A PDA and Kalman Smoothing Approach to Multiuser Detection in Asynchronous CDMA

David N. Pham, Jie Luo, K.R. Pattipati, and P.K. Willett
ECE Dept., Univ. of Connecticut
Storrs, CT 06269
Email: krishna@engr.uconn.edu

Abstract - *The optimal maximum-likelihood multi-user detection problem in synchronous Code-Division Multiple Access (CDMA) is NP-hard; the asynchronous version of the problem is super-exponential. The computational difficulty has driven research into suboptimal algorithms that provide reliable decisions and ensure polynomial computational costs. The Probabilistic Data Association (PDA) method, originally developed for tracking, and recently adapted for multiuser detection in synchronous CDMA, is extended to asynchronous CDMA where a Kalman smoother is employed to track correlated noise arising from the outputs of a decorrelator. The estimates from the tracker, coupled with an iterative PDA, result in impressively low bit error rates. Computer simulations show that the new scheme significantly outperforms the Decision Feedback detector. The algorithm has $O(K^3)$ complexity per time frame, where K is the number of users.*

Keywords: Tracking, filtering, estimation, PDA

1 Introduction

In Code-Division Multiple Access (CDMA) systems, users simultaneously transmit information in the form of antipodal binary sequences over a common channel by modulating their signature waveforms. The received signal is the sum of all user signals in the presence of additive noise. The conventional method of demodulation consists of passing the signal through a bank of filters, where each filter is matched to a user's signature waveform. In practical CDMA systems, where users transmit asynchronously, the signature waveforms have nonzero correlations; this results in multiple access interference (MAI). As a consequence of MAI, the conventional detector performs poorly in situations where the user energies are substantially different.

The exponential computational complexity of the optimal Maximum-Likelihood (ML) detector [1] for asynchronous CDMA and the unreliable performance of the conventional detector under realistic conditions has motivated research into improved sub-optimal multiuser detectors that yield low bit error rates and that ensure polynomial computational costs. Examples include the decorrelator [1], the Minimum Mean Square Error (MMSE) detector [2], the Decision Feedback (DF) detector [3] [5], and the multistage detector [4]. All of these detectors provide bit error rates that are lower than the conventional detector. The decorrelator, among the poorest of the aforementioned detectors in terms of error probability, decorrelates the information bits (thus solving the MAI problem in a way that would be optimal were the MAI Gaussian) by inverting the channel. However, the transformation enhances and correlates background noise. For this reason, there is a substantial gap in performance between the ML detector and the decorrelator. The gap between the DF detector, one of the more efficient detection schemes, and the ML detector is still significant. The DF detector makes its decisions one bit at a time by using past decisions and matched filter outputs [3].

In this paper, a new multiuser detection scheme for coherent demodulation in asynchronous CDMA channels is introduced. The detection scheme uses the Probabilistic Data Association (PDA) [6] method, recently developed for multiuser detection in synchronous CDMA [7], in conjunction with a Kalman smoother for tracking the correlated noise arising from the outputs of a decorrelator. Simulation results show that the new strategy significantly outperforms the DF detector [3] [5]. The algorithm has $O(K^3)$ complexity per time frame, where K is the number of users.

The rest of the paper is organized as follows. In section 2, the asynchronous CDMA model is given, and the decorrelator and the DF detector are reviewed. The PDA multiuser detector for synchronous CDMA is reviewed in section 3. In section 4, the PDA and Kalman Smoother multiuser detector for the asynchronous case is presented. Simulation results are provided in section 5. The paper concludes with a summary in section 6.

2 Decorrelator and DF Detectors

The discrete-time equivalent model for the matched-filter outputs at the receiver of a K-user asynchronous CDMA channel is described in the z domain [1] by the K-length vector

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{A}\boldsymbol{b}(z) + \boldsymbol{n}(z) \quad (1)$$

where $\boldsymbol{A}$ is a diagonal matrix whose i-th diagonal element, a_i, is the square root of the received signal energy per bit of the i-th user; $\boldsymbol{b}(z)$ is the z transform of the bit sequence transmitted by the K active users; $\boldsymbol{n}$ is colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}(z)$; and $\boldsymbol{R}(z)$ is the signature correlation matrix which is composed of three parts [1]

$$\boldsymbol{R}(z) = \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1} \qquad (2)$$

In (2), $\boldsymbol{R}[0]$ is a symmetric matrix with unity diagonal elements and whose off-diagonal elements represent the correlations between user signatures at the same time index; and $\boldsymbol{R}[1]$ is a singular matrix whose elements represent the signature correlations relating to successive time frames. The $(i, j)^{\text{th}}$ element of $\boldsymbol{R}[1]$ is denoted by $R[1]_{ij}$. Since user signal i in time frame k cannot simultaneously be correlated with that of j in time frame $k - 1$ and in time frame $k + 1$, we have $R[1]_{ij}\, R[1]_{ji} = 0$.

The decorrelated model for asynchronous CDMA in the z domain is obtained by multiplying both sides of (1) from the left by $\boldsymbol{R}^{-1}(z)$

$$\tilde{\boldsymbol{y}}(z) = \boldsymbol{A}\boldsymbol{b}(z) + \boldsymbol{x}(z) \qquad (3)$$

where $\boldsymbol{x}(z)$ is colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}^{-1}(z)$. The corresponding time-domain representation of (3) is

$$\tilde{\boldsymbol{y}}(k) = \boldsymbol{A}\boldsymbol{b}(k) + \boldsymbol{x}(k) \qquad (4)$$

where $\boldsymbol{b}(k) \in \{-1, +1\}^K$

The decorrelating detector estimates $\boldsymbol{b}(k)$ via

$$\hat{\boldsymbol{b}}(k) = \text{sgn}(\tilde{\boldsymbol{y}}(k)) \qquad (5)$$

In the absence of any background noise (i.e., $\sigma^2 = 0$), the decorrelator provides error free demodulation.

The decorrelating DF detector [3] [5] for asynchronous CDMA uses two filters: an anticausal feedforward filter $\boldsymbol{G}(z)$ which takes matched filter outputs as its inputs and a causal feedback filter $\boldsymbol{B}(z)$ which takes past decisions as its inputs (Fig. 1).

The DF detector operates one bit at a time. The users are ordered according to some criteria (e.g., arrival time, received energy) and then demodulated sequentially. It is shown in [3] that the correlation matrix $\boldsymbol{R}$(z) can be factored as

$$\boldsymbol{R}(z) = (\boldsymbol{F}[0] + \boldsymbol{F}[1]z)^T (\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}) \qquad (6)$$

where $\boldsymbol{F}[0]$ is a lower triangular matrix and $\boldsymbol{F}[1]$ is an upper triangular matrix [1]. $\boldsymbol{G}(z)$ is related to the factorized matrices via

$$\boldsymbol{G}(z) = \left(\boldsymbol{F}[0] + \boldsymbol{F}[1]z\right)^{-T} \qquad (7)$$

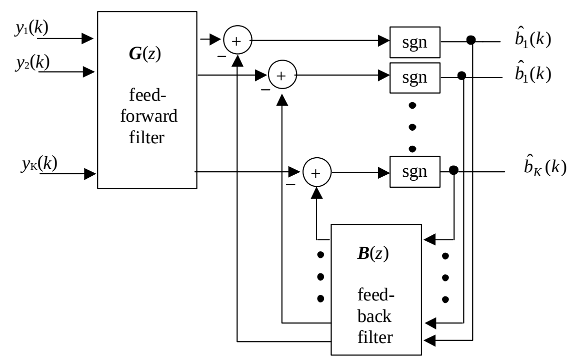

Figure 1: Block diagram of Decision Feedback Detector

Multiplying both sides of (1) by $\boldsymbol{G}(z)$ results in the white noise model

$$\tilde{\tilde{\boldsymbol{y}}}(z) = (\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1})\boldsymbol{A}\boldsymbol{b}(z) + \boldsymbol{w}(z) \qquad (8)$$

where $\tilde{\tilde{\boldsymbol{y}}}(z) = \boldsymbol{G}(z)\boldsymbol{y}(z)$ and $\boldsymbol{w}(z)$ is white Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{I}$. The time domain representation of (8) is

$$\tilde{\tilde{\boldsymbol{y}}}(k) = \boldsymbol{F}[0]\boldsymbol{b}(k) + \boldsymbol{F}[1]\boldsymbol{b}(k-1) + \boldsymbol{w}(k) \qquad (9)$$

We define $\boldsymbol{s}(k)$ as

$$\boldsymbol{s}(k) = \tilde{\tilde{\boldsymbol{y}}}(k) - \boldsymbol{F}[1]\hat{\boldsymbol{b}}(k-1) \qquad (10)$$

Since $\boldsymbol{F}[0]$ is lower triangular, user 1 has no interference from other users when $\boldsymbol{b}(k-1)$ is known (the ideal case). The estimate for user 1 is

$$\hat{b}_1(k) = \text{sgn}(s_1(k)) \qquad (11)$$

where s_1 is the first element of $\boldsymbol{s}$. User $i > 1$ has interference due to users $j \in \{1, 2, \ldots, i-1\}$. Provided that all past decisions are correct (the ideal case), user interference is eliminated through successive cancellation [1]. The estimate for user i is

$$\hat{b}_i(k) = \text{sgn}\left(s_i(k) - \sum_{j=1}^{i-1} F[0]_{ij}\, a_j \hat{b}_j(k) \right) \qquad (12)$$

In vector notation, (12) becomes

$$\hat{\boldsymbol{b}}(k) = \mathrm{sgn}\left(\boldsymbol{s}(k) - (\boldsymbol{F}[0] - \mathrm{diag}\,\boldsymbol{F}[0])\boldsymbol{A}\hat{\boldsymbol{b}}(k)\right)$$

$$\hat{\boldsymbol{b}}(k) = \mathrm{sgn}\left(\tilde{\tilde{\boldsymbol{y}}}(k) - \begin{pmatrix}(F[0] - \mathrm{diag}\,F[0])\boldsymbol{A}\hat{\boldsymbol{b}}(k) + \\ F[1]\boldsymbol{A}\hat{\boldsymbol{b}}(k-1)\end{pmatrix}\right) \quad (13)$$

The corresponding representation of (13) in the z domain is

$$\hat{\boldsymbol{b}}(z) = \mathrm{sgn}\left(\tilde{\tilde{\boldsymbol{y}}}(z) - \boldsymbol{B}(z)\boldsymbol{A}\hat{\boldsymbol{b}}(z)\right) \quad (14)$$

where the feedback filter $\boldsymbol{B}(z)$ is given by

$$\boldsymbol{B}(z) = \boldsymbol{F}[0] - \mathrm{diag}\,\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1} \quad (15)$$

The performance of the DF detector is dependent upon user ordering. The user ordering and time labeling criteria proposed in [8] results in an error probability that is remarkably close to the ideal DF detector. Despite the improvement of [8], the gap in error probability between the DF detector and the optimal detector is still considerable.

3 PDA Multiuser Detector for Synchronous CDMA

Similar to the system model of (1), the discrete-time equivalent model for the matched-filter outputs at the receiver of a K-user synchronous CDMA channel is given in the time domain by the K-length vector [1]

$$\boldsymbol{y} = \boldsymbol{RAb} + \boldsymbol{n} \quad (16)$$

where $\boldsymbol{R}$ is the symmetric signature correlation matrix with unity diagonal elements and $\boldsymbol{n}$ is a real-valued zero-mean Gaussian random vector with a covariance matrix $\sigma^2\boldsymbol{R}$.

Multiplying by $(\boldsymbol{RA})^{-1}$ from the left on both sides of (16), we obtain

$$\begin{aligned}\boldsymbol{A}^{-1}\tilde{\boldsymbol{y}} &= \boldsymbol{b} + \boldsymbol{A}^{-1}\boldsymbol{x} \\ &= b_i\boldsymbol{e}_i + \textstyle\sum_{j\neq i} b_j\boldsymbol{e}_j + \boldsymbol{A}^{-1}\boldsymbol{x}\end{aligned} \quad (17)$$

where $\tilde{\boldsymbol{y}} = \boldsymbol{R}^{-1}\boldsymbol{y}$; $\boldsymbol{x} = \boldsymbol{R}^{-1}\boldsymbol{n}$; variable $b_i \in \{-1, +1\}$ represents the ith element of vector $\boldsymbol{b}$; and $\boldsymbol{e}_i$ is a column vector whose ith element is 1 and whose other elements are 0.

The PDA method suggests that we treat the decision variables $\boldsymbol{b}$ as binary random variables. For any user i, associate a probability P_{bi} with user signal b_i to express the current estimation of the probability that $b_i = 1$ and $1 - P_{bi}$ is the corresponding estimate for $b_i = -1$. Now, for an arbitrary user signal b_i, treat the other user signals b_j, $(j \neq i)$ as binary random variables and the effective Gaussian noise interfering with user i is

$$\boldsymbol{N}_i = \textstyle\sum_{j\neq i} b_j\boldsymbol{e}_j + \boldsymbol{A}^{-1}\boldsymbol{x} \quad (18)$$

The mean and covariance of (18) is given by

$$\begin{aligned}E[\boldsymbol{N}_i] &= \sum_{j\neq i}\boldsymbol{e}_j(2P_{bj} - 1) \\ \mathrm{cov}[\boldsymbol{N}_i] &= \sum_{j\neq i}[4P_{bj}(1 - P_{bj})\boldsymbol{e}_j\boldsymbol{e}_j^T] \\ &\quad + \sigma^2(\boldsymbol{ARA})^{-1}\end{aligned} \quad (19)$$

By defining the variables

$$\begin{aligned}\boldsymbol{\theta}_i &= E[\boldsymbol{N}_i] - \boldsymbol{A}^{-1}\tilde{\boldsymbol{y}} \\ \boldsymbol{\Omega}_i &= \mathrm{cov}[\boldsymbol{N}_i]\end{aligned} \quad (20)$$

we form the likelihood ratio of the belief that $b_i = 1$.

$$\begin{aligned}\frac{P_{bi}}{1 - P_{bi}} &= \frac{\exp\left(-(\boldsymbol{\theta}_i + \boldsymbol{e}_i)^T\boldsymbol{\Omega}_i^{-1}(\boldsymbol{\theta}_i + \boldsymbol{e}_i)/2\right)}{\exp\left(-(\boldsymbol{\theta}_i - \boldsymbol{e}_i)^T\boldsymbol{\Omega}_i^{-1}(\boldsymbol{\theta}_i - \boldsymbol{e}_i)/2\right)} \\ &= \exp\left(-2\boldsymbol{\theta}_i^T\boldsymbol{\Omega}_i^{-1}\boldsymbol{e}_i\right)\end{aligned} \quad (21)$$

The basic form of the multistage (iterative) PDA detector is as follows:

(1) Sort users according to the user ordering criterion proposed for the decision feedback detector in [5] (specifically Theorem 1of [5]).

(2) $\forall i$, initialize the probabilities as $P_{bi} = 0.5$. Initialize the stage counter $m = 1$.

(3) Initialize the user counter $i = 1$.

(4) Based on the current value of P_{bj} $(j \neq i)$ for user i, update P_{bi} using (6).

(5) If $i < K$, let $i = i + 1$ and goto step 4.

(6) If $\forall i$, P_{bi} has converged, goto step 7. Otherwise let $m = m + 1$ and return to step 3.

(7) $\forall i$, make a decision on user signal i via

$$b_i = \begin{cases}1 & P_{bi} \geq 0.5 \\ -1 & P_{bi} < 0.5\end{cases} \quad (22)$$

Computationally efficient numerical schemes for updating *(4)* are presented in [7].

4 PDA-Kalman Smoother Detector for Asynchronous CDMA

The new detection scheme is based on the asynchronous decorrelated model in (4). The idea behind this scheme lies in the dynamics governing the correlated noise $\boldsymbol{x}(k)$ in (4). Since the covariance of $\boldsymbol{x}(z)$ is $\sigma^2\boldsymbol{R}^{-1}(z)$, then from (6), $\boldsymbol{x}(z)$ can be transformed to white noise via:

$$(\boldsymbol{F}[0]+\boldsymbol{F}[1]z^{-1})\boldsymbol{x}(z)=\boldsymbol{w}(z)$$
$$\boldsymbol{x}(z)=-\boldsymbol{F}[0]^{-1}\boldsymbol{F}[1]z^{-1}\boldsymbol{x}(z)+\boldsymbol{F}[0]^{-1}\boldsymbol{w}(z) \tag{23}$$

where $\boldsymbol{w}(z)$ is a white Gaussian noise vector with zero mean and covariance $\sigma^2\boldsymbol{I}$. The corresponding time-domain representation of (23) is

$$\boldsymbol{x}(k)=-\boldsymbol{F}[0]^{-1}\boldsymbol{F}[1]\boldsymbol{x}(k-1)+\boldsymbol{F}[0]^{-1}\boldsymbol{w}(k) \tag{24}$$

Provided that the conditional mean and covariance of the correlated noise $\boldsymbol{x}(k)$ are known, the PDA method of (9) can be applied to obtain $\boldsymbol{b}(k)$ with only a few modifications: equation (19) is now given by

$$E[\boldsymbol{N}_i(k)]=\sum_{j\neq i}\boldsymbol{e}_j(2P_{bj}(k)-1)+\boldsymbol{A}^{-1}\hat{\boldsymbol{x}}(k\mid k+1)$$
$$\mathrm{cov}[\boldsymbol{N}_i(k)]=\sum_{j\neq i}[4P_{bj}(k)(1-P_{bj}(k))\boldsymbol{e}_j\boldsymbol{e}_j^T] + \boldsymbol{A}^{-1}\boldsymbol{P}(k\mid k+1)\boldsymbol{A}^{-T} \tag{25}$$

where $\hat{\boldsymbol{x}}(k\mid k+1)$ and $\boldsymbol{P}(k\mid k+1)$ denote the one-step smoothed conditional mean and covariance of $\boldsymbol{x}(k)$; and equations (20) and (21) remain the same with the exception that time index k is now added to each variable. Treating (24) as the state equation and (4) as the measurement equation, a Kalman smoother is used in conjunction with the PDA method of [7] to compute $\hat{\boldsymbol{x}}(k\mid k+1)$ and $\boldsymbol{P}(k\mid k+1)$.

Let $\boldsymbol{\mu}(k+1)$ and $\boldsymbol{\Phi}(k+1)$ denote the estimate and conditional covariance of $\boldsymbol{b}(k+1)$ obtained by the PDA method of [7] using the predicted state estimate $\hat{\boldsymbol{x}}(k+1\mid k)$ and the predicted state covariance $\boldsymbol{P}(k+1\mid k)$ in place of $\hat{\boldsymbol{x}}(k\mid k+1)$ and $\boldsymbol{P}(k\mid k+1)$ in (25) without making the hard decision at the end after the user probabilities at time $k+1$ have converged. $\boldsymbol{\mu}(k+1)$ and $\boldsymbol{\Phi}(k+1)$ are computed via

$$\boldsymbol{\mu}(k+1)=\sum_i(2P_{bi}(k+1)-1)\boldsymbol{e}_i$$
$$\boldsymbol{\Phi}(k+1)=\sum_i 4P_{bi}(k+1)(1-P_{bi}(k+1))\boldsymbol{e}_i\boldsymbol{e}_i^T \tag{26}$$

The predicted state estimate and state covariance at $k = 0$ is obtained by recognizing that $\boldsymbol{x}(0)$ is

$$\boldsymbol{x}(0)=\boldsymbol{F}[0]^{-1}\boldsymbol{w}(0) \tag{27}$$

Equation (27) is based on the assumption that because the first information packet arrives at $k = 0$, the output of the decorrelator is zero prior to that time. Hence, $\boldsymbol{x}(-1)$ is zero. From (27), the predicted state estimate and state covariance at $k = 0$ is

$$\hat{\boldsymbol{x}}(0\mid -1)=\boldsymbol{0}$$
$$\boldsymbol{P}(0\mid -1)=\sigma^2\boldsymbol{F}[0]^{-1}\boldsymbol{F}[0]^{-T} \tag{28}$$

The initial state estimate and covariance for the Kalman filter is

$$\hat{\boldsymbol{x}}(0\mid 0)=\tilde{\boldsymbol{y}}(0)-\boldsymbol{A}\boldsymbol{\mu}(0)$$
$$\boldsymbol{P}(0\mid 0)=(\boldsymbol{I}-\boldsymbol{W})\boldsymbol{P}(0\mid -1) \tag{29}$$

where $\boldsymbol{W}=\boldsymbol{P}(0\mid -1)\left[\boldsymbol{P}(0\mid -1)+\boldsymbol{A}\boldsymbol{\Phi}(0)\boldsymbol{A}^T\right]^{-1}$.

The procedure for initializing the detector is illustrated in Figure 2.

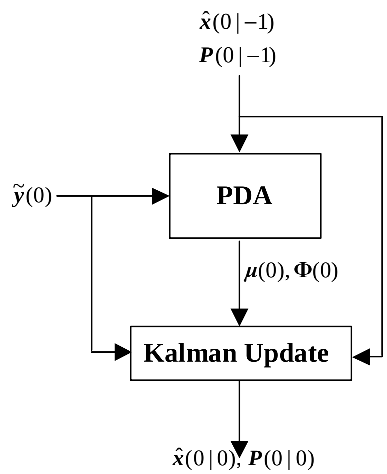

Figure 2: Initialization of PDA-Kalman Smoother Detector

In order to compute the updated state estimate $\hat{\boldsymbol{x}}(k+1\mid k+1)$ and the updated state covariance $\boldsymbol{P}(k+1\mid k+1)$, the mean and covariance of the measurement noise at time $k+1$ is required. In (4), $\boldsymbol{Ab}(k+1)$ is treated as Gaussian noise with mean and covariance

$$E[\boldsymbol{Ab}(k+1)]=\boldsymbol{A}\boldsymbol{\mu}(k+1)$$
$$\mathrm{cov}[\boldsymbol{Ab}(k+1)]=\boldsymbol{A}\boldsymbol{\Phi}(k+1)\boldsymbol{A}^T \tag{30}$$

After the updated state estimate $\hat{\boldsymbol{x}}(k+1\mid k+1)$ and the covariance $\boldsymbol{P}(k+1\mid k+1)$ have been computed, one-step smoothing is performed to obtain $\hat{\boldsymbol{x}}(k\mid k+1)$ and $\boldsymbol{P}(k\mid k+1)$. The one-step smoothing is [9]

$$\hat{\boldsymbol{x}}(k \mid k+1) = \hat{\boldsymbol{x}}(k \mid k) + \boldsymbol{C}\left[\hat{\boldsymbol{x}}(k+1 \mid k+1) - \hat{\boldsymbol{x}}(k+1 \mid k)\right]$$
$$\boldsymbol{P}(k \mid k+1) = \boldsymbol{P}(k \mid k) + \boldsymbol{C}\left[\boldsymbol{P}(k+1 \mid k+1) - \boldsymbol{P}(k+1 \mid k)\right]\boldsymbol{C}^{T} \quad (31)$$

where $\boldsymbol{C} = -\boldsymbol{P}(k \mid k)\boldsymbol{F}[1]^{T}\,\boldsymbol{F}[0]^{-T}\,\boldsymbol{P}(k+1 \mid k)$.

Using the PDA method of [7] (with the modifications), the final decision on $\boldsymbol{b}(k)$ is made. (Note that in obtaining $\boldsymbol{b}(k)$, the user probabilities at time index k are reinitialized to 0.5 before PDA is applied). The procedure to obtain the final decision on $\boldsymbol{b}(k)$ is summarized in the flow diagram of Figure 3. The detector starts at time index $k = 0$. We also remark that any of the algorithms discussed in [10] for synchronous CDMA can be substituted in place of the last PDA stage.

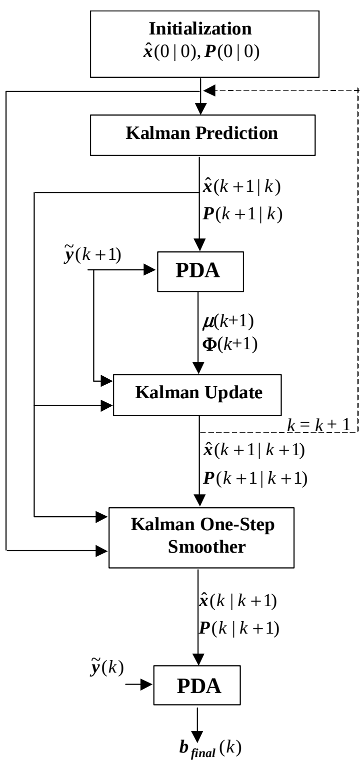

Figure 3: Block Diagram of PDA-Kalman Smoother Detector

5 Simulation Results

In this section, we compare the performance of the Decorrelator, the DF detector, and the PDA-Kalman Smoother detector. The optimal user ordering and time labeling rule proposed in [8] is applied to both the DF and PDA-Kalman Smoother detectors. A performance lower bound is also provided by the ideal optimal detector that assumes no error propagation. The simulation was performed on an overloaded system of 20 users and 15-length randomly generated codes. The time delays of the user signals are random and uniformly distributed within a symbol duration and we use the system model introduced in [11] to generate the signature correlation matrix. The square roots of user signal powers are generated randomly $w_i \sim N(4.5, 4)$ and are limited to the range of [3,6] to avoid domination of results by deep fade errors.

Figure 4 shows the performance comparison of different detectors. The performance of the new detector is significantly better than the DF detector and reasonably close to the performance lower bound. (Note that the lower bound provided by the ideal optimal detector is not necessarily reachable.)

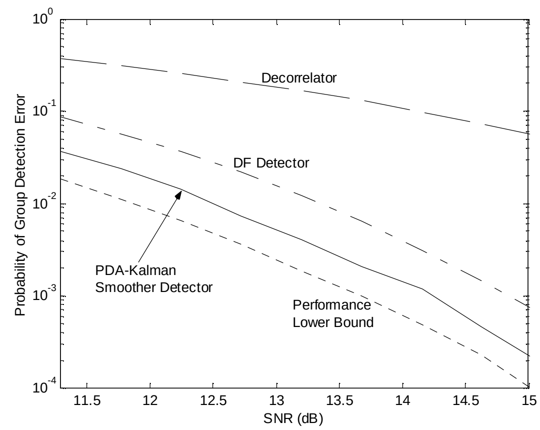

Figure 4: Performance Comparison, 20-users, 100,000 Monte-Carlo Runs

In Figure 5, we examined the effect of smoothing by eliminating the smoothing stage from the detector and using $\hat{\boldsymbol{x}}(k \mid k)$ and $\boldsymbol{P}(k \mid k)$ to obtain the final decision on $\boldsymbol{b}(k)$. At every SNR value, the detector had a higher error probability than in the case when smoothing was employed. In our simulations, we also observed that smoothing beyond one step resulted in no discernible improvement.

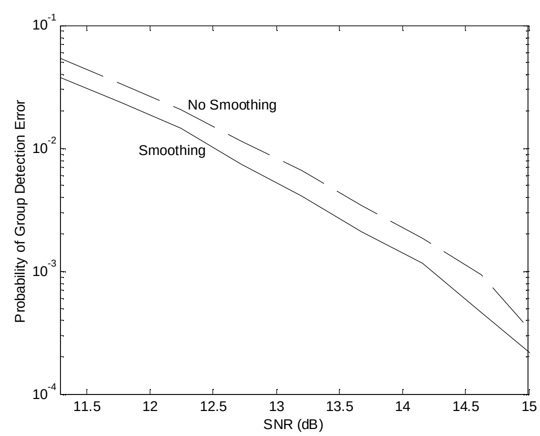

Figure 5: Effect of Smoothing Performance Comparison, 20-users, 100,000 Monte-Carlo Runs

6 Conclusions

Transformation to a decorrelated signal model for asynchronous CDMA results in a vector noise process that evolves according to a linear Gaussian system. This would suggest the use of a Kalman technique to estimate it; however, unfortunately, this noise is added to a very non-Gaussian process by the users' bits.

It turns out that the iterative "Gaussianization" by the PDA detector [7] of these other users makes it a natural fit with the Kalman smoother; the Kalman smoother, in its turn, benefits PDA by reducing the effective noise.

The resulting algorithm amounts to a one-step soft look-ahead to time $k+1$ by the PDA, with an accompanying Kalman smoother to mitigate Gaussian noise at time k. (Deeper look-aheads are possible, but have shown little improvement compared to their added complexity.)

Simulation results show that the performance of the PDA-Kalman Smoother detector, in terms of the probability of group detection error, is significantly better than the decorrelator and the DF detectors.

References

[1] S. Verdu, *Multiuser Detection*, Cambridge University Press, New York, 1998.

[2] U. Madhow, M. Honig, *MMSE Interference Suppression for Direct-Sequence Spread Spectrum CDMA*, IEEE Trans. Comm., Vol. 42, pp. 3178-3188, Dec. 1994.

[3] A. Duel Hallen, *A Family of Decision-Feedback for Asynchronous Code-Division Multiple-Access Channels*, IEEE Trans. on Comm., Vol. 43, pp. 421-433, Feb./Mar./Apr.1995

[4] M. Varanasi, *Multistage Detection for Asynchronous Code-Division Multiple-Access Communications*, IEEE Trans. Comm., vol. 38, pp. 509-519, Apr. 1990.

[5] M. Varanasi, *Decision Feedback Multiuser Detection: A Systematic Approach*, IEEE Trans. on Info. Theory, Vol. 45, pp. 219-240, Jan. 1999.

[6] Y. Bar-Shalom, E. Tse, *Tracking in A Clusttered Environment with Probabilistic Data Association*, Automatica, Vol. 11, pp. 451-460, Sep. 1975.

[7] J. Luo, K. Pattipati, P. Willett, F. Hasegawa, *Near-Optimal Multiuser Detection in Synchronous CDMA using Probabilistic Data Association*, IEEE Comm. Lett., Vol. 5, pp. 361-363, Sept. 2001.

[8] J. Luo, K. Pattipati, P. Willett, F. Hasegawa, *Optimal User Ordering and Time Labeling for Decision Feedback Detection in Asynchronous CDMA*, IEEE ICASSP 2002, Orlando, FL, May 2002.

[9] Y. Bar-Shalom, X.-Rong Li, T. Kirubarajan, *Estimation with Applications to Tracking and Navigation*, John Wiley & Sons, Inc. New York, 2001.

[10] F. Hasegawa, J. Luo, K. Pattipati, P. Willet, *Speed and Accuracy Comparison of Techniques to Solve a Binary Quadratic Programming Problem with Application to Synchronous CDMA*, IEEE CDC, Orlando, FL, Dec. 2001

[11] M. Pursley, *Performance Evaluation for Phase-Coded Spread-Spectrum Multiple-Access Communication-Part I: System Analysis*, IEEE Trans. on Comm., Vol. COM-25, pp. 795-799, Aug. 1977.