A sub-optimal soft decision PDA method for binary quadratic programming

J. Luo, K. R. Pattipati and P. K. Willett

ECE Dept., University of Connecticut
Storrs, CT 06269
Email: Krishna@engr.uconn.edu

Abstract[1]

Binary Quadratic Programming (BQP) problems arise frequently in digital communication systems where online solutions are required. The Multiuser Detection (MUD) problem in Code Division Multiple Access (CDMA) communications, studied in this paper, is one such example. Due to the NP-hard nature of the BQP problem arising in MUD, only sub-optimal methods with polynomial complexities can be realistically considered. In this paper, a sub-optimal algorithm based on the idea of Probabilistic Data Association (PDA) is proposed. By treating the detection parameters as binary random variables, and by approximating the multi-modal Gaussian mixture by a single Gaussian noise, the PDA method provides near-optimal solution with a computational complexity of $O(N^3)$, where N is the problem size. Several other algorithms for the MUD problem are also considered and compared in terms of computational efficiency and the degree of sub-optimality. Although all of the problems studied in this paper come from the domain of MUD in CDMA, we are testing the PDA method on general BQP problems. Further results will be reported in our future research work.

1 Introduction

Multiuser Detection (MUD), aimed at suppressing the Multiple Access Interference (MAI) in Code Division Multiple Access communications, has been widely discussed in the literature for over a decade [9]. When user signals assume only binary values, solving the MUD problem corresponds to a Binary Quadratic Programming (BQP) problem. Generally, the BQP problem is NP-hard and exponentially complex in the problem size [1] (the number of users in the context of MUD). Therefore, only sub-optimal algorithms with polynomial complexities are considered in practice.

The linear detectors, including the decorrelator [1], the Minimum Mean Square Error (MMSE) detector [3], were proposed in mid 1980's. When the number of users is N, the overall computational load for a linear detector is $O(N^2)$. The decision-driven multiuser detectors, including the multistage detector [4], the decision feedback detector (DFD) [5][10], and the group detector [7][16], were proposed in the early 1990's. Unlike the linear detectors, decision-driven multiuser detectors make decisions sequentially with weak users utilizing the decisions on strong user signals to mitigate the MAI. The performance of the class of decision-driven multiuser detectors is significantly better than the linear detectors. However, it is well known that the performances of decision-driven detectors are affected by the order of the users [9]. Fortunately, the optimal user ordering for DFD and GDFD are reported and can be found offline [10][16]. Therefore, the online-computational-load for decision-driven multiuser detectors remains $O(N^2)$.

In addition to linear and decision-driven multiuser detectors, many other algorithms have also been developed for the MUD problem during the past several years. Among them are the multistage detector using a genetic algorithm [14], an adaptive Bayesian detector using Gibbs sampling [13], and semi-definite relaxation [17]. These methods solve the BQP problem from different points of view. The performance of these methods is generally good. Neverthless, their computational costs, although still polynomial, are higher than the linear and decision-driven detectors. The semi-definite relaxation method provides a near-optimal performance with a complexity of $O(N^{3.5})$ [17]. Since the key issue in MUD is to solve a positive definite BQP problem, general optimization techniques, such as the coordinate descent search [15], the tabu search [11], the simulated annealing method [12], the Lagrangian relaxation method [12], the roll-out method [18], and the Boltzmann machine [8] can be applied. Related work is reported in [20] and the corresponding performance comparisons are given. Further more, as shown in this paper, exploring the multiuser detection algorithms also provides alternative sub-optimal approaches for the general BQP problem.

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The Probabilistic Data Association (PDA) is one of the most successful methods in target tracking [6]. The key feature of PDA is to represent the unknown binary parameters as binary random variables, together with a repeated approximation of a multimodal Gaussian mixture as a single Gaussian with a matched mean and covariance. Although this is an unjustifiable step to some extent, it is difficult to argue with its good performance and low complexity. In CDMA multiuser detection, similar idea can be applied. Simulations show that the PDA method provides performance that is indistinguishable from optimal with a computational complexity of $O(N^3)$.

Portions of the results shown in this paper will also appear in [19].

2 System Model and Its Relation to General BQP

A discrete-time equivalent model for the matched-filter outputs at the receiver of a synchronous CDMA channel is given by the N-length vector [10]

$$y = Hb + v \tag{1}$$

where $b \in \{-1,+1\}^N$ denotes the N-length vector of bits transmitted by the N active users. $H = W^{1/2} R W^{1/2}$ is a nonnegative definite signature waveform correlation matrix; R is the symmetric normalized correlation matrix with unit diagonal elements; W is a diagonal matrix whose k-th diagonal component, w_k is the received signal energy per bit of the k-th user; and v is a colored Gaussian noise with zero mean and a covariance matrix of $\sigma^2 H$, where σ^2 is the covariance of the white Gaussian noise before the matched filter [10].

When all the user signals are equally probable, the optimal solution for (1) is the output of a Maximum Likelihood (ML) detector

$$\Phi_{ML} : \hat{b} = \arg\min_{b \in \{-1,+1\}^N} \left(b^T H b - 2 y^T b \right) \tag{2}$$

which corresponds to a BQP problem and minimizes the probability that not all users' decisions are correct. Generally, the BQP problem in (2) is NP-hard [1]. Therefore, previous research has focused on developing easily implementable and effective multiuser detectors.

Before presenting the proposed algorithm, the following proposition shows that all BQP problems can be equivalently written as (2).

Proposition 1:

(a) Any binary quadratic programming problem can be equivalently converted to a binary quadratic $\{-1,+1\}$ programming problem. In other words, the following two minimization problems are equivalent:

$$\Phi_1 : \hat{b} = \arg\min_{\tilde{b}_i \in \{\alpha_i, \beta_i\}} \left(\tilde{b}^T \tilde{H} \tilde{b} - 2 \tilde{y}^T \tilde{b} \right) \quad (i = 1, \Lambda, N)$$

$$\Phi_2 : \hat{b} = \arg\min_{b \in \{-1,+1\}^N} \left(b^T H b - 2 y^T b \right)$$

and the following two maximization problems are also equivalent:

$$\Phi_1 : \hat{b} = \arg\max_{\tilde{b}_i \in \{\alpha_i, \beta_i\}} \left(\tilde{b}^T \tilde{H} \tilde{b} - 2 \tilde{y}^T \tilde{b} \right) \quad (i = 1, \Lambda, N)$$

$$\Phi_2 : \hat{b} = \arg\max_{b \in \{-1,+1\}^N} \left(b^T H b - 2 y^T b \right)$$

Here $\tilde{b}_i$ denotes the ith component of column vector $\tilde{b}$, and $\alpha_i < \beta_i \ \forall i$ are arbitrary real numbers.

(b) Any binary quadratic maximization problem can be converted to a binary quadratic minimization problem, i.e., the following two problems are equivalent:

$$\Phi_1 : \hat{b} = \arg\max_{\tilde{b} \in \{-1,+1\}^N} \left(\tilde{b}^T \tilde{H} \tilde{b} - 2 \tilde{y}^T \tilde{b} \right)$$

$$\Phi_2 : \hat{b} = \arg\min_{b \in \{-1,+1\}^N} \left(b^T H b - 2 y^T b \right)$$

(d) Any binary quadratic minimization problem Φ_1 can be converted to a positive definite binary quadratic minimization problem Φ_2

$$\Phi_1 : \hat{b} = \arg\min_{\tilde{b} \in \{-1,+1\}^N} \left(\tilde{b}^T \tilde{H} \tilde{b} - 2 y^T \tilde{b} \right) \quad (i = 1, \Lambda, N)$$

$$\Phi_2 : \hat{b} = \arg\min_{b \in \{-1,+1\}^N} \left(b^T H b - 2 y^T b \right)$$

where $\tilde{H}$ is an arbitrary symmetric matrix, while H is a symmetric positive definite matrix.

Proof: The problem in part (a) of proposition 1 can be easily proved by assigning

$$b_i = \frac{2\tilde{b}_i - (\alpha_i + \beta_i)}{\beta_i - \alpha_i} \tag{3}$$

$$H = (\mathrm{B} - \mathrm{A}) \tilde{H} (\mathrm{B} - \mathrm{A}) \tag{4}$$

$$y = \tilde{H}(\alpha + \beta) + (\mathrm{B} - \mathrm{A}) \tilde{y} \tag{5}$$

where A and B are diagonal matrices whose ith diagonal elements are α_i and β_i, respectively.

Part (b) can be proved by assigning

$$H = -\tilde{H} \tag{6}$$

$$y = -\tilde{y} \tag{7}$$

In order to prove part (c), we use the fact that $\tilde{b}^T \tilde{b} = N$ to obtain

$$\tilde{b}^T \tilde{H} \tilde{b} - 2 y^T \tilde{b} = \tilde{b}^T \left(\tilde{H} - \lambda I \right) \tilde{b} - 2 y^T \tilde{b} + \lambda N \tag{8}$$

Here I is an $N \times N$ identity matrix. By selecting λ to be a number less than the minimum eigenvalue of

$\tilde{H}$, and assigning $H = \tilde{H} - \lambda I$, we can see that part (c) of proposition 1 holds.

3 A Sub-optimal Algorithm Using Probabilistic Data Association

A. The Basic Algorithm

PDA considers undecided binary parameters as binary random variables. By approximating a multimodal Gaussian mixture by a single Gaussian noise with matched mean and covariance, the probability masses of the binary variables are updated with each successive new observation so that they converge to ones with smaller variances. The converged mass functions are used in making decisions on the binary parameters. In the binary quadratic minimization problem in (2), we can also use a similar idea. Rewrite the original system model (1) as

$$\tilde{y} = H^{-1} y = b + \tilde{v} \tag{10}$$

where $\tilde{v} = H^{-1} v$ is Gaussian with zero mean and covariance, $\sigma^2 H^{-1}$. We call (10) the "decorrelated model" since $\tilde{y}$ is in fact a normalized version of the decorrelator output before the hard decisions. For any user *i*, we associate a probability $P_b(i)$ with user signal b_i to express the current belief on its value, i.e., $P_b(i)$ is the current estimate of the probability that $b_i = 1$, and $1 - P_b(i)$ is the corresponding estimate for $b_i = -1$. Now, for an arbitrary user signal b_i, treat other user signals $\{b_j | j \neq i\}$ as binary random variables and treat $\sum_{j \neq i} b_j e_j + \tilde{v}$ as the "effective" noise, where e_j is a column vector whose *j*th component is 1 and whose other components are 0. Consequently, $P(b_i = 1 | \tilde{y}, \{P_b(j)\}_{j \neq i})$ and $P(b_i = -1 | \tilde{y}, \{P_b(j)\}_{j \neq i})$ can be obtained from (10). This provides a way to update iteratively the probabilities associated with the user signals. Based on the decorrelated system model, the basic form of the proposed multistage PDA detector proceeds as follows:

(1) Sort users according to the user-ordering criterion proposed for the decision feedback detector in [10] (specifically Theorem 1 of [10]).

(2) Initialize the probabilities as $P_b(i) = 0.5, \forall i$. Initialize the stage counter $k = 1$.

(3) Initialize the user counter $i = 1$.

(4) Based on the current values of $P_b(j)$ $(j \neq i)$, update $P_b(i)$ via

$$P_b(i) = P(b_i = 1 | \tilde{y}, \{P_b(j)\}_{j \neq i}) \tag{11}$$

(5) If $i < N$, let $i = i + 1$ and goto step (4).

(6) If $\forall i$, $P_b(i)$ has converged, goto step (7). Otherwise, let $k = k + 1$ and return to step (3).

(7) $\forall i$, make a decision on user signal *i* via

$$b_i = \begin{cases} 1 & P_b(i) \geq 0.5 \\ -1 & P_b(i) < 0.5 \end{cases} \tag{12}$$

In the above procedure, the computation of (11) is evidently exponential in the number of users. In order to avoid the expensive computation, we use the Gaussian approximation recommended by PDA. Define

$$v_i = \sum_{j \neq i} b_j e_j + \tilde{v} \tag{13}$$

Approximate v_i by a single Gaussian random variable with matched mean and covariance matrix; that is,

$$E[v_i] = \sum_{j \neq i} e_j (2P_b(j) - 1) \tag{14}$$

$$Cov[v_i] = \sum_{j \neq i} [4P_b(j)(1 - P_b(j))] e_j e_j^T + \sigma^2 H^{-1} \tag{15}$$

Now, define

$$\theta_i = \sum_{j \neq i} e_j (2P_b(j) - 1) - \tilde{y} \tag{16}$$

$$\Omega_i = \sum_{j \neq i} [4P_b(j)(1 - P_b(j))] e_j e_j^T + \sigma^2 H^{-1} \tag{17}$$

We obtain

$$\frac{P_b(i)}{1 - P_b(i)} = \exp\{-2\theta_i^T \Omega_i^{-1} e_i\} \tag{18}$$

B. Refinements

B.1 Speed-Up: Matrix Arithmetic

In addition to the Gaussian approximation idea, further simplifications on the computational load can be obtained by applying the Sherman-Morrison-Woodbury formula [2] on the calculation of Ω_i^{-1}. Define auxiliary variables

$$\theta = \sum_j e_j (2P_b(j) - 1) - \tilde{y} \tag{19}$$

$$\Omega = \sum_j [4P_b(j)(1 - P_b(j))] e_j e_j^T + \sigma^2 H^{-1} \tag{20}$$

Using Sherman-Morrison-Woodbury formula, we have

$$\theta_i = \theta - e_i (2P_b(i) - 1) \tag{21}$$

$$\Omega_i^{-1} = \Omega^{-1} + \frac{4P_b(i)(1-P_b(i))\Omega^{-1}e_ie_i^T\Omega^{-1}}{1-4P_b(i)(1-P_b(i))e_i^T\Omega^{-1}e_i} \quad (22)$$

and

$$\theta = \theta_i + e_i(2P_b(i)-1) \quad (23)$$

$$\Omega^{-1} = \Omega_i^{-1} - \frac{4P_b(i)(1-P_b(i))\Omega_i^{-1}e_ie_i^T\Omega_i^{-1}}{1+4P_b(i)(1-P_b(i))e_i^T\Omega_i^{-1}e_i} \quad (24)$$

By keeping the updated versions of θ and Ω^{-1}, we can divide step 3 into three sub-steps. In sub-step 1, we calculate θ_i and Ω_i^{-1} using (21) and (22). Sub-step 2 obtains the updated $P_b(i)$ using (18). In sub-step 3, we use the new $P_b(i)$ and update θ and Ω^{-1} using (23) and (24). The overall computational complexity for one stage of the PDA method is then reduced to $O(N^3)$.

B.2 Speed-Up: Successive Cancellation

In the basic form of the PDA method, the overall complexity can be high if one or two users show a slow convergence. In our computer simulations, in most of the cases, more than 70% of the users will converge in the first stage. Thus to avoid the high computational load caused by a small number of slowly converging users, we introduce the idea of successive cancellation among the stages.

After the *k*th stage, define G to be the group of "converged" users that satisfy

$$\forall i \in G,\ P_b(i) \in [0,\varepsilon] \cup [1-\varepsilon,1] \quad (25)$$

where ε is a small positive number. Make decisions on the users in G via

$$\forall i \in G,\ b_i = sign(P_b(i) - 0.5) \quad (26)$$

Denote $\overline{G}$ to be the complement of G, i.e., the group of "non-converged" users. By canceling the MAI, the decorrelated system model for the users in $\overline{G}$ can be reformulated as

$$(H_{\overline{G}\overline{G}})^{-1}(y_{\overline{G}} - H_{\overline{G}G}b_G) = b_{\overline{G}} + \tilde{v}_{\overline{G}} \quad (27)$$

Here $H_{\overline{G}\overline{G}}$ denotes the sub-block matrix of H that only contains the columns and rows corresponding to users in $\overline{G}$. $\tilde{v}_G$ is the colored Gaussian noise of the sub-system with zero mean and covariance matrix $\sigma^2(H_{\overline{G}\overline{G}})^{-1}$. Consequently, in the $(k+1)$st stage, we apply the PDA detection procedure only on the sub-system model.

B.3 Performance: Bit-Flipping

In computer simulations, we note that when optimal and PDA solutions to (2) differ, they usually disagree in one element only. Thus, as an inexpensive way to improve the PDA detector, we add a "bit-flip" stage after PDA has converged. This corresponds to a one-step coordinate descent [15].

4 Computer Simulations

In this section, we use several computer simulations to show the performance and computational cost of the PDA method. The ML detector, the Decorrelator [1], the Decorrelator-based DFD [12], and the Semi-definite relaxation method [17] are compared. In the successive cancellation part of the PDA method, we set $\varepsilon = \frac{10^{-2}}{4SNR}$ where SNR is the signal to noise ratio. For the Semi-definite Relaxation algorithm, the number of randomization is set to 20.

The first example is a typical CDMA MUD problem over band-efficient channel. Assume we have 29 users. The signature sequences are randomly chosen from length-31 Gold codes [9]. The signal amplitudes are randomly and independently generated by $\sqrt{w_{ii}} \sim N(4.5,4)\ \forall i$, and are limited within a range of $[2,7]$ ($N(.)$ represents the Normal distribution). The H matrix in (1) is generated via $H = W^{1/2}RW^{1/2}$. The performance comparison of different algorithms is shown in Figure 1.

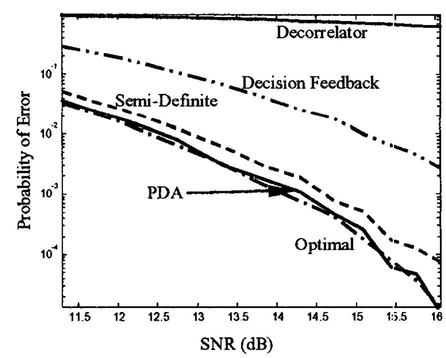

Figure 1. 29-users, length-31 Gold codes as signature sequences, 100000 Monte-Carlo runs

In the second example, we fix the SNR to be 12dB. The signature sequences are randomly generated and the ratio between the spreading factor (the length of the signature sequence) and the number of users is fixed at 1.2. Let the number of users vary from 3 to 60. Figure 2 shows the worst case computational complexity measured in terms of the number of multiplications plus number of additions of the PDA detector and of the Semi-definite Relaxation method. The online complexity of the Decorrelator and the DFD are known to be $O(N^2)$ and the complexity of the ML detector is exponential in the number of users. For comparison purposes, we do not show the computational costs of these methods in Figure 2. It is known that the computational cost of the Semi-definite Relaxation method is $O(N^{3.5})$. Therefore, we claim that the computational cost for the PDA method is significantly less than $O(N^{3.5})$. Simulations show the complexity is in fact $O(N^3)$.

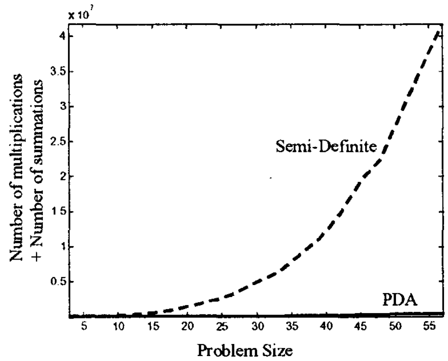

Figure 2. Comparison of worst-case computational costs. Random signature sequences, spreading factor=1.2*N*, SNR=12dB are assumed for PDA and Semi-definite Relaxation.

In the third example, we apply the above multiuser detection algorithms to general binary quadratic programming problems. We fix the problem size at 15. The symmetric matrix H is generated randomly with all the elements uniformly distributed between $[-2,+2]$. The y vector is also randomly generated with components uniformly distributed in $[-8,+8]$. We use a randomly generated flag $f=\{-1,+1\}$ to determine whether it is a binary quadratic maximization problem or it is a binary quadratic minimization problem (maximize (minimize) $(b^T Hb-2y^T b)$ when $f=+1$ ($f=-1$)). In order to convert a general binary quadratic minimization problem to a positive definite binary quadratic minimization problem, we apply the method shown in the proof of proposition 1. In (9), we choose $\lambda=\frac{1}{N}tr(\tilde{H})-\left\|\tilde{H}-\frac{1}{N}tr(\tilde{H})I\right\|_1$, where $\|A\|_1$ denotes the 1-norm of matrix A, $tr(A)$ denotes the trace of matrix A, and I is $N\times N$ identity matrix. Other settings are identical to that in example 1. However, for the PDA method, there is an additional problem. As opposed to the CDMA MUD problem, the parameters SNR and σ^2 in the MUD system model are not available in general BQP problems. In our computer simulation, we select SNR and σ^2 using the following procedure.

First convert a general BQP problem to a positive definite binary quadratic minimization problem (as described above), and write the minimization problem as

$$\Phi:\hat{b}=\underset{b\in\{-1,+1\}^N}{\arg\min}\left(b^T Hb-2y^T b\right) \tag{27}$$

Estimate the average signal power by averaging the absolute values of the diagonal elements of H, i.e.,

$$P_s=\frac{1}{N}\sum_{i=1}^{N}|h_{ii}| \tag{28}$$

Suppose $L^T L=H$ is the Cholesky decomposition of H. Randomly generate a decision vector b, calculate the white noise vector (assume that the system yields the MUD system model) by

$$n=\left(L^T\right)^{-1}y-Lb \tag{29}$$

Estimate σ^2 by

$$\sigma^2=\frac{1}{16N}\sum_{i=1}^{N}n_i^{\,2} \tag{30}$$

The SNR is computed by $SNR=\frac{P_s}{\sigma^2}$.

For the purpose of a fair comparison, we add a final bit-flip stage to all the compared methods. The simulation result is obtained from 10000 Mont-Carlo runs. Figure 3 shows the box plot of the normalized error, which is defined as

$$\text{Normalized Error}=\left|\frac{f(b)-f(b^*)}{f(b^*)}\right| \tag{31}$$

where $f()$ represents the cost function, b^* is the optimal solution and b is the solution given by the sub-optimal algorithm. Table 1 shows, for different algorithms, the percentage of problems where optimal solution was found, and the percentage of problems where the normalized error is less than 5%. In most of the cases, the normalized error for the PDA method is very small, and it rarely exceeds 20% in 10000 Monte-Carlo runs.

In addition, recall that the user ordering needs to be done offline for the DFD algorithm. For general BQP problem, however, the computational cost for the user ordering must be taken into consideration. Therefore, the overall computational cost for DFD is $O(N^3)$ instead of $O(N^2)$.

5 Conclusions

A new method based on the idea of Probabilistic

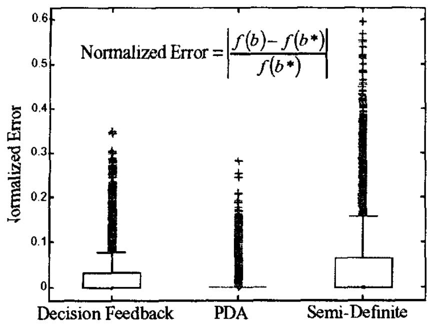

Figure 3. Box plot of the normalized error

	DFD	PDA	Semi-Definite
Optimum Solution Found	59.08%	76.61%	49.60%
Normalized Error < 5%	82.54%	94.39%	71.24%

Table 1. Comparison of Optimality

Data Association is proposed for multiuser detection in synchronous CDMA communications. Computer simulation shows that the PDA method provides near optimal performance with an overall computational complexity of $O(N^3)$. It is also shown that any BQP problem can be equivalently written in the form of a CDMA multiuser detection problem. Therefore, all the multiuser detection algorithms can be used to solve the general BQP problems. The PDA method as well as other sub-optimal multiuser detectors will be tested on a variety of large-scale binary quadratic programming problems in our future research.

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