the user ordering and a tight MSE lower bound on the optimal cost, the proposed algorithm reduces the average computational cost significantly.

The optimal algorithm is proposed and studied in section 4.1. As a by-product, a suboptimal “Any-time” algorithm is proposed in section 4.2. Simulation results are shown in section 4.3.

4.1 Optimal Algorithm Based on Branch and Bound

Since $\boldsymbol{H}^{-1} = \tilde{\boldsymbol{L}}^{-1}\tilde{\boldsymbol{L}}^{-T}$, from the white noise model, the objective function in (121) can be equivalently written as

$$\Phi_{ML} : \hat{\boldsymbol{b}} = \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left\| \tilde{\boldsymbol{L}}\boldsymbol{b} - \tilde{\boldsymbol{y}} \right\|_2^2 \tag{122}$$

Define $\boldsymbol{D} = \tilde{\boldsymbol{L}}\boldsymbol{b}$, and denote the kth component of $\boldsymbol{D}$ and $\tilde{\boldsymbol{y}}$ by D_k and $\tilde{y}_k$, respectively. Consequently, we have

$$\begin{aligned} \phi_{ML} : \hat{\boldsymbol{b}} &= \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left\| \boldsymbol{D} - \tilde{\boldsymbol{y}} \right\|_2^2 \\ &= \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \sum_{k=1}^{K} (D_k - \tilde{y}_k)^2 \end{aligned} \tag{123}$$

Here, since $\tilde{\boldsymbol{L}}$ is a lower triangular matrix, D_k depends only on $(b_1, b_2, ..., b_k)$. When the decisions for the first k users are fixed, the term

$$\xi_k = \sum_{i=1}^{k} (D_i - \tilde{y}_i)^2 \tag{124}$$

can serve as a lower bound of (123). It can be easily seen that the lower bound is in fact an unconstrained MMSE solution and is achievable when the binary constraints on $(b_{k+1}, ..., b_K)$ are disregarded. The branch and bound tree search to find the minimum value of $\|\boldsymbol{D} - \tilde{\boldsymbol{y}}\|_2^2$ is described below.

Similar to a general branch and bound method [Bk99], the algorithm maintains a node list called $OPEN$, and a scalar called $UPPER$, which is equal to the minimum feasible cost found so far, i.e., the "Current-Best" solution. Define k to be the level of a node (virtual root node has level 0). Label the branches with $D_k(b_1, b_2, ..., b_{k+1})$, which connect the two nodes $(b_1, ..., b_k)$ and $(b_1, ..., b_{k+1})$. The node $(b_1, ..., b_k)$ is labeled with the lower bound ξ_k. Also, define $\mathbf{v}_k = \sum_{i=1}^{k} \left[b_i * (\text{the } i\text{th column of } \tilde{\boldsymbol{L}}) \right] - \tilde{\boldsymbol{y}}$, denote $[\mathbf{v}_k]_j$ as the jth component of vector $\mathbf{v}_k$, and l_{ij} as the (i, j)th element of $\boldsymbol{L}$. The branch and bound algorithm proceeds as follows.

1) Precompute $\tilde{\boldsymbol{y}} = {\boldsymbol{L}^{-1}}^T \boldsymbol{y}$;

2) Initialize $k = 0$. $\mathbf{v}_k = -\tilde{\boldsymbol{y}}$, $\xi_k = 0$, $UPPER = +\infty$ and $OPEN = NULL$;

3) Set $k = k + 1$. Choose the node in level k such that $b_k = -sign\left([\mathbf{v}_{k-1}]_k\right)$. If $k < K$, append the node with $b_k = sign\left([\mathbf{v}_{k-1}]_k\right)$ to the end of the $OPEN$ list;

4) Compute $\mathbf{v}_k = \mathbf{v}_{k-1} + b_k * (\text{the } k\text{th column of } \tilde{\boldsymbol{L}})$;

5) Compute $\xi_k = \xi_{k-1} + (D_k - \tilde{y}_k)^2 = \xi_{k-1} + (\mathbf{v}_k)_k^2$;

6) If $\xi_k \geq UPPER$ and the $OPEN$ list is not empty, drop this node. Pick the node from the end of the $OPEN$ list, set k equal to the level of this node and go to step 4;

7) If $\xi_k < UPPER$, $k = K$ and the $OPEN$ list is not empty, update the "Current-Best" solution and $UPPER = \xi_k$. Pick the node from the end of the $OPEN$ list, set k equal to the level of this node and go to step 4;

8) If $\xi_k < UPPER$ and $k \neq K$, go to step 3;

9) If $\xi_k < UPPER$, $k = K$ and the $OPEN$ list is empty, update the "current-best" solution and $UPPER = \xi_k$;

10) For all other cases, stop and report the "current-best" solution.

Example 1: The following 3-user example illustrates the procedure. The system is given by

$$\begin{aligned}\boldsymbol{H} &= \begin{bmatrix} 4.25 & 0.85 & 0.57 \\ 0.85 & 3.0 & 1.6 \\ 0.57 & 1.6 & 2.0 \end{bmatrix} \\ &= \begin{bmatrix} 2.0 & 0 & 0 \\ 0.3 & 1.3 & 0 \\ 0.4 & 1.1 & 1.4 \end{bmatrix}^T \begin{bmatrix} 2.0 & 0 & 0 \\ 0.3 & 1.3 & 0 \\ 0.4 & 1.1 & 1.4 \end{bmatrix} \end{aligned} \tag{125}$$

Assume the source signal is $\boldsymbol{b} = [1, -1, 1]^T$, the noise vector multiplied by $\boldsymbol{W}$ is $\boldsymbol{Wn} = [0.81, 1.93, -0.22]^T$, hence $\boldsymbol{Wy} = [4.78, 1.38, 0.75]^T$. Figure 17 shows the branch-and-bound tree structure.

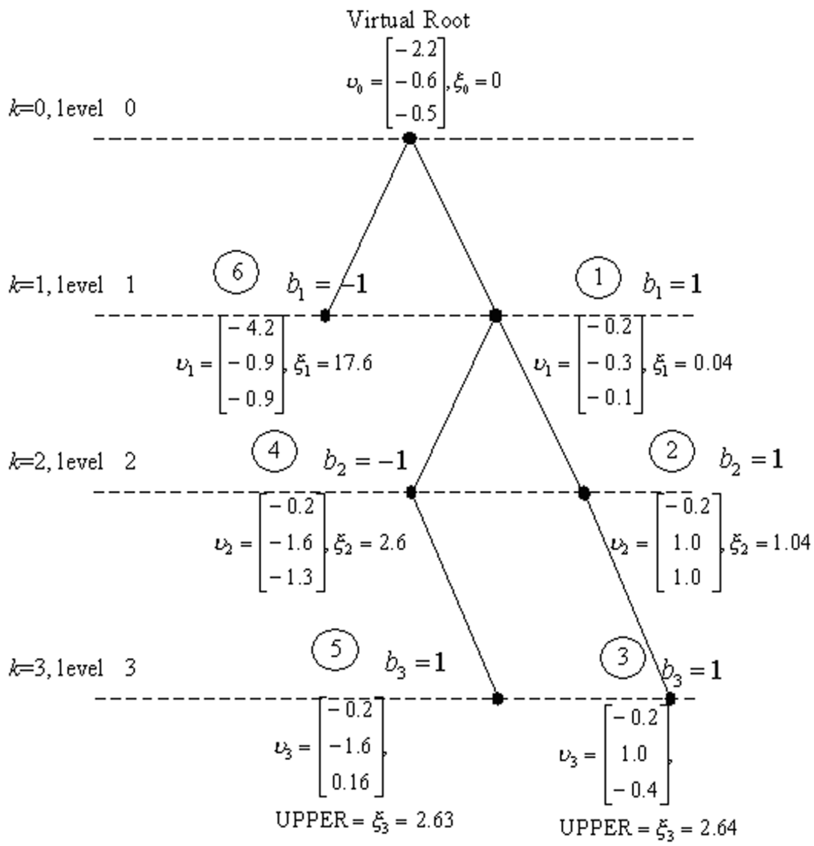

Figure 17: Example of the depth-first Branch-and-Bound algorithm

In step 1), we precompute $\tilde{\boldsymbol{y}} = (\boldsymbol{L}^{-1})^T \boldsymbol{y} = (\tilde{\boldsymbol{L}}^{-1})^T \boldsymbol{Wy} = [2.2, 0.6, 0.5]^T$. Then, initialize $k = 0$, $\mathbf{v_0} = [-2.2, -0.6, -0.5]^T$, $\xi_0 = 0$, $UPPER = +\infty$, $OPEN = NULL$. In step 3), let $k = 1$, choose the node with $b_1 = -sign(-2.2) = 1$ (node 1 in Figure 17). Add node 5 to the $OPEN$ list. Update $\mathbf{v}_1 = [-0.2, -0.3, -0.1]^T$, $\xi_1 = 0.04$. Since $\xi_1 < UPPER$, goto step 3. This leads us to node 2. Add node

4 to the end of the $OPEN$ list. Then, since the next level is the bottom level, from step 3, we know node 3 gives better result than node $(1, 1, -1)$. Therefore, without changing the $OPEN$ list, we go to node 3 (which is the first feasible solution and, as shown later, it also corresponds to the D-DF solution) and update $UPPER = \xi_3 = 2.64$. In step 6, we pick node 4 from the end of the $OPEN$ list. Go to node 5, and obtain $\xi_3 = 2.63 < UPPER$, which means that node 5 is a better solution. Update $UPPER = 2.63$ and pick node 6 from the $OPEN$ list. For node 6, since $\xi_1 = 17.6 > UPPER$, we drop this node. Now the $OPEN$ list is empty, the algorithm stops and reports node 5 as the optimal solution.

The above algorithm is a branch and bound method with depth-first search. The computational cost for step 1) is $\frac{K(K+1)}{2}$ multiplications and $\frac{K(K-1)}{2}$ additions. In step 3), since b_k can only take known discrete values, $\tilde{\boldsymbol{L}}$ can be precomputed and stored; hence, only $K - k + 1$ additions are needed to obtain $\boldsymbol{v}_k$. Step 5) needs 1 addition and 1 multiplication. Notice that step 1) is outside the branch-and-bound search. To update the lower bound for a node on level $K - k + 1$ $(k = 1, ..., K)$, only $k + 1$ additions and 1 multiplication is needed. In addition, the computation for finding the first feasible solution (also the optimal solution in the noise-free case) requires $\frac{K(K+3)}{2}$ multiplications and $K(K + 1)$ additions.

Proposition 1: The first feasible solution obtained from the above depth-first search is the solution of D-DFD method.

Proof: From step 3), when we branch, we first go to the node with a smaller lower bound value. In the above Branch-and-Bound method, suppose $(b_1, ..., b_{k-1})$ has already been fixed by the branch, the choice of b_k for the branch and bound method can be described by

$$\tilde{\boldsymbol{b}} = \arg \min_{\substack{\boldsymbol{b}_k \in \{-1,+1\} \\ \boldsymbol{b}_{k+1},...,\boldsymbol{b}_K \in (-\infty,\infty)}} (\boldsymbol{b} - \boldsymbol{H}^{-1}\mathbf{W}\boldsymbol{y})^T H (\boldsymbol{b} - \boldsymbol{H}^{-1}\mathbf{W}\boldsymbol{y})$$
$$b_k = \tilde{b}_k \tag{126}$$

Notice that in (126), $(b_1, ..., b_{k-1})$ is fixed and we only have a binary constraint on b_k. The choice of b_k for D-DF method, however, is given by

$$\tilde{\boldsymbol{b}} = \arg \min_{b_k,...,b_K \in (-\infty,\infty)} (\boldsymbol{b} - \boldsymbol{H}^{-1}\mathbf{W}\boldsymbol{y})^T \boldsymbol{H}(b - \boldsymbol{H}^{-1}\mathbf{W}\boldsymbol{y})$$
$$b_k = sign(\tilde{b}_k) \tag{127}$$

Figure 18 shows the difference between the above two choices. The ellipses here represent the level curves of the objective function. For the D-DFD method, the decision on b_k is made by comparing the lengths $|AO|$ and $|BO|$. While for the proposed branch-and-bound method, the decision on b_k is made by comparing the lengths $|CO|$ and $|DO|$. Since the triangles AOC and BOD are similar, (126) and (127) are equivalent.

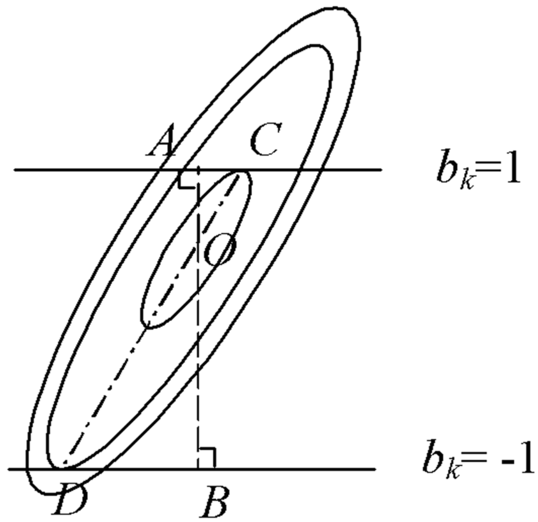

Figure 18: Comparison of D-DF detector and Branch-Bound decisions on b_k

Example 1 - continued : In the above example, on node 1, the user expurgated channel for the D-DF method is represented by

$$\begin{bmatrix} w_{22}y_2 \\ w_{33}y_3 \end{bmatrix} - \begin{bmatrix} 0.85 \\ 0.57 \end{bmatrix} = \begin{bmatrix} 3.0 & 1.6 \\ 1.6 & 2.0 \end{bmatrix} \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} w_{22}n_2 \\ w_{33}n_3 \end{bmatrix} \tag{128}$$

According to (127), the decision on b_2 for D-DFD is made by

$$\begin{aligned} \tilde{\mathbf{b}} &= \begin{bmatrix} 3.0 & 1.6 \\ 1.6 & 2.0 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 1.38 \\ 0.75 \end{bmatrix} - \begin{bmatrix} 0.85 \\ 0.57 \end{bmatrix} \right\} = \begin{bmatrix} 0.22 \\ -0.09 \end{bmatrix} \\ b_2 &= sign(\tilde{b}_2) = 1 \end{aligned} \tag{129}$$

which is consistent with the depth-first direction of branch-and-bound algorithm. However, as shown in the example, D-DF method failed to find the optimal solution.

Recall that in the branch-and-bound algorithm, the computational cost required to obtain the first feasible solution (also the solution of D-DF detector) is much less than the computational cost of a conventional linear detector. Evidently, any further computations will result in better accuracy than the D-DF detector (unless the D-DF solution is already optimal).

4.2 "Any-Time" Suboptimal Algorithm

Although the average computational cost may not be very high, the computation for the worst case is still exponential in the number of users since the ML solution is generally NP hard. Hence the optimal algorithm is not implementable when the number of users is large. When a strict limitation on computational cost exists, the "current-best" solution in the above branch and bound method can serve as a sub-optimal alternative to the NP hard optimal solution.

Define the sub-opimal detector that explores the sub-tree under and including level $K-k+1$ to be Φ_{BB-k} $(k = 1, ..., K)$. From the above analysis of the computational cost, the worst-case computation for Φ_{BB-k} is given by

$$\begin{aligned} \text{Multiplications} \quad & \leq \frac{K(K+3)}{2} + 3 * 2^{k-1} - k - 2 \\ \text{Additions} \quad & \leq K(K+1) + 5 * 2^{k} \\ & - \frac{(k+3)(k+4)}{2} \end{aligned} \tag{130}$$

To derive the SE measure for Φ_{BB-k}, define $P(i|1, ..., i-1)$ to be the event that the decision on user i is correct $(i = 1, ..., K-k)$, given all the decisions on users $j < i$ are correct. Consider the ML solution (122). Substitute (10) into

(122), denote the true source signal by $\boldsymbol{b_0}$, to obtain

$$\Phi_{ML} : \hat{\boldsymbol{b}} = \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left\| \tilde{\boldsymbol{L}}\boldsymbol{b} - \tilde{\boldsymbol{y}} \right\|$$
$$= \arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \left\| \tilde{\boldsymbol{L}}(\boldsymbol{b} - \boldsymbol{b_0}) - \tilde{\boldsymbol{n}} \right\| \tag{131}$$

Since $\tilde{\boldsymbol{n}}$ can be viewed as K independent zero mean Gaussian noise variables with covariance matrix $\sigma^2 I$. Assuming that the decisions on $(b_1, ..., b_{i-1})$ are correct, the lower bound ξ_i can be expressed as

$$\xi_i = \sum_{j=1}^{i} (D_j - \tilde{y}_j)^2 = \sum_{j=1}^{i-1} \tilde{n}_j^2 + [\tilde{l}_{ii}(b_i - b0_i) - \tilde{n}_i]^2 \tag{132}$$

and $P(i|1, ..., i-1)$ is given by

$$P(i|1, ..., i-1) = Q(\frac{|\tilde{l}_{ii}|}{\sigma}) \tag{133}$$

Also, similar to (27), define the minimum distance among users $K-k+1, ..., K$ by

$$d_{min-k} = \sqrt{\min_{\substack{\mathbf{e} \in \{-1,0,1\}^K - \{0\} \\ e_1, ..., e_{K-k} = 0}} \{\boldsymbol{e}^T \boldsymbol{H} \boldsymbol{e}\}} \tag{134}$$

Given that the decisions on users $1, ..., K-k$ are correct, the group detection error of Φ_{BB-k} can be approximated by

$$P(K-k+1, ...K|1, ..., K-k) \approx Q\left(\frac{d_{min-k}}{\sigma}\right) \tag{135}$$

Therefore, the overall group decision error of Φ_{BB-k} can be expressed as

$$P_{\boldsymbol{\Phi}_{BB-k}}(e) \approx 1 - \{\prod_{j=1}^{K-k} [1 - Q\left(\frac{|\tilde{l}_{jj}|}{\sigma}\right)]\}[1 - Q\left(\frac{d_{min-k}}{\sigma}\right)] \tag{136}$$

The SE is then given by

$$E(\Phi_{BB-k}) = \min_{i=1,...,K-k} (\tilde{l}_{ii}^2, d_{min-k}^2) \tag{137}$$

Furthermore, from the definitions of (134) and (27), we have

$$d_{min-k}^2 \geq d_{min}^2 = E(\Phi_{ML}) \geq E(\Phi_{BB-k}) \tag{138}$$

$E(\Phi_{BB-k})$ can then be denoted by

$$E(\Phi_{BB-k}) = min(d_{min}^2, \min_{i=1,...,K-k} \tilde{l}_{ii}^2) \tag{139}$$

Evidently, the performance and even the average computational cost of the above sub-optimal method are also affected by the detection order of the users. For the D-DF method, a user ordering algorithm is proposed in [Va99] as follows,

Order Algorithm: Order users as follows: select the first user in the new order (denote this user's index as i_1) as

$$i_1 = arg \min_{j=1,...,K} [\boldsymbol{H}^{-1}]_{jj} \tag{140}$$

For $k = 2, ..., K$, form a new matrix $\hat{\boldsymbol{H}}$ to be part of $\boldsymbol{H}$ that only contains the components $\{h_{ij}\}$ $(i, j \in \{\{1, ..., K\} - \{i_1, ..., i_{k-1}\}\})$. Find

$$\hat{i}_k = arg \min_{j=1,...,K-k+1} [\hat{\boldsymbol{H}}^{-1}]_{jj} \tag{141}$$

and let i_k equal to the user corresponding to $\hat{i}_k$.

Proposition 2: When ordering users by the order algorithm, the SE $E(\Phi_{BB-k})$ of all $k = 1, ..., K$ are maximized simultaneously.

We ignore the proof here since it is actually a special case to the GDF detector with $|G|_{max} = 1$.

4.3 Simulation Results

According to (130), the computational complexity for the sub-optimal detector Φ_{BB-k} is exponential in k. However, since we assume $\boldsymbol{H}$ to be known, the SE of the proposed "current-best" sub-optimal solutions can be found offline by (139). The following simulation results show that, in some cases, a small amount of extra computation can significantly improve the performance of the system, when compared with the Φ_{D-DF} (which is the same as Φ_{BB-1}).

Example 1 - continued: In the previous example, since users 2 and 3 are strongly correlated, we expect that $E(\phi_{BB-2})$ will be a significant improvement over $E(\phi_{D-DF})$. The SE for different detectors can be obtained via (26) (139), $E(\phi_{ML}) = 1.8$, $E(\phi_{BB-2}) = 1.8$, $E(\phi_{D-DF}) = 1.69$. The simulation result is given in Figure 19, which is consistent with the theoretical analysis.

Example 2: Now suppose we have 50 users. We use binary signature sequences of length 55. The signature sequences are generated such that $5 - 10$ users are correlated with each other (The maximum correlation among users is set to be around 0.85). The energy of each user is generated randomly between $[1, 4.5]$. In these cases, the proposed sub-optimal algorithm outperforms the D-DFD method significantly. In the situations when only a small number of users are correlated, the sub-optimal algorithms can even reach the performance bound of the optimal detector with marginal increase in computational cost over the D-DFD method. Figure 20 shows the simulation results of one of these examples.

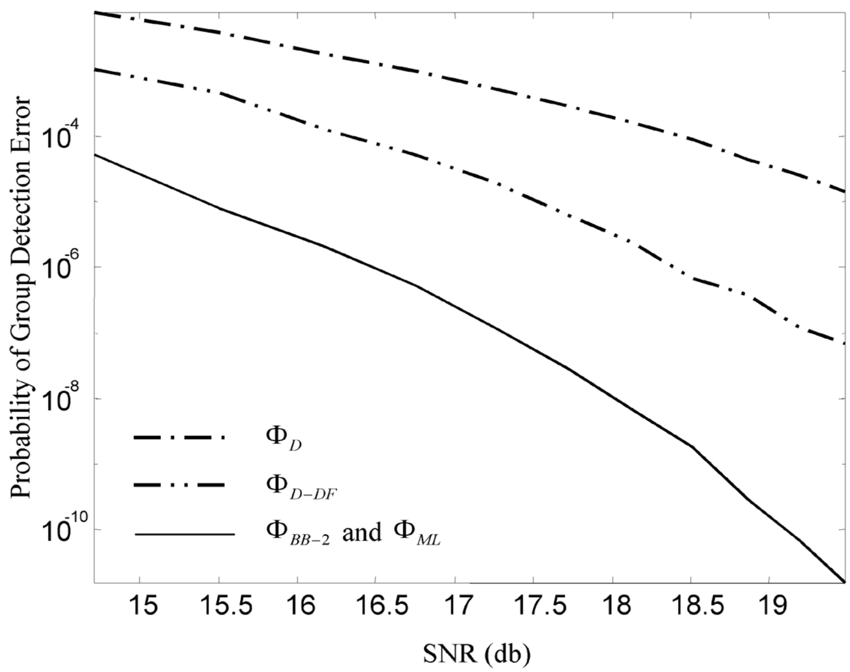

Figure 19: Performance of various methods (3 users, 10000 Monte-Carlo runs)

The comparison of the computational cost for the group detection of different algorithms is given in Table 3.

	ϕ_D		ϕ_{D-DF}		ϕ_{BB-5}			
SNR (db)	$\times$	$+$	$\times$	$+$	$\times$ Average	$+$ Average	$\times$ Maximum	$+$ Maximum
17	2500	2450	1325	2550	1329.1	2568.5	1366	2674
18.5	2500	2450	1325	2550	1329	2568.1	1366	2674
19.5	2500	2450	1325	2550	1329	2568	1366	2674
21.1	2500	2450	1325	2550	1329	2568	1366	2674
21.8	2500	2450	1325	2550	1329	2568	1366	2674

Table 3: Comparison of Computational Cost (50 users, spreading factor 55, 10000 Monte-Carlo runs, $\times$ = number of multiplications, $+$ = number of additions)

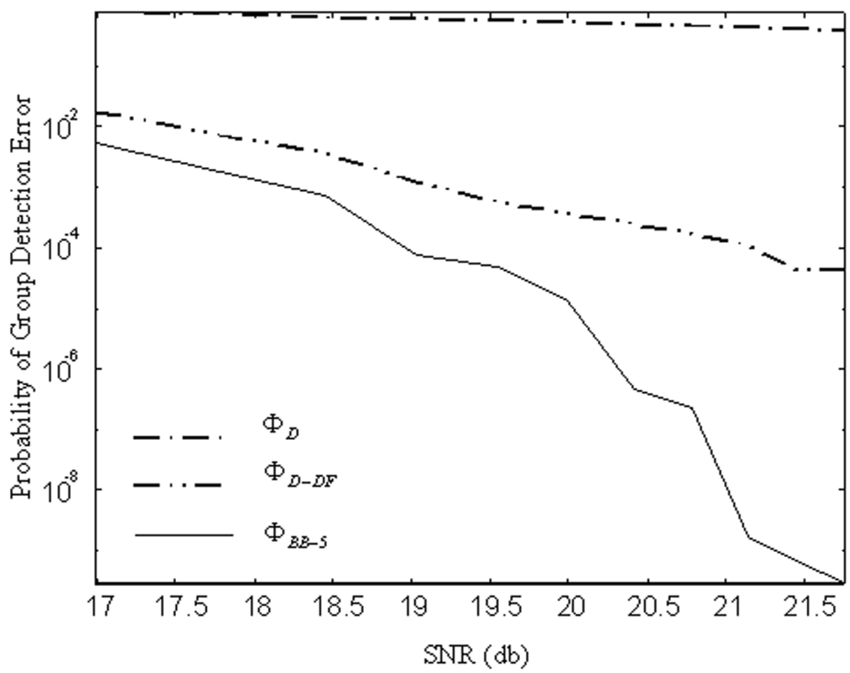

Figure 20: Performance of various methods (50 users, spreading factor 55, 10000 Monte-Carlo runs)

Example 3: In another 8 user example, the H matrix is randomly generated as

$$\boldsymbol{H} = \begin{bmatrix} 3.0 & -0.4 & 1.4 & -0.5 & 0.4 & -0.3 & 0.3 & -0.6 \\ -0.4 & 1.9 & -0.8 & 0.0 & 0.7 & 0.6 & -0.5 & 0.2 \\ 1.4 & -0.8 & 2.8 & -1.8 & 0.8 & -0.0 & 0.0 & -0.3 \\ -0.5 & 0.0 & -1.8 & 2.6 & -1.6 & -0.6 & -0.6 & -0.3 \\ 0.4 & 0.7 & 0.8 & -1.6 & 2.2 & 1.2 & -0.0 & 0.2 \\ -0.3 & 0.6 & -0.0 & -0.6 & 1.2 & 1.4 & -0.0 & 0.2 \\ 0.3 & -0.5 & 0.0 & -0.6 & -0.0 & -0.0 & 1.2 & -0.2 \\ -0.6 & 0.2 & -0.3 & -0.3 & 0.2 & 0.2 & -0.2 & 1.0 \end{bmatrix} \tag{142}$$

The users have already been ordered by the order algorithm. The Symmetric Energy for the ML detector is

$$E(\phi_{ML}) = d_{min}^2 = 1.0 \tag{143}$$

The SE for various sub-optimal detectors are (for $E(\phi_{BB-1})$ through $E(\phi_{BB-7})$)

$$\{0.87, 0.87, 0.87, 0.87, 0.87, 0.89, 1.0\} \tag{144}$$

In this example, the computational cost for improving the performance from D-DFD is high. In addition, even the SE of the ML detector does not differ much from that of D-DFD. Hence, D-DFD is an efficient detector in this case. Figure 21 shows the probability of error for group detection.

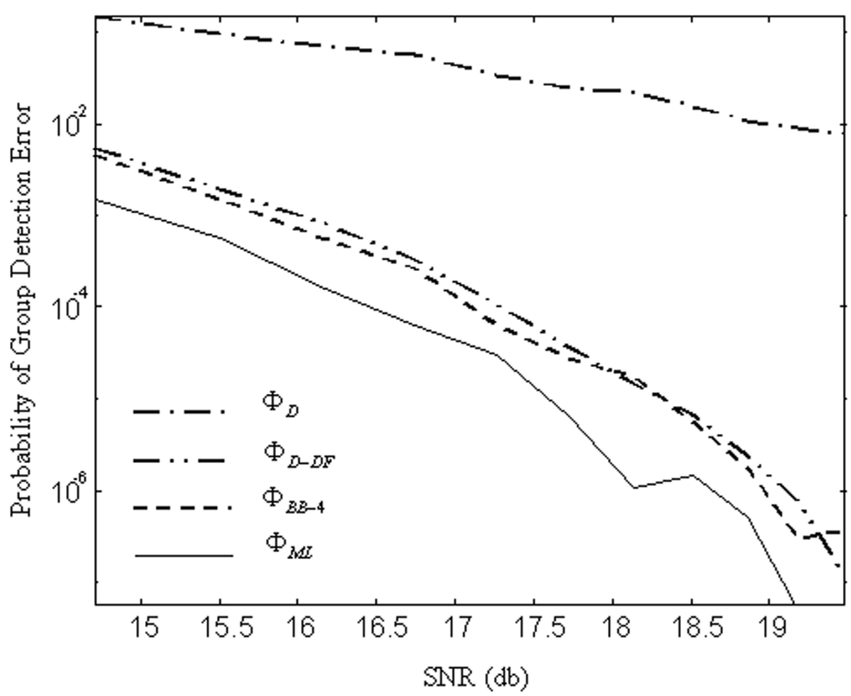

Figure 21: Performance of various methods (8 users, 10000 Monte-Carlo runs)

4.4 Conclusions

The proposed branch-and-bound algorithm shows that, in addition to the D-DF method, there exists a class of sub-optimal methods that provides "any-time" sub-optimal solutions to the user signals. Given a CDMA system, the performance (measured by the SE), the computational bound and even the distribution of computational cost for the proposed sub-optimal algorithms can be estimated offline via (130) and (139). In addition, the detection sequence provided by the ordering algorithm is proved to be optimal for all the sub-optimal algorithms. The proposed algorithm can be easily extended to finite-alphabet signals instead of binary ones.

Chapter 5

Multiuser Detection Using Probabilistic Data Association (PDA)

Although DF is one of the most efficient methods, in many cases the gap between the probability of error of the DF detector and that of a ML detector is still large. Due to advances in hardware computational capabilities, finding multiuser detection algorithms that achieve close-to-optimal performance, while maintaining outstanding computational efficiency, has been attractive to researchers as well as to industry. Many advanced detection algorithms have already been proposed in the past several years. However, not all of them have been successful. As described in Chapter 1, multiuser detection is a problem that combines both the features of binary programming and the features of statistical estimation. It is not a pure estimation problem since the binary nature of the signals must be considered and the probability of detection error must be low enough to satisfy the quality of service requirements of practical communication systems. It is not a pure optimization problem either since the observation vector in the cost function stems from a statistical model. Therefore, ignoring any of these features will reduce the efficiency of the multiuser detection.

Probabilistic Data Association (PDA) [BT75] [BL95] is a highly successful approach to target tracking in the case that measurements are unlabeled and may be spurious. Its key feature is a repeated conversion of a multimodal Gaussian mixture probability structure to a single Gaussian with matched mean and covariance. This is a bold and to some extent unjustifiable step, but it is difficult to argue with good performance and low complexity. Now, in the CDMA case the true probability function is also a Gaussian mixture, and complexity is also the issue. It is natural to extend PDA and apply the Gaussian "forcing" idea to multiuser detection; whereas in the tracking application this forcing occurs once per scan and there is no revisit, in CDMA it occurs for each user, and there is iteration. Instead of fixing the binary signal variables at ± 1, PDA employs a soft MAI cancellation by increasing the covariance of the effective noise based on the uncertainty in the other user signals. When the binary variables converge to the true value, the covariance approaches that of the original noise.

Actually, the PDA multiuser detection turns out to be very successful. In both synchronous and asynchronous CDMA, PDA detector achieves near-optimal performance with $O(K^3)$ computations. The soft-decision feature makes it very flexible and easy to extend to more realistic environments.

The chapter is organized as follows. The PDA detector for synchronous CDMA is proposed and analyzed in section 5.1. The synchronous overloaded system is discussed in section 5.2. The PDA method is slightly modified to avoid taking the inverse of a singular matrix. By considering the asynchronous system as a (very big) synchronous system, the PDA detector is directly extended to asynchronous CDMA in section 5.3. It is then modified to a sliding window precessing to reduce the detection delay on the output. Unlike the methods introduced in chapter 2, the theoretical performance analysis for the PDA detector is currently unavailable. However, analysis on computational complexity is given and simulation results on a range of situations are presented to show the outstanding performance and the computational efficiency of the PDA detector.

5.1 PDA multiuser detector for Synchronous non-overloaded system

The matched-filter output of the synchronous CDMA system can be represented by [Vd98]

$$\boldsymbol{y} = \boldsymbol{RWb} + \boldsymbol{n} \tag{145}$$

Multiplying by $\boldsymbol{W}^{-1}\boldsymbol{R}^{-1}$ on both sides of (145) from the left, the system model can be reformulated as

$$\ddot{\boldsymbol{y}} = \boldsymbol{b} + \ddot{\boldsymbol{n}} = b_k\boldsymbol{e}_k + \sum_{j\neq k} b_j\boldsymbol{e}_j + \ddot{\boldsymbol{n}} \tag{146}$$

where $\ddot{y} = \boldsymbol{W}^{-1}\boldsymbol{R}^{-1}y$, $\ddot{\boldsymbol{n}} = \boldsymbol{W}^{-1}\boldsymbol{R}^{-1}\boldsymbol{n}$, and $\boldsymbol{e}_k$ is a column vector whose kth component is 1 and whose other components are 0. We call (146) "the decorrelated model", since $\ddot{\boldsymbol{y}}$ is in fact a normalized version of the decorrelator output before the hard decision.

5.1.1 The Basic Algorithm

In the CDMA system model (146), we treat the decision variables b as binary random variables. For any user k, we associate a probability $P_b(k)$ with user signal b_k to express the current belief on its value; i.e., $P_b(k)$ is the current estimate of the probability that $b_k = 1$, and $1 - P_b(k)$ is the corresponding estimate for $b_k = -1$. Now, for an arbitrary user signal b_k, treat the other user signals b_j $(j \neq k)$ as binary random variables and treat $\sum_{j \neq k} b_j \mathbf{e}_j + \ddot{\boldsymbol{n}}$ as the effective noise. Consequently, $p(b_k = 1|\ddot{\mathbf{y}}, \{P_b(j)\}_{j \neq k})$ and $p(b_k = -1|\ddot{\mathbf{y}}, \{P_b(j)\}_{j \neq k})$ can be obtained from (146); they serve as updated information on user signal b_k. Based on the decorrelated model, the basic form of the proposed multistage PDA detector is as follows.

(1) Sort users according to the user ordering criterion proposed for the decision feedback detector in [Va99] (specifically Theorem 1 of [Va99]).

(2) $\forall k$, initialize the probabilities as $P_b(k) = 0.5$. Initialize the stage counter $i = 1$.

(3) Initialize the user counter $k = 1$

(4) Based on the current values of $P_b(j)$ $(j \neq k)$ for user k, update $P_b(k)$ by

$$P_b(k) = P\{b_k = 1|\ddot{\mathbf{y}}, \{P_b(j)\}_{j \neq k}\} \tag{147}$$

(5) If $k < K$, let $k = k + 1$ and goto step (4)

(6) If $\forall k$, $P_b(k)$ has converged, goto step (7). Otherwise, let $i = i + 1$ and return to step (3).

(7) $\forall k$, make a decision on user signal k via

$$b_k = \begin{cases} 1 & P_b(k) \geq 0.5 \\ -1 & P_b(k) < 0.5 \end{cases} \tag{148}$$

In the above procedure, the computational cost of obtaining $P_b(k) = P\{b_k = 1|\ddot{\boldsymbol{y}}, \{P_b(j)\}_{j\neq k}\}$ is evidently exponential in the number of users. Define

$$\boldsymbol{N}_k = \sum_{j\neq k} b_j \boldsymbol{e}_j + \ddot{\boldsymbol{n}} \tag{149}$$

from (146). Here is the key: to avoid the computational cost of $P_b(k)$, the PDA idea from [BL95] recommends that $\boldsymbol{N}_k$ be *approximated as a Gaussian noise* with matched mean and covariance; that is, we use

$$\begin{aligned} E[\boldsymbol{N}_k] &= \sum_{j\neq k} \boldsymbol{e}_j(2P_b(j) - 1) \\ Cov[\boldsymbol{N}_k] &= \sum_{j\neq k} \left[4P_b(j)(1 - P_b(j))\boldsymbol{e}_j\boldsymbol{e}_j^T\right] \\ &\quad +\sigma^2(\boldsymbol{W}^T\boldsymbol{R}\boldsymbol{W})^{-1} \end{aligned} \tag{150}$$

Now, defining

$$\begin{aligned} \theta_k &= \sum_{j\neq k} \boldsymbol{e}_j(2P_b(j) - 1) - \ddot{\boldsymbol{y}} \\ \Omega_k &= \sum_{j\neq k} \left[4P_b(j)(1 - P_b(j))\boldsymbol{e}_j\boldsymbol{e}_j^T\right] \\ &\quad +\sigma^2(\boldsymbol{W}^T\boldsymbol{R}\boldsymbol{W})^{-1} \end{aligned} \tag{151}$$

we obtain

$$\frac{P_b(k)}{1-P_b(k)} = exp\left\{-2\theta_k^T\Omega_k^{-1}\mathbf{e}_k\right\} \tag{152}$$

5.1.2 Refinements

5.1.2.1 Speed-Up: Matrix Arithmetic

Although the computation in step 3 is no longer exponential, direct calculation of Ω_k^{-1} for each user is still expensive. Further simplifications can be made by defining auxiliary variables

$$\begin{aligned}
\theta &= \sum_j \mathbf{e}_j(2P_b(j)-1) - \ddot{\boldsymbol{y}} = \theta_k + \mathbf{e}_k(2P_b(k)-1) \\
\Omega &= \sum_j \left[4P_b(j)(1-P_b(j))\mathbf{e}_j\mathbf{e}_j^T\right] + \sigma^2(\boldsymbol{W}^T\boldsymbol{R}\boldsymbol{W})^{-1} \\
&= \Omega_k + 4P_b(k)(1-P_b(k))\mathbf{e}_k\mathbf{e}_k^T
\end{aligned} \tag{153}$$

The Sherman-Morrison-Woodbury formula [Hg89] yields

$$\begin{aligned}
\theta_k &= \theta - \mathbf{e}_k(2P_b(k)-1) \\
\Omega_k^{-1} &= \Omega^{-1} + \frac{4P_b(k)(1-P_b(k))\Omega^{-1}\mathbf{e}_k\mathbf{e}_k^T\Omega^{-1}}{1-4P_b(k)(1-P_b(k))\mathbf{e}_k^T\Omega^{-1}\mathbf{e}_k}
\end{aligned} \tag{154}$$

$$\begin{aligned}
\theta &= \theta_k + \mathbf{e}_k(2P_b(k)-1) \\
\Omega^{-1} &= \Omega_k^{-1} - \frac{4P_b(k)(1-P_b(k))\Omega_k^{-1}\mathbf{e}_k\mathbf{e}_k^T\Omega_k^{-1}}{1+4P_b(k)(1-P_b(k))\mathbf{e}_k^T\Omega_k^{-1}\mathbf{e}_k}
\end{aligned} \tag{155}$$

By keeping the updated versions of θ and Ω^{-1}, we can divide step 3 into three sub-steps. In sub-step 1, we calculate θ_k and Ω_k^{-1} using (154). Sub-step 2 obtains the updated $P_b(k)$ using (152). In sub-step 3, we use the new $P_b(i)$ and update θ

and Ω using (155). The overall computation of step 3 is then reduced to $O(K^2)$, and the overall complexity of each stage in the PDA detector is now $O(K^3)$.

5.1.2.2 Speed-Up: Successive Cancellation

Since the number of stages in the PDA detector is not fixed, the overall complexity can be high if one or two users show a slow convergence. In fact, computer simulation experience has shown that, in most cases, more than $\frac{2}{3}$ of users will converge during the first stage. Thus, to simplify further, we introduce successive cancellation among the stages.

After the ith stage, define G to be the group of users that satisfy

$$\forall j \in G,\, P_b(j) \in [0, \epsilon] \cup [1-\epsilon, 1] \tag{156}$$

where ϵ is a small positive number. Denote $\bar{G}$ to be the complement of G. Make decisions that

$$\forall j \in G,\, b_j = sign(P_b(j) - 0.5) \tag{157}$$

Buy canceling the MAI, the decorrelated system model for the users in $\bar{G}$ can be formulated as

$$\boldsymbol{W}^{-1}_{\bar{G}\bar{G}}\boldsymbol{R}^{-1}_{\bar{G}\bar{G}}\boldsymbol{y}_{\bar{G}} - \boldsymbol{W}^{-1}_{\bar{G}\bar{G}}\boldsymbol{R}^{-1}_{\bar{G}\bar{G}}\boldsymbol{R}_{\bar{G}G}\boldsymbol{W}_{GG}\boldsymbol{b}_G = \boldsymbol{b}_{\bar{G}} + \ddot{\boldsymbol{n}}_{\bar{G}} \tag{158}$$

Here $\boldsymbol{R}_{\bar{G}\bar{G}}$ denotes the sub-block matrix of $\boldsymbol{R}$ that only contains the columns and rows corresponding to users in $\bar{G}$. $\ddot{\boldsymbol{n}}_{\bar{G}}$ is the colored Gaussian noise of the sub-system with zero mean and covariance matrix $\sigma^2(\boldsymbol{W}_{\bar{G}\bar{G}}\boldsymbol{R}_{\bar{G}\bar{G}}\boldsymbol{W}_{\bar{G}\bar{G}})^{-1}$. Consequently, in the $(i+1)$st stage, we apply the PDA detection procedure only on the sub-system model.

5.1.2.3 Performance: Coordinate Descent

It has been noted that when optimal and PDA solutions to (121) differ, they usually disagree in one element only. Thus, as an inexpensive way to improve the PDA detector, we add a coordinate descent search ("bit flip") [LLPW00] after PDA has converged.

5.1.3 Computer Simulation Results

In this section, we use several computer simulation examples to show the performance and the computational cost of the PDA detector. Besides the proposed PDA detector, the Decorrelating Detector [LV89], the Decision Feedback Detector [Va99], the Semi-definite Relaxation method [MDWLC00] and the optimal Maximum Likelihood detector [LPWL00] are compared in the examples. In the successive cancellation part of the PDA detector, we set $\epsilon = \frac{10^{-2}}{4SNR}$ where SNR is the signal to noise ratio. For the Semidefinite Relaxation algorithm, the number of randomizations is set to 20.

In the first 29-user example, we use length-31 Gold codes as the signature sequences. The user signal amplitudes are randomly and independently generated by $w_{kk} \sim N(4.5, 4)$, $\forall k$, and are limited within the range of $[2, 7]$ ($N(.)$ represents the Normal distribution). Figure 22 shows the performance comparison based on 100000 Monte-Carlo runs with importance sampling.

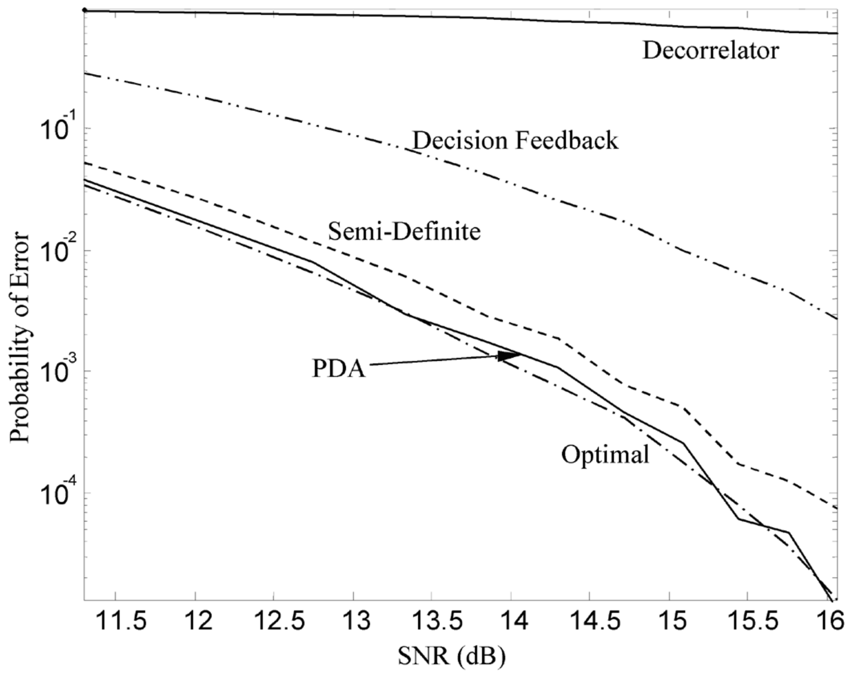

Figure 22: 29-users, length-31 Gold codes as signature sequences, 100000 Monte-Carlo runs

In the second example, we fix the SNR to be 12 dB. The signature sequences are randomly generated and the ratio between the spreading factor and the number of users is fixed at 1.2. Let the number of users vary from 3 to 60. Figure 23 shows the worst case computational complexity measured in terms of the number of multiplications plus number of additions of the PDA detector and of the Semidefinite Relaxation method. It is known that the computational cost of the Semidefinite Relaxation method is $O(K^{3.5})$ [MDWLC00]. Therefore, we claim that the computational cost for the PDA detector is significantly less than

$O(K^{3.5})$. Simulation results show that the computational cost is in fact $O(K^3)$.

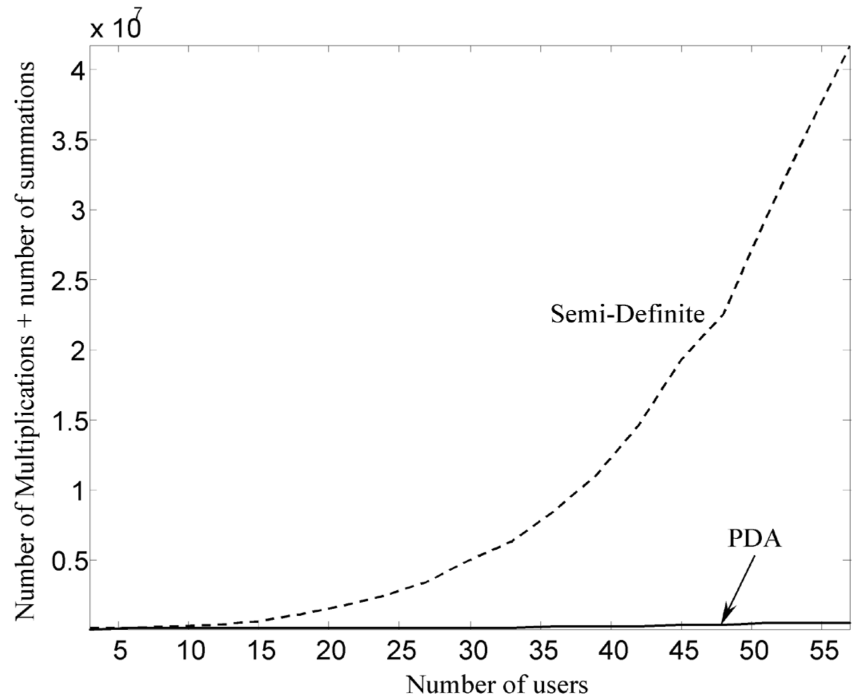

Figure 23: Comparison on the worst case computational costs, random signature sequences, spreading factor=1.2K, SNR=12 dB

5.1.4 Conclusions

A new algorithm based on the idea of Probabilistic Data Association is proposed for the multiuser detection in synchronous CDMA communications. Simulation results show that the PDA detector provides near-optimal performance, with the overall computational cost $O(K^3)$, where K is the number of users.

5.2 PDA detector for Synchronous Overloaded System

In synchronous CDMA, having more users than the signature length results in a singular correlation matrix. However, with a careful design of the correlations and with the help of the binary feature of user signals, good performance in a slightly-overloaded system can still be achieved [SVM00] [VMS00]. However, the singularity of the correlation matrix does make the direct implementation of many multiuser detectors impossible.

Define the length of the signature sequence as N, which is also known as the spreading factor [Vd98]. When the system is overloaded ($K > N$), $\boldsymbol{R}$ becomes singular. The optimal solution of (121) may not be unique even when the noise is not present. This evidently results in an unavoidably high probability of error in multiuser detection. Nevertheless, with a careful design of the signature sequences and the correlations, [SVM00] [VMS00] show that it is possible to avoid multi-solutions and still achieve good performance in slightly overloaded systems.

We first consider the optimal solution. Suppose the chip matched-filter is available at the receiver side, the system model of the chip matched-filer output can be represented by

$$\boldsymbol{x} = \boldsymbol{S}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{z} \tag{159}$$

where $\boldsymbol{S}$ is a $N \times K$ matrix whose kth column, $\boldsymbol{s}_k$, is the normalized signature of the k-th user. Since the symbol matched-filter output satisfies

$$\boldsymbol{y} = \boldsymbol{S}^T\mathbf{x} \tag{160}$$

We obtain

$$\boldsymbol{R} = \boldsymbol{S}^T\boldsymbol{S}$$
$$\boldsymbol{n} = \boldsymbol{S}^T\boldsymbol{z} \tag{161}$$

Assume that the chip matched-filter output is given. Then the optimal decision for system model (159) is given by

$$\begin{aligned}\Phi_{ML} : \hat{\boldsymbol{b}} &= arg \min_{\boldsymbol{b}\in\{-1,+1\}^K} \left(\boldsymbol{b}^T\boldsymbol{W}\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{W}\boldsymbol{b} - 2\boldsymbol{x}^T\boldsymbol{S}\boldsymbol{W}\boldsymbol{b}\right)\\ &= arg \min_{\boldsymbol{b}\in\{-1,+1\}^K} \left(\boldsymbol{b}^T\boldsymbol{W}\boldsymbol{R}\boldsymbol{W}\boldsymbol{b} - 2\boldsymbol{y}^T\boldsymbol{W}\boldsymbol{b}\right)\end{aligned} \tag{162}$$

which is exactly the same as (121).

Furthermore, since $b_k^2 = 1$, $\forall k$, the ML detector can also be equivalently written as,

$$\Phi_{ML} : \hat{\boldsymbol{b}} = arg \min_{\boldsymbol{b}\in\{-1,+1\}^K} \left[\boldsymbol{b}^T\boldsymbol{W}(\boldsymbol{R}+\Lambda)\boldsymbol{W}\boldsymbol{b} - 2\boldsymbol{y}^T\boldsymbol{W}\boldsymbol{b}\right] \tag{163}$$

where Λ is an arbitrary diagonal matrix with positive diagonal components. Evidently, $\boldsymbol{R}+\Lambda$ is positive definite. Therefore, the branch-and-bound-based optimal algorithm proposed in [LPWL00] can be applied with minor modifications.

5.2.1 Modifying the PDA method

For PDA detector, two issues need to be addressed. The first one is the user ordering in (step 1). Since $\boldsymbol{L}^{-1}$ does not exist, the original white noise model is no longer valid. However, since PDA works with soft MAI cancellations, the performance is less sensitive to user ordering than the DF detector. Therefore, as a small modification, we use $\boldsymbol{R}+\sigma^2\boldsymbol{I}$ instead of $\boldsymbol{R}$ in the user ordering algorithm. Although the actual values of the elements in $\boldsymbol{R}+\sigma^2\boldsymbol{I}$ may not be reliable when σ^2 is small, the resulting user order is good enough for PDA to achieve near-optimal performance. The other issue is the probability update in (step 4). Again, since $\boldsymbol{R}$ does not exist, the decorrelator model is no longer valid. Therefore, the original PDA method must be modified to avoid taking the inverse of a singular matrix.

Similar to the analysis of the ML detector, assume that the chip matched-filter outputs are available. Rewrite (159) as

$$\boldsymbol{x} = \boldsymbol{s}_k w_{kk} b_k + \sum_{j \neq k} \boldsymbol{s}_j w_{jj} b_j + \boldsymbol{z} \tag{164}$$

Define the effective noise to user k as

$$\boldsymbol{N}_k = \sum_{j \neq k} \boldsymbol{s}_j w_{jj} b_j + \boldsymbol{z} \tag{165}$$

The mean and covariance matrix of $\boldsymbol{N}_k$ are

$$\begin{aligned} E[\boldsymbol{N}_k] &= \sum_{j \neq k} \boldsymbol{s}_j w_{jj}(2P_{bj} - 1) \\ Cov[\boldsymbol{N}_k] &= \sum_{j \neq k} 4P_{bj}(1 - P_{bj}) w_{jj}^2 \boldsymbol{s}_j \boldsymbol{s}_j^T + \sigma^2 \boldsymbol{I} \end{aligned} \tag{166}$$

Similarly, define

$$\begin{aligned} \theta_k &= \sum_{j \neq k} \boldsymbol{s}_j w_{jj}(2P_{bj} - 1) - \boldsymbol{x} \\ \Omega_k &= \sum_{j \neq k} 4P_{bj}(1 - P_{bj}) w_{jj}^2 \boldsymbol{s}_j \boldsymbol{s}_j^T + \sigma^2 \boldsymbol{I} \end{aligned} \tag{167}$$

The updated probability $P_b(k)$ is given by

$$\frac{P_{bk}}{1 - P_{bk}} = exp\{-2\theta_k^T \Omega_k^{-1} \boldsymbol{s}_k w_{kk}\} \tag{168}$$

Now, define auxiliary variables

$$\begin{aligned}
\mu &= [w_{11}(2P_{b1}-1),\ldots,w_{KK}(2P_{bK}-1)]^T \\
\Sigma &= diag(4P_{b1}(1-P_{b1})w_{11}^2,\ldots,4P_{bK}(1-P_{bK})w_{KK}^2) \\
\theta &= \boldsymbol{S}\mu - \boldsymbol{x} \\
\Omega &= \boldsymbol{S}\Sigma\boldsymbol{S}^T + \sigma^2\boldsymbol{I}
\end{aligned} \tag{169}$$

Define G to be the group of users such that $\forall j \in G$, $P_{bj}(1-P_{bj}) \neq 0$. Also assume user $k \in G$. Define

$$G_k = G \setminus \{\text{user } k\} \tag{170}$$

Since for any $j \notin G$, $P_{bj}(1-P_{bj}) = 0$, we have

$$\Omega_k = \boldsymbol{S}_{G_k}\boldsymbol{D}_{G_kG_k}\boldsymbol{S}_{G_k}^T + \sigma^2\boldsymbol{I} \tag{171}$$

Here $\boldsymbol{S}_{G_k}$ denotes the $\boldsymbol{S}$ matrix that only contains the columns corresponding to users in G_k; and $\Sigma_{G_kG_k}$ represents the Σ matrix that only contains the columns and rows corresponding to users in G_k. Using the matrix inverse lemma, we have

$$\Omega_k^{-1} = \frac{1}{\sigma^2}\boldsymbol{I} - \frac{1}{\sigma^4}\boldsymbol{S}_{G_k}\left(\frac{1}{\sigma^2}\boldsymbol{S}_{G_k}^T\boldsymbol{S}_{G_k} + \Sigma_{G_kG_k}^{-1}\right)^{-1}\boldsymbol{S}_{G_k}^T \tag{172}$$

It is easy to see from (161) that

$$\begin{aligned}
\boldsymbol{S}_{G_k}^T\boldsymbol{S}_{G_k} &= \boldsymbol{R}_{G_kG_k} \\
\theta_k^T\boldsymbol{s}_k &= \mathbf{r}_{\{k\}G_k}^T\mu_{G_k} - y_k \\
\theta_k^T\boldsymbol{S}_{G_k} &= \mu_{G_k}^T\boldsymbol{R}_{G_kG_k} - \boldsymbol{y}_{G_k}^T \\
\boldsymbol{S}_{G_k}^T\boldsymbol{s}_k &= \mathbf{r}_{\{k\}G_k}
\end{aligned} \tag{173}$$

Therefore,

$$\theta_k^T \Omega_k^{-1} \mathbf{s}_k = \frac{1}{\sigma^2}(\mathbf{r}_{\{k\}G_k}^T \mu_{G_k} - y_k)$$
$$-\frac{1}{\sigma^4}\mathbf{r}_{\{k\}G_k}^T \left(\frac{1}{\sigma^2}\boldsymbol{R}_{G_kG_k} + \Sigma_{G_kG_k}^{-1}\right)^{-1} (\boldsymbol{R}_{G_kG_k}\mu_{G_k} - \boldsymbol{y}_{G_k}) \tag{174}$$

Similar to the optimal detector case, chip matched-filter outputs do not appear in the final result. Therefore, only the symbol matched-filters are required.

Furthermore, as shown in [LPWF01] for the original PDA detector, the complexity of computing (174) can also be reduced to $O(K^2)$ per user. Define,

$$\Xi_{GG} = \frac{1}{\sigma^2}\boldsymbol{R}_{GG} + \Sigma_{GG}^{-1} \tag{175}$$

Since user $k \in G$, we have

$$\Xi_{GG}^{-1} = \begin{bmatrix} \Xi_{G_kG_k}^{-1} & \frac{1}{\sigma^2}\mathbf{r}_{G_k\{k\}} \\ \frac{1}{\sigma^2}\mathbf{r}_{G_k\{k\}}^T & \frac{1}{\sigma^2} + \Sigma_{kk}^{-1} \end{bmatrix}^{-1} =$$
$$\begin{bmatrix} (\Xi_{G_kG_k} - \frac{1}{\sigma^4}\mathbf{r}_{G_k\{k\}}\mathbf{r}_{G_k\{k\}}^T)^{-1} & -\Xi_{G_kG_k}^{-1}\mathbf{r}_{G_k\{k\}}\Delta^{-1} \\ -\Delta^{-1}\mathbf{r}_{G_k\{k\}}^T\Xi_{G_kG_k}^{-1} & \Delta^{-1} \end{bmatrix} \tag{176}$$

where

$$\Delta = \frac{1}{\sigma^2} + \Sigma_{kk}^{-1} - \frac{1}{\sigma^4}\mathbf{r}_{G_k\{k\}}^T\Xi_{G_kG_k}^{-1}\mathbf{r}_{G_k\{k\}} \tag{177}$$

is the Schur complement of $\Xi_{G_kG_k}$.

Evidently, if we always keep the updated version of Ξ_{GG}^{-1}, (174) as well as P_{bk} can be obtained with $O(|G|^2)$ computations, where $|G|$ denotes the number of users in G. If the updated P_{bk} satisfies $P_{bk}(1-P_{bk}) \neq 0$, we can update Ξ_{GG}^{-1} using Sherman-Morrison formula. If $P_{bk}(1-P_{bk}) = 0$, we can invoke the successive cancellation idea, make decision on b_k immediately. Consequently, only $\Xi_{G_kG_k}^{-1}$, which can also be obtained from Ξ_{GG}^{-1} in $O(|G|^2)$ computations, is needed in further updates. Although successive cancellation is not necessary for non-overloaded systems and is introduced to reduce the computational complexity [LPWF01], it is required for overloaded system to avoid numerical error.

The PDA detector for overloaded system can be summarized as follows:

(1) Sort users according to the user ordering criterion proposed for the decision feedback detector in [Va99] (substitute $\boldsymbol{R}$ by $\boldsymbol{R}+\sigma^2\boldsymbol{I}$).

(2) $\forall k$, initialize the probabilities as $P_b(k) = 0.5$; initialize G to be the set of all K users; and initialize threshold γ with a small positive number.

(3) Initialize

$$\Xi_{GG}^{-1} = [\frac{1}{\sigma^2}\boldsymbol{R}_{GG} + \Sigma_{GG}^{-1}]^{-1} \tag{178}$$

(4) Initialize $k = 1$

(5) If user $k \in G$, obtain $\boldsymbol{r}_{G_k\{k\}}^T \Xi_{G_kG_k}^{-1}$ from (176). Obtain $\theta_k^T \Omega_k^{-1} \boldsymbol{s}_k$ from (174), and then obtain the updated probability $\hat{P}_b(k)$ by

$$\hat{P}_b(k) = \frac{exp(-2\theta_k^T \Omega_k^{-1} \boldsymbol{s}_k w_{kk})}{1 + exp(-2\theta_k^T \Omega_k^{-1} \boldsymbol{s}_k w_{kk})} \tag{179}$$

If user $k \notin G$, goto step (8).

(6) If $\hat{P}_b(k)(1-\hat{P}_b(k)) > \gamma$, update Ξ_{GG}^{-1} by

$$\Xi_{GG}^{-1} = \Xi_{GG}^{-1} - \frac{\delta_k w_{kk}^2 [\Xi_{GG}^{-1}]_k [\Xi_{GG}^{-1}]_k^T}{1+\delta_k w_{kk}^2 [\Xi_{GG}^{-1}]_{kk}} \tag{180}$$

where $\delta_k = 4\hat{P}_{bk}(1-\hat{P}_{bk}) - 4P_{bk}(1-P_{bk})$. And set $P_b(k) = \hat{P}_b(k)$

(7) If $\hat{P}_b(k)(1-\hat{P}_b(k)) \leq \gamma$, make decision on user k via

$$b_k = \begin{cases} 1 & \hat{P}_b(k) \geq 0.5 \\ -1 & \hat{P}_b(k) < 0.5 \end{cases} \tag{181}$$

Subtract the interference of user k from the matched-filter output by updating

$$\boldsymbol{y} = \boldsymbol{y} - \mathbf{r}_k b_k w_{kk} \tag{182}$$

From (176), define $\boldsymbol{A} = [\Xi_{GG}^{-1}]_{G_k G_k}$, let $G = G_k$ and update

$$\Xi_{GG}^{-1} = \boldsymbol{A} - \frac{\boldsymbol{A}\mathbf{r}_{\{k\}G}\mathbf{r}_{\{k\}G}^T\boldsymbol{A}}{\sigma^4 + \mathbf{r}_{\{k\}G}^T\boldsymbol{A}\mathbf{r}_{\{k\}G}} \tag{183}$$

(8) $k = k+1$. If $k \leq K$, goto step (5). Otherwise, perform a coordinate descent search as proposed in [LPWF01], output final decisions and stop.

5.2.2 Simulation Results

In this section, we compare the performances of the PDA detector, the MMSE detector, the MMSE-based DF detector [Va99] and the optimal detector. The Decorrelator and the Decorrelator-based DF detector are not included since they require $\boldsymbol{R}^{-1}$, which does not exist for a overloaded system.

The first example is similar to example 1 in [SVM00] but of a smaller size. Suppose we have 5 users and the signature length is 4. The first 4 users use orthogonal signature sequences generated from Walsh-Hadamard codes. The 5th user is a TDMA user, whose signature sequence is $\boldsymbol{s}_5 = [1, 0, 0, 0, 0]^T$. The user signal powers are set to 4. Noting that the user signature sequences are normalized to have unit two-norm, the correlation matrix is then given by

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 0 & 0.5 \\ 0 & 0 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 1 \end{bmatrix} \tag{184}$$

Figure 24 gives the performance comparisons of different multiuser detectors. The curve labeled "iterative detector" refers to the performance of the iterative detection method proposed in [SVM00]. We can see that the performance of PDA method is close to that of the optimal algorithm and is also significantly better than other methods.

In the second example, we have 7 users with a signature length of 5. The signatures are Welch Bound Equality (WBE) sequences generated from the iterative procedure introduced in [UY01]. The user signal powers are again set to 4. The performance comparisons are given in Figure 25.

Although PDA achieves near optimal performance in most of the cases, in this example, the performance of the PDA detector is significantly worse than

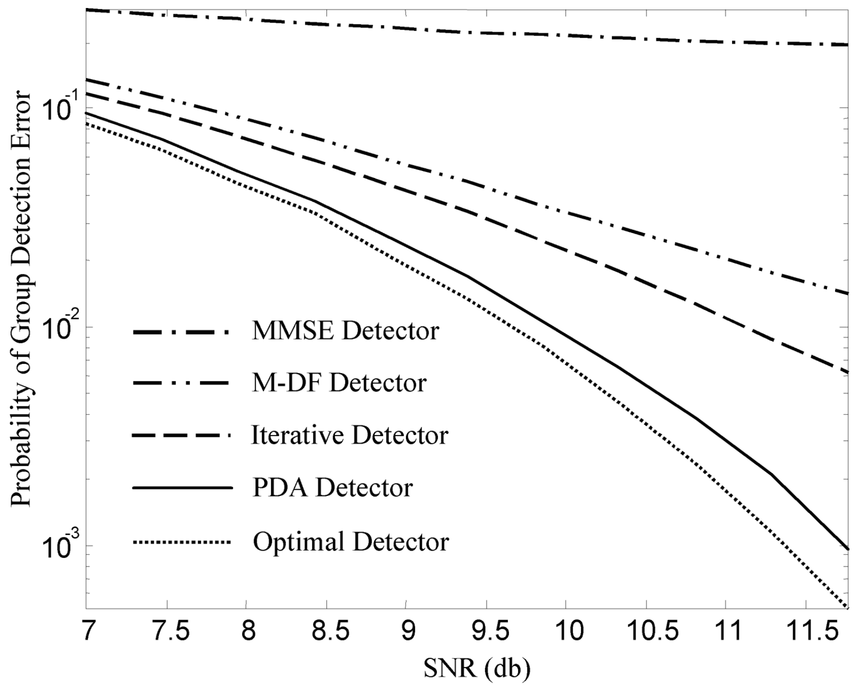

Figure 24: Performance comparison, 5 users, spreading factor=4, 200000 Monte-Carlo runs

the optimal detector. However, it is still better than the MMSE detector and the MMSE-based DF detector. The WBE sequences are shown to be the optimal signature set that maximizes the theoretical *sum capacity* [RM84], and is therefore important to the synchronous overloaded CDMA system. Further improvment of the PDA performance with WBE signature sequences, however, is an open research topic and will be addressed in our future research.

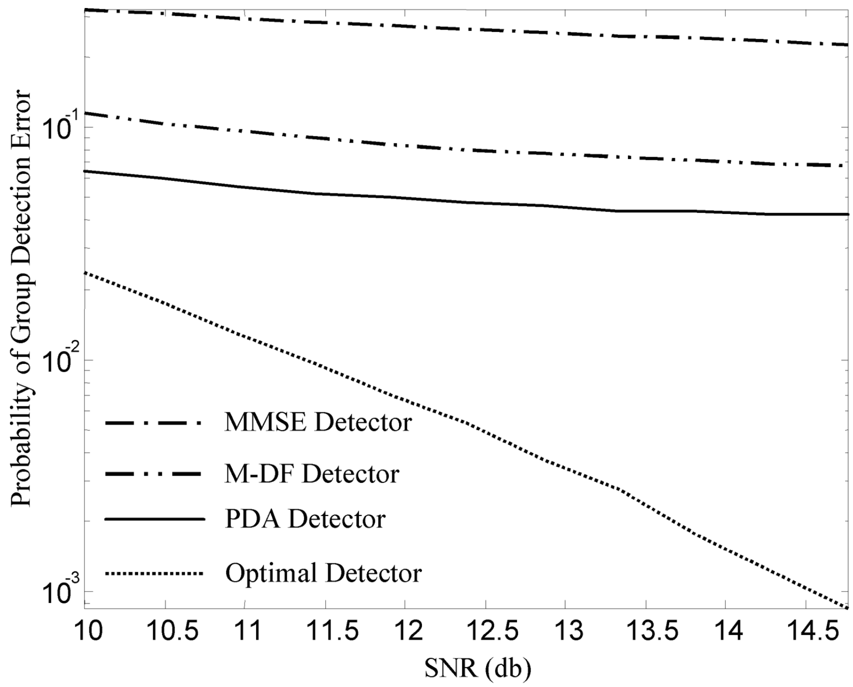

Figure 25: Performance comparison, 7 users, length-5 WBE signature sequences, 200000 Monte-Carlo runs

5.2.3 Conclusions

PDA detector has been extended to synchronous overloaded system. With the help of successive cancellation, taking inverse of a singular matrix is avoided. Simulation results show that PDA outperforms the MMSE detector and the MMSE-based DF detector, the performance is also close to optimal in many situations.

5.3 PDA Detector for Asynchronous CDMA

The asynchronous CDMA system can be described in the z domain by [Vd98]

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{W}\boldsymbol{b}(z) + \boldsymbol{n}(z) \tag{185}$$

Here $\boldsymbol{R}(z)$ is the correlation matrix, which can be factored as

$$\boldsymbol{R}(z) = (\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}) \tag{186}$$

Applying the anticausal feed-forward filter $(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)^{-1}$ to both sides of (185), we obtain the white noise model [Dh95]

$$\tilde{\boldsymbol{y}}(z) = (\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1})\boldsymbol{W}\boldsymbol{b}(z) + \tilde{\boldsymbol{n}}(z) \tag{187}$$

The corresponding time-domain representation of the white noise model is

$$\tilde{\boldsymbol{y}}(i) = \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(i) + \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(i-1) + \tilde{\boldsymbol{n}}(i) \tag{188}$$

Suppose there are overall M time frames in the transmission. We can view the asynchronous system as an MK-user synchronous system and rewrite (188) as

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{W}}\tilde{\boldsymbol{b}} + \tilde{\boldsymbol{N}} \tag{189}$$

Here

$$\begin{aligned}
\tilde{\boldsymbol{Y}} &= [\tilde{\boldsymbol{y}}(0)^T, \tilde{\boldsymbol{y}}(1)^T, \ldots, \tilde{\boldsymbol{y}}(M-1)^T]^T \\
\tilde{\boldsymbol{b}} &= [\boldsymbol{b}(0)^T, \boldsymbol{b}(1)^T, \ldots, \boldsymbol{b}(M-1)^T]^T \\
\tilde{\boldsymbol{N}} &= [\tilde{\boldsymbol{n}}(0)^T, \tilde{\boldsymbol{n}}(1)^T, \ldots, \tilde{\boldsymbol{n}}(M-1)^T]^T \\
\tilde{\boldsymbol{W}} &= \begin{bmatrix} \boldsymbol{W} & \boldsymbol{0} & \ldots \\ \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \ldots & \boldsymbol{0} & \boldsymbol{W} \end{bmatrix}
\end{aligned} \tag{190}$$

and

$$\tilde{\boldsymbol{L}} = \begin{bmatrix} \boldsymbol{F}[0] & \boldsymbol{0} & \dots & \dots \\ \boldsymbol{F}[1] & \boldsymbol{F}[0] & \boldsymbol{0} & \dots \\ \boldsymbol{0} & \dots & \dots & \dots \\ \dots & \boldsymbol{0} & \boldsymbol{F}[1] & \boldsymbol{F}[0] \end{bmatrix} \tag{191}$$

is the Cholesky decomposition matrix of the equivalent synchronous system.

5.3.1 Direct Extension

Apparently, the computational cost of directly applying the PDA method to the equivalent MK-user system is $O\{(MK)^3\}$, which can be very high if M is not small. Fortunately, due to the special structure of the Cholesky decomposition matrix $\tilde{\boldsymbol{L}}$, the probability update in the PDA method can be simplified.

Consider updating the probability associated with user k in time frame i. From (188), we have

$$\begin{aligned} P_{bk}(i) &= P\left\{b_k(i) = 1 \,\middle|\, \tilde{\mathbf{Y}}, \{P_{bj}(i)\}_{j \neq k}, \{P_{bl}(m)\}_{m \neq i}\right\} \\ &= P\left\{ b_k(i) = 1 \,\middle|\, \begin{matrix} \tilde{\boldsymbol{y}}(i), & \tilde{\boldsymbol{y}}(i+1) \\ \{P_{bj}(i)\}_{j \neq k}, & \{P_{bl}(m)\}_{m=i-1,i+1} \end{matrix} \right\} \end{aligned} \tag{192}$$

Therefore, to update the probability $P_{bk}(i)$, only observation vectors, $\tilde{\boldsymbol{y}}(i)$ and $\tilde{\boldsymbol{y}}(i+1)$, are required. The corresponding observation model from (188) is

$$\begin{bmatrix} \tilde{\boldsymbol{y}}(i) \\ \tilde{\boldsymbol{y}}(i+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}[0] \\ \boldsymbol{F}[1] \end{bmatrix} \boldsymbol{W}\boldsymbol{b}(i) + \begin{bmatrix} \boldsymbol{F}[1]\,\boldsymbol{W}\boldsymbol{b}(i-1) \\ \boldsymbol{F}[0]\,\boldsymbol{W}\boldsymbol{b}(i+1) \end{bmatrix}$$

$$+\begin{bmatrix} \tilde{\boldsymbol{n}}(i) \\ \tilde{\boldsymbol{n}}(i+1) \end{bmatrix} \tag{193}$$

For user signal $b_k(i)$, define the effective noise as

$$\begin{aligned} \boldsymbol{N}_k(i) = & \sum_{j\neq k} \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix} w_{jj} b_j(i) + \begin{bmatrix} \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(i-1) \\ \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(i+1) \end{bmatrix} \\ & + \begin{bmatrix} \tilde{\boldsymbol{n}}(i) \\ \tilde{\boldsymbol{n}}(i+1) \end{bmatrix} \end{aligned} \tag{194}$$

where $\boldsymbol{f}_j[0]$ and $\boldsymbol{f}_j[1]$ denote the jth columns of $\boldsymbol{F}[0]$ and $\boldsymbol{F}[1]$, respectively. Consequently, we have

$$\begin{aligned} E[\boldsymbol{N}_k(i)] &= \sum_{j\neq k} \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix} w_{jj}(2P_{bj}(i)-1) \\ &+ \sum_{j=1}^{K} \begin{bmatrix} \boldsymbol{f}_j[1] w_{jj}(2P_{bj}(i-1)-1) \\ \boldsymbol{f}_j[0] w_{jj}(2P_{bj}(i+1)-1) \end{bmatrix} \\ Cov[\boldsymbol{N}_k(i)] &= \sigma^2 \boldsymbol{I} \\ &+ \sum_{j\neq k} 4P_{bj}(i)(1-P_{bj}(i))w_{jj}^2 \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix}^T \\ &+ \sum_{j=1}^{K} 4P_{bj}(i-1)(1-P_{bj}(i-1))w_{jj}^2 \begin{bmatrix} \boldsymbol{f}_j[1]\boldsymbol{f}_j[1]^T & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \\ &+ \sum_{j=1}^{K} 4P_{bj}(i+1)(1-P_{bj}(i+1))w_{jj}^2 \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{f}_j[0]\boldsymbol{f}_j[0]^T \end{bmatrix} \end{aligned} \tag{195}$$

Approximating $\boldsymbol{N}_k(j)$ by a Gaussian noise with matched mean and covariance, it is easy to see that the computational load for updating $P_{bk}(i)$ is $O(K^2)$. Therefore, the overall computational cost of the PDA detector is $O(MK^3)$, i.e., $O(K^3)$ per time frame. This is the same as in the case of synchronous CDMA.

5.3.2 PDA with Sliding Processing Window

Since PDA updates the associated probabilities iteratively, in the above batch method, PDA can do iterations and make decisions on user signals only when the entire transmitted data has been received. This can consequently cause significant delays at the receiver.

Suppose we are only interested in decisions on user signal vector $\boldsymbol{b}(i)$. Consider a truncated processing window that contains user signal vectors $\boldsymbol{b}(m)$, $(i-h \leq m \leq i+h)$, i.e., the width of the processing window is $2h+1$. Due to the limited error propagation in practical systems, it is reasonable to assume that, if h is large enough, the effects of values of user signals outside the processing window on the decisions of $\boldsymbol{b}(i)$ are negligible. Therefore, when making decisions on $\boldsymbol{b}(i)$, one can apply the PDA method and perform iterations only within the truncated processing window.

Notice that the processing windows of user signals in successive time frames differ only slightly. Hence, we can use the probabilities from a processing window as the initial conditions of the PDA method for the next processing window to further simplify the iterative updates. This modifies the truncated-window PDA to a sliding-window PDA. The detailed procedure is described below:

(1) Sort users according to the user ordering and time labeling criterion proposed for decision feedback detector in [LPWF01].

(2) $\forall k$ and $\forall i$, initialize the probabilities as $P_{bk}(i) = 0.5$. Initialize the window counter $m = 1$.

(3) Initialize the time frame counter $i = \max\{1, m-h\}$.

(4) Initialize the user counter $k = 1$.

(5) Based on the current values of the associated probabilities, update $P_{bk}(i)$ according to (192).

(6) If $k < K$, let $k = k+1$ and goto step (5).

(7) If $i < \min\{M, m+h\}$, let $i = i+1$ and goto step (4).

(8) If $i > h$, $\forall i$, make a decision on user signal $b_k(m-h)$ via

$$b_k(m-h) = \begin{cases} 1 & P_{bk}(m-h) \geq 0.5 \\ -1 & P_{bk}(m-h) < 0.5 \end{cases} \tag{196}$$

(9) If $i < M+h$, let $i = i+1$ and goto step (3). Otherwise, stop.

The relations between the indices k, i and m in the above procedure are further illustrated in Figure 26.

Apparently, the computational complexity of the above PDA detector is $O((2h+1)K^3)$ per time frame.

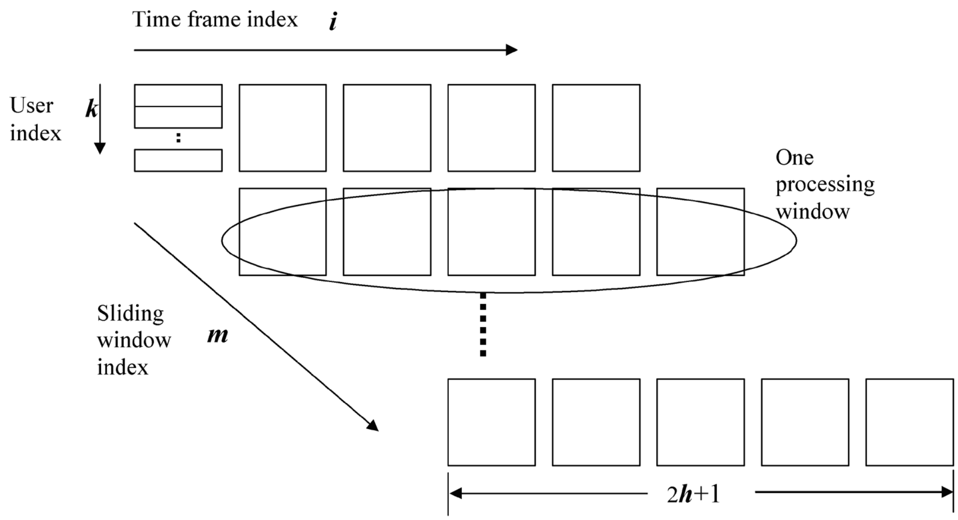

Figure 26: Illustration of the sliding-window PDA

5.3.3 Simulation Results

In this section, we compare the performances of the Decorrelator, the DF detector and the PDA detector in various situations. The optimal user ordering and time labeling rule proposed in [LPWF01] is applied to both the DF and the PDA detectors. By clairvoyantly plugging the true values of $\boldsymbol{b}(i-1)$ and $\boldsymbol{b}(i+1)$ into (193) and applying the ML detection for synchronous CDMA, a performance bound is also provided.

Example 1: In the first 3-user example, the correlation matrices $R[0]$, $R[1]$ and the square roots of user signal powers W are randomly chosen as

$$\boldsymbol{R}[0] = \begin{bmatrix} 1.0 & -0.27 & -0.49 \\ -0.27 & 1.0 & 0.55 \\ -0.49 & 0.55 & 1.0 \end{bmatrix}$$

$$\boldsymbol{R}[1] = \begin{bmatrix} 0 & 0 & 0 \\ -0.06 & 0 & 0 \\ 0.16 & -0.01 & 0 \end{bmatrix}$$
$$\boldsymbol{W} = diag(4.48, 4.36, 4.1) \tag{197}$$

The width of the processing window for the PDA detector is chosen to be 3. Figure 27 shows the performance comparison of different algorithms obtained from a simulation of 1000000 Monte-Carlo runs. Similar to the synchronous case [LPW01], the probability of error of the PDA detector is very close to the performance lower bound.

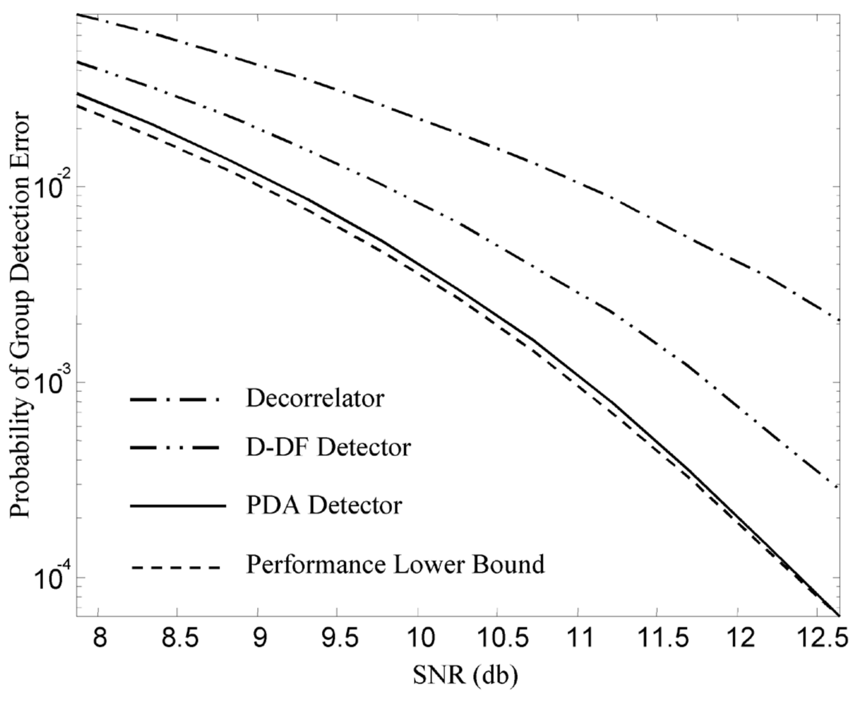

Figure 27: Performance comparison, 3-users, 1000000 Monte-Carlo runs.

Example 2: The second example is an overloaded system with 30 users and 15-length Gold codes as signature sequences. Although the number of users is increased to 30, the width of the processing window for the PDA detector remains at 3. The time delays of the user signals are random and uniformly-distributed within a symbol duration and we use the system model introduced in [Ps77] to generate the signature correlation matrix. The square roots of user signal powers are generated randomly by $w_{kk} \sim N(4.5, 4)$ ($N(.)$ represents the Gaussian distribution) and are limited within the range of $[3, 6]$. Figure 28 shows the performance comparison of different detectors. The performance of the PDA detector is significantly better than the decorrelator and the DF detector. It is also close to the performance lower bound (notice that the performance lower bound is not necessarily reachable even by the optimal ML detector).

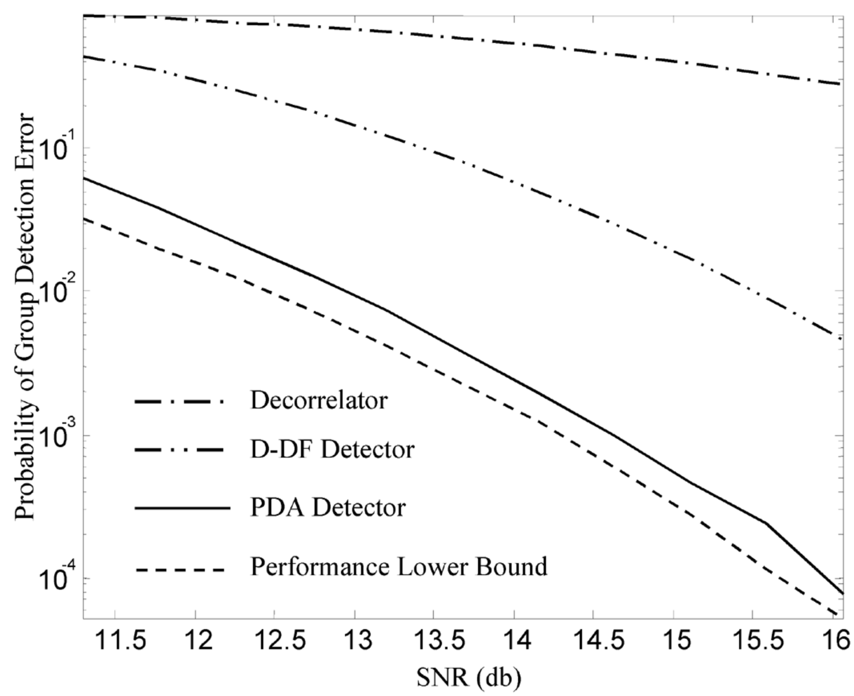

Figure 28: Performance comparison, 30-users, 15-length Gold codes as signature sequences, 1000000 Monte-Carlo runs.

5.3.4 Conclusions

The PDA method proposed in [LPW01] has been extended to the multiuser detection over asynchronous CDMA communication channels. With a sliding window of width $2h+1$, the computational complexity of the proposed PDA detector is shown to be $O((2h+1)K^3)$ per time frame where K is the number of users. Simulation results show that the performance of the PDA detector, in terms of the probability of group detection error, is significantly better than the decorrelator and the DF detector; and is also close to the performance lower bound in both regular and overloaded systems.

Chapter 6

Summary and Future Research Directions

6.1 Summary

The performances of Decision Feedback (DF) related multiuser detector are improved. The optimal user partitioning and ordering for the Group Decision Feedback (GDF) detector in synchronous CDMA is found. A fast optimal algorithm based on user ordering and Branch-and-Bound search is proposed. Compared with the optimal algorithm in the literature, the average computational cost is significantly reduced. The time labeling issue is solved for DF detector in asynchronous CDMA. The optimal time labeling and user ordering is derived.

In addition, a new multiuser detection algorithm based on the idea of Probabilistic Data Association (PDA) is proposed. It is shown that the PDA detector achieves near-optimal performance in most of the cases in both synchronous and asynchronous systems with $O(K^3)$ complexity where K is the number of users.

The soft-output feature of the PDA method makes it extremely flexible and easy to extend to multiuser detection problems in more realistic communication settings.

6.2 Future Research Directions

In this section, we present several ideas to extend the proposed methods in our future research. The delayed multiuser detection in synchronous CDMA is proposed in section 6.2.1. PDA multiuser detection over a flat Rayleigh fading channel is studied in section 6.2.2. The combination of multiuser detection and channel coding is briefly introduced in section 6.2.3.

6.2.1 Delayed Multiuser Detection

Linear detectors and some of the decision-driven detectors have fixed numbers of steps to obtain the detection results. Consequently, the computational complexities and the detection performances of these algorithms are relatively easy to analyze. However, most advanced detection algorithms, especially those stemming from an optimization viewpoint, involve iterations and global search, which makes the actual computational costs dynamic and observation-dependent. The fast optimal and suboptimal "Any-time" algorithms based on Branch-and-Bound search proposed in chapter 3 is one such example. The average computational costs of the proposed algorithms are significantly less than the worst case ones; they also decrease when SNR increases.

The synchronous CDMA system model is given by

$$\boldsymbol{y} = \boldsymbol{R}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{n} \tag{198}$$

where $\boldsymbol{n}\ N(0, \sigma^2\boldsymbol{R})$ is the colored Gaussian noise. Denote the probability of error and the computational cost of a multiuser detector Φ for a specific time frame as $Pe_{\boldsymbol{\Phi}}(\sigma)$ and $C_{\boldsymbol{\Phi}}(\boldsymbol{y})$, respectively. Assume that the actual probability of error (that takes computational resources into account) per time frame is $\bar{P}e_{\boldsymbol{\Phi}}(\sigma)$ and the hardware computational capacity per time frame is H. Since a conventional model of multiuser detection problem assumes that the decisions in one time frame must be made before obtaining the matched filter outputs of the next time frame, $H \geq \max_{\boldsymbol{y}} C_{\boldsymbol{\Phi}}(\boldsymbol{y})$ must be satisfied in order to avoid computational overflow.

For linear detectors, $C_{\boldsymbol{\Phi}}(\boldsymbol{y}) = C_{\boldsymbol{\Phi}} = constant$. Therefore, the actual probability of error of the system satisfies

$$\bar{P}e_{\boldsymbol{\Phi}}(\sigma) = \begin{cases} Pe_{\boldsymbol{\Phi}}(\sigma) & H \geq C_{\boldsymbol{\Phi}} \\ 1 & H < C_{\boldsymbol{\Phi}} \end{cases} \tag{199}$$

However, for detector with a dynamic computational cost, $C_{\boldsymbol{\Phi}}(\boldsymbol{y})$ becomes a random variable since $\boldsymbol{y}$ is random. Thus, a detection error can be caused either by a decision error of the algorithm when $C_{\boldsymbol{\Phi}}(\boldsymbol{y}) \leq H$, or by a computational overflow when $C_{\boldsymbol{\Phi}}(\boldsymbol{y}) > H$. Therefore, the actual probability of error (assume that the detector reports an decision error in case of computational overflow) becomes

$$\bar{P}e_{\boldsymbol{\Phi}}(\sigma) = Pe_{\boldsymbol{\Phi}}(\sigma)[1 - P(C_{\boldsymbol{\Phi}}(\boldsymbol{y}) > H)] + P(C_{\boldsymbol{\Phi}}(\boldsymbol{y}) > H) \tag{200}$$

where $P(C_{\Phi}(\boldsymbol{y}) > H)$ denotes the probability that $C_{\Phi}(\boldsymbol{y}) > H$.

Nevertheless, a computational overflow does not necessarily mean a detection error if we do not have the constraint that the detection must be made before obtaining the matched filter outputs of the next time frame. For example, if there is no constraint on the delay of the output, the problem becomes an offline detection problem and $\bar{Pe}_{\Phi}(\sigma) = Pe_{\Phi}(\sigma)$ since no computational overflow will occur. Evidently, the actual probability of error in the range $[Pe_{\Phi}, Pe_{\Phi}(\sigma) * [1 - P(C_{\Phi}(\boldsymbol{y}) > H)] + P(C_{\Phi}(\boldsymbol{y}) > H)]$ is expected if a certain amount of delay on the output is allowed. This is the main idea of the delayed multiuser detection.

Now, suppose we allow a delay of m time frames on the output. The relationship between the input, the output and the actual computational cost for each time frame is illustrated in Figure 29.

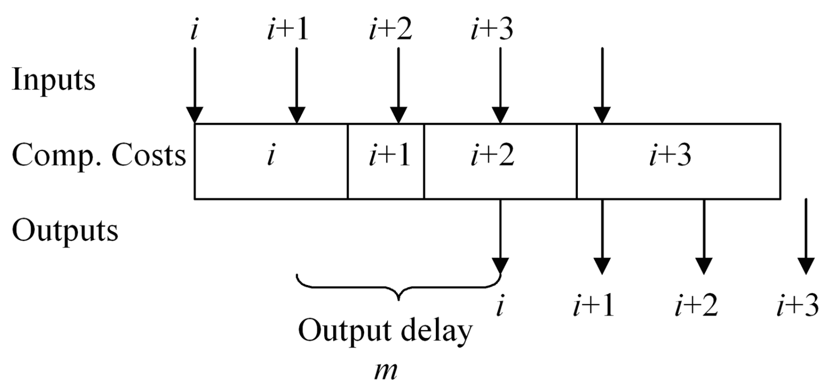

Figure 29: Illustration of the delayed multiuser detection

Define $C_{\Phi}^{(i)}(\boldsymbol{y})$ to be the computational cost of the detection of i successive time frames. There will be no computational overflow if $C_{\Phi}^{(i)}(\boldsymbol{y}) \leq (i+m)H$ is satisfied $\forall i$. Consider the fast optimal algorithm proposed in chapter 3, where

$C_{\mathbf{\Phi}}^{(i)}(\boldsymbol{y})$ is a function of σ and decreases to $i[\min_{\boldsymbol{y}} C_{\mathbf{\Phi}}^{(1)}(\boldsymbol{y})]$ when $\sigma \to 0$. Therefore, asymptotically, the probability of computational overflow can be represented by

$$P^{(m)}(\text{Overflow}) = \sum_{i=1}^{\infty} P(C_{\mathbf{\Phi}}^{(i)}(\boldsymbol{y}) \leq (i+m)H) \tag{201}$$

Assume that, when a computational overflow event occurs, we terminate the computation and report an error on detection of $m+1$ successive time frames. The overall probability of error is given by

$$\bar{P}e_{\mathbf{\Phi}}(\sigma) = Pe_{\mathbf{\Phi}}(\sigma) * [1 - P^{(m)}(\text{Overflow})] + (m+1)P^{(m)}(\text{Overflow}) \tag{202}$$

Assume the actual detection error is dominated, asymptotically, by the overflow so that

$$\lim_{\sigma \to \mathbf{0}} \frac{Pe_{\mathbf{\Phi}}(\sigma)}{P^{(m)}(\text{Overflow})} = 0 \tag{203}$$

The delayed multiuser detection improves $\bar{P}e_{\mathbf{\Phi}}(\sigma)$ asymptotically when

$$\lim_{\sigma \to \mathbf{0}} \frac{P^{(m)}(\text{Overflow})}{P^{(0)}(\text{Overflow})} = 0 \tag{204}$$

is satisfied.

6.2.2 Multiuser Detection Over Flat Rayleigh Fading Channels

In wireless communications, the multipath effect is caused by signal reflection from objects located between or around the transmitter and the receiver. In addition, the moving of the mobile receiver or transmitter causes Doppler shift. These make the received power of the signals random and time varying, a phenomenon called channel fading. Channel fading is a critical issue in wireless communications, and has been widely studied for decades.

When the reflectors are located around the receiver and there is no line-of-sight from the transmitter and the mobile receiver, the received signal amplitudes yield the Rayleigh distribution, and thus this is a Rayleigh fading channel. Commonly used models that simulate the Rayleigh fading channel are the second order AR model [Lb93], the Jakes model [Jk74] and the modified Jakes model [DBC93]. Assuming that the received signal amplitudes are slowly time-varying and are non-frequency selective, [Lb93] shows that the signal amplitudes can be estimated by a Kalman filter using a second order AR model. In [Wx96], a method that combines the DF detector with a Kalman filter is proposed for the multiuser detection over flat Rayleigh fading channels.

Define $\boldsymbol{w}(i)$ to be the $K \times 1$ vector whose kth element $w_k(i)$ is the received signal amplitude for user k in time frame i. Define $\boldsymbol{B}(i)$ to be a diagonal matrix whose diagonal components are corresponding binary user signals in time frame i. The synchronous CDMA model can be alternatively written as

$$\boldsymbol{y}(i) = \boldsymbol{R}\boldsymbol{B}(i)\boldsymbol{w}(i) + \boldsymbol{n}(i) \tag{205}$$

From the second AR model of the Rayleigh fading channel, we have [Wx96]

$$\begin{bmatrix} \boldsymbol{w}(i+1) \\ \boldsymbol{w}(i) \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_1 & \boldsymbol{A}_2 \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{w}(i) \\ \boldsymbol{w}(i-1) \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{0} \end{bmatrix} \mathbf{v}(i) \tag{206}$$

Here $\boldsymbol{A}_1$, $\boldsymbol{A}_2$ are diagonal matrices whose diagonal elements A_{1k}, A_{2k} are defined by

$$A_{1k} = -2r_d cos(2\pi f_d' T), \qquad A_{2k} = r_d^2 \tag{207}$$

where f_d' is the spectral peak frequency, and r_d is the pole radius which corresponds to the steepness of the peaks of the power spectrum. $\mathbf{v}(i)$ is the driven noise vector whose kth element is zero mean Gaussian with a variance of

$$\sigma^2_{v_k(i)} = \frac{[(1+r_d^2)^2 - 4r_d^2 cos^2(2\pi f_d' T)](1-r_d^2)}{1+r_d^2} \tag{208}$$

Assume $\boldsymbol{B}(j)$, $\forall j \leq i$ are known. A Kalman filter can be formed using (206) as the state equation and (205) as the observation equation. With the Kalman filter, a prediction on $\mathbf{w}(i+1)$ can be obtained as

$$\mathbf{w}(i+1) \sim N(\hat{\mathbf{w}}(i+1), \Sigma(i+1)) \tag{209}$$

Then, multiuser detection on $\boldsymbol{b}(i+1)$ can be applied given (209) and $\boldsymbol{y}(i+1)$. The decision, $\hat{\boldsymbol{b}}(i+1)$, is then fedback to the Kalman filter for propagating to the next time frame assuming no detection error. The multiuser detection scheme is illustrated in Figure 30

In the multiuser detection part of the above procedure, the ML detection becomes

$$\Phi_{ML} : \hat{\boldsymbol{b}}(i+1) = arg \max_{\boldsymbol{b}(i+1) \in \{-1,1\}^K} f_y(\boldsymbol{y}(i+1)|\boldsymbol{b}(i+1)) \tag{210}$$

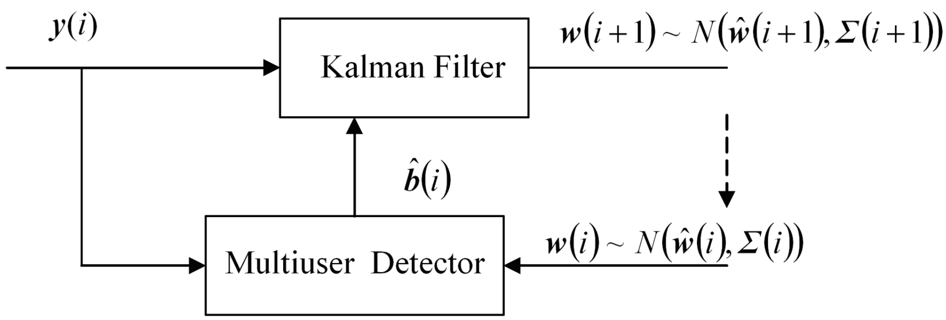

Figure 30: Multiuser Detection over Fading Channels

and the key step is to obtain, given (209),

$$f(\boldsymbol{y}(i+1)|\boldsymbol{b}(i+1)) = \int_{\mathbf{W}(i+1)} f_y(\boldsymbol{y}(i+1)|\boldsymbol{b}(i+1), \mathbf{w}(i+1)) f_w(\mathbf{w}(i+1)) d\mathbf{w}(i+1) \tag{211}$$

Unfortunately, no closed-form solution is available unless $\Sigma(i+1)$ is diagonal.

A similar problem is studied in [VV96], where it is proposed to consider $\boldsymbol{w}(i+1) \sim N(\hat{\mathbf{w}}(i+1), \tilde{\Sigma}(i+1))$ in multiuser detection. Here $\tilde{\Sigma}(i+1)$ is a diagonal matrix whose diagonal elements are equal to those of $\Sigma(i+1)$. Consequently, (211) becomes

$$f(\boldsymbol{y}(i+1)|\boldsymbol{b}(i+1)) = N(\boldsymbol{B}(i+1)\hat{\mathbf{w}}(i+1), \sigma^2(\frac{\tilde{\Sigma}(i+1)}{\sigma^2} + \boldsymbol{R}^{-1})) \tag{212}$$

which is equivalent to a non-fading system with $\hat{\mathbf{w}}(i+1)$ as the signal amplitudes and $(\frac{\tilde{\Sigma}(i+1)}{\sigma^2} + \boldsymbol{R}^{-1}))$ as the correlation matrix.

In asynchronous CDMA, we start from the white noise model. The observation equation for the Kalman filter is changed to

$$\tilde{\boldsymbol{y}}(i) = \boldsymbol{F}[0]\boldsymbol{B}(i)\mathbf{w}(i) + \boldsymbol{F}[1]\boldsymbol{B}(i-1)\mathbf{w}(i-1) + \tilde{\boldsymbol{n}}(i) \tag{213}$$

Similar to (212), by considering diagonal correlation on $\hat{\boldsymbol{w}}(i+1)$ and $\hat{\boldsymbol{w}}(i)$, we obtain

$$\begin{aligned} f(\boldsymbol{y}(i+1)|\boldsymbol{b}(i+1), \boldsymbol{b}(i)) &= N(\boldsymbol{F}[0]\boldsymbol{B}(i+1)\hat{\boldsymbol{w}}(i+1) + \boldsymbol{F}[1]\boldsymbol{B}(i)\hat{\boldsymbol{w}}(i), \Xi(i+1)) \\ \Xi(i+1) &= \sigma^2(\frac{\boldsymbol{F}[0]\tilde{\Sigma}(i+1)\boldsymbol{F}[0]^T + \boldsymbol{F}[1]\tilde{\Sigma}(i)\boldsymbol{F}[1]^T}{\sigma^2} + \boldsymbol{I}) \end{aligned} \quad (214)$$

Therefore, the system is equivalent to a non-fading one with modified parameters.

In synchronous CDMA, the extension of the proposed PDA detector to multiuser detection over Rayleigh fading channel, using the above methods, is straightforward. However, since asynchronous PDA involves sliding window processing, further study is required to combine PDA with the Kalman filter idea.

6.2.3 Combined Multiuser Detection with Channel Coding

Although the multiuser detection problem is already NP hard, it has been shown that combining the multiuser detection with channel coding will significantly improve the overall system performance. In the current literature, related research has been reported in [Mh98] [RSAA98] [WP99]. However, the computational costs for these methods are usually very high.

As shown in chapter 4, the proposed PDA algorithm achieves close to optimal performance in most cases with a relatively low complexity. It is natural to combine the proposed PDA method with “Turbo-coding” [BGT93] since both of them work in probability space and give “soft-outputs”, and both are reported to achieve near optimal performance. An illustration of the combination is given in Figure 31.

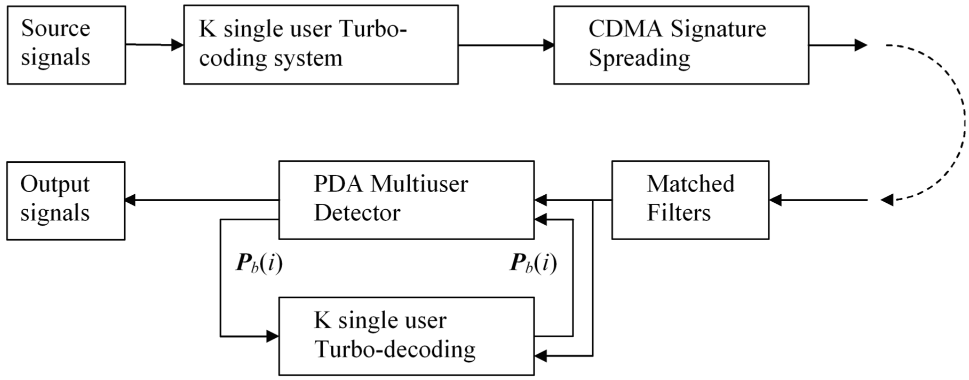

Figure 31: Combining PDA multiuser detection with Turbo coding

In this procedure, the probability updates are iterated between the PDA multiuser detector and the Turbo-decoder. Decisions are made according to the probabilities after convergence. An outstanding performance with relatively low computational complexity is expected.

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