Near-Optimal Multiuser Detection in Synchronous CDMA Using Probabilistic Data Association

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***Abstract*—A Probabilistic Data Association (PDA) method is proposed in this letter for multiuser detection over synchronous code-division multiple-access (CDMA) communication channels. PDA models the undecided user signals as binary random variables. By approximating the inter-user interference (IUI) as Gaussian noise with an appropriately elevated covariance matrix, the probability associated with each user signal is iteratively updated. Computer simulations show that the system usually converges within three to four iterations, and the resulting probability of error is very close to that of the optimal maximum-likelihood (ML) detector. Further modifications are also presented to significantly reduce the computational cost.**

***Index Terms*—Code division multiple access, multiuser detection, probabilistic data association.**

I. INTRODUCTION

THE MULTIUSER detection (MUD) problem in synchronous code-division multiple-access (CDMA) communication systems has been widely studied in the past decade. Since the computation of an optimal maximum-likelihood (ML) detector is exponential in the number of users, suboptimal solutions are proposed to provide reliable decisions with relatively low computational cost. Among them are the conventional decorrelator [1], the decision feedback detector (DFD) [3], [6], the multistage detector [2] and the group detector [5], [8]. Although DFD is one of the most efficient methods, in most cases the gap between the probability of error of the DFD and that of a ML detector is still large. The DFD can be considered a special case of a group detector with unity maximum group size, and the performance gap becomes narrower as this is increased. However, despite the improvements described in [8], finding the optimal group assignment and user ordering remains expensive.

The probabilistic data association (PDA) filter [4] is a highly successful approach to target tracking in the case that measurements are unlabeled and may be spurious. Its key feature is a repeated conversion of a multimodal Gaussian mixture probability structure to a single Gaussian with matched mean and covariance. This is a bold and to some extent unjustifiable step, but it is difficult to argue with good performance and low complexity. Now, in the CDMA case the true probability function is also a Gaussian mixture, and complexity is also the issue. We thus propose to apply the Gaussian "forcing" idea; whereas in the tracking application this forcing occurs once per scan and there is no revisit, in CDMA it occurs for each user, and there is iteration. Instead of fixing the binary signal variables at ± 1, the PDA employs a soft IUI cancellation by increasing the covariance of the effective noise based on the uncertainty in the other user signals. When the binary variables converge to the true value, the covariance approaches to that of the original noise. This, we believe, helps PDA avoid local minima. Simulation results show that the PDA detector gives a probability of error very close to and often indistinguishable from that of the optimal ML detector. Simulations also show that the worst case computational cost of the PDA method is $O(K^3)$, where K is the number of users.

Manuscript received March 30, 2001. The associate editor coordinating the review of this letter and approving it for publication was Prof. S. W. Kim. This work was supported by the Office of Naval Research under Contract N00014-98-1-0465, N00014-00-1-0101, and by NUWC under Contract N66604-01-1-1125.

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Publisher Item Identifier S 1089-7798(01)06463-8.

II. THE MULTISTAGE PDA DETECTOR

A. Problem Formulation

A discrete-time model for the matched-filter outputs at the receiver of a CDMA channel is given by the K-length vector [1]

$$y = RWb + n \tag{1}$$

where $b \in \{-1, +1\}^K$ denotes the K-length vector of bits transmitted by the K active users; R is the symmetric normalized signature correlation matrix with unit diagonal elements; and W is a diagonal matrix whose diagonal elements are the signal amplitudes of the corresponding users. Here n is a colored Gaussian noise vector with zero mean and covariance matrix $E[nn^T] = \sigma^2 R$, where σ^2 is the power of the white noise before the matched filter.

When all the user signals are equally probable, the optimal solution of (1) is the output of a ML detector [1]

$$\phi_{\mathrm{ML}}\colon \hat{b} = \arg \min_{b \in \{-1,+1\}^K} \left(b^T WRWb - 2y^T Wb \right). \tag{2}$$

It is known that obtaining the ML solution is generally NP-hard [1], unless the signature correlation matrix has a special structure.

Multiplying by $W^{-1}R^{-1}$ on both sides of (1) from the left, the system model can be reformulated as

$$\tilde{y} = b + \tilde{n} = b_i e_i + \sum_{j \neq i} b_j e_j + \tilde{n} \tag{3}$$

where $\tilde{y} = W^{-1}R^{-1}y$ and $\tilde{n} = W^{-1}R^{-1}n$. The variable b_i represents the ith element of vector b; e_i is a column vector whose ith component is 1 and whose other components are 0. We call (3) "the decorrelated model," since $\tilde{y}$ is in fact a normalized version of the decorrelator output before the hard decision.

B. The Basic Algorithm

In the CDMA system model (3), we treat the decision variables b as binary random variables. For any user i, we associate a probability $P_b(i)$ with user signal b_i to express the current belief on its value, i.e., $P_b(i)$ is the current estimate of the probability that $b_i = 1$, and $1 - P_b(i)$ is the corresponding estimates for $b_i = -1$. Now, for an arbitrary user signal b_i, treat the other user signals b_j $(j \neq i)$ as binary random variables and treat $\sum_{j\neq i} b_j e_j + \tilde{n}$ as the effective noise. Consequently, $p(b_i = 1|\tilde{y}, \{P_b(j)\}_{j\neq i})$ and $p(b_i = -1|\tilde{y}, \{P_b(j)\}_{j\neq i})$ can be obtained from (3); they serve as updated information on user signal b_i. Based on the decorrelated model, the basic form of the proposed multistage PDA detector is as follows.

1) Sort users according to the user ordering criterion proposed for the decision feedback detector in [6] (specifically [6, theorem 1]).
2) $\forall i$, initialize the probabilities as $P_b(i) = 0.5$. Initialize the stage counter $k = 1$.
3) Initialize the user counter $i = 1$.
4) Based on the current value of $P_b(j)$ $(j \neq i)$ for user i, update $P_b(i)$ by
$$P_b(i) = P\{b_i = 1|\tilde{y}, \{P_b(j)\}_{j\neq i}\}. \tag{4}$$
5) If $i < K$, let $i = i + 1$ and goto step 4).
6) If $\forall i$, $P_b(i)$ has converged, goto step 7). Otherwise, let $k = k + 1$ and return to step 3).
7) $\forall i$, make a decision on user signal i via
$$b_i = \begin{cases} 1, & P_b(i) \geq 0.5 \\ -1, & P_b(i) < 0.5. \end{cases} \tag{5}$$

In the above procedure, the computational cost of obtaining $P_b(i) = P\{b_i = 1|\tilde{y}, \{P_b(j)\}_{j\neq i}\}$ is evidently exponential in the number of users. Define
$$N_i = \sum_{j\neq i} b_j e_j + \tilde{n} \tag{6}$$
from (3). Here is the key: to avoid the computational cost of $P_b(i)$, the PDA idea from [4] recommends that N_i be *approximated as a Gaussian noise* with matched mean and covariance; that is, we use
$$E[N_i] = \sum_{j\neq i} e_j(2P_b(j) - 1)$$
$$\mathrm{Cov}[N_i] = \sum_{j\neq i} \left[4P_b(j)(1 - P_b(j))e_j e_j^T\right] + \sigma^2\left(W^T RW\right)^{-1}. \tag{7}$$

Now, defining
$$\theta_i = \sum_{j\neq i} e_j(2P_b(j) - 1) - \tilde{y}$$
$$\Omega_i = \sum_{j\neq i} \left[4P_b(j)(1 - P_b(j))e_j e_j^T\right] + \sigma^2\left(W^T RW\right)^{-1} \tag{8}$$

we obtain
$$\frac{P_b(i)}{1 - P_b(i)} = \exp\{-2\theta_i^T \Omega_i^{-1} e_i\}. \tag{9}$$

C. Refinements

1) Speed-Up—Matrix Arithmetic: Although the computation in step 3) is no longer exponential, direct calculation of Ω_i^{-1} for each user is still expensive. Further simplifications can be made by defining auxiliary variables
$$\theta = \sum_j e_j(2P_b(j) - 1) - \tilde{y} = \theta_i + e_i(2P_b(i) - 1)$$
$$\Omega = \sum_j \left[4P_b(j)(1 - P_b(j))e_j e_j^T\right] + \sigma^2\left(W^T RW\right)^{-1}$$
$$= \Omega_i + 4P_b(i)(1 - P_b(i))e_i e_i^T. \tag{10}$$

The Sherman–Morrison–Woodbury formula [9] yields
$$\theta_i = \theta - e_i(2P_b(i) - 1)$$
$$\Omega_i^{-1} = \Omega^{-1} + \frac{4P_b(i)(1 - P_b(i))\Omega^{-1} e_i e_i^T \Omega^{-1}}{1 - 4P_b(i)(1 - P_b(i))e_i^T \Omega^{-1} e_i} \tag{11}$$
$$\theta = \theta_i + e_i(2P_b(i) - 1)$$
$$\Omega^{-1} = \Omega_i^{-1} - \frac{4P_b(i)(1 - P_b(i))\Omega_i^{-1} e_i e_i^T \Omega_i^{-1}}{1 + 4P_b(i)(1 - P_b(i))e_i^T \Omega_i^{-1} e_i}. \tag{12}$$

By keeping the updated versions of θ and Ω^{-1}, we can divide step 3) into three sub-steps. In sub-step 1, we calculate θ_i and Ω_i^{-1} using (11). Sub-step 2 obtains the updated $P_b(i)$ using (9). In sub-step 3, we use the new $P_b(i)$ and update θ and Ω using (12). The overall computation of step 3) is then reduced to $O(K^2)$, and the overall complexity of each stage in the PDA detector is now $O(K^3)$.

2) Speed-Up—Successive Cancellation: Since the number of stages in the PDA detector is not fixed, the overall complexity can be high if one or two users show a slow convergence. In fact, computer simulation shows that, in most cases, more than 2/3 of users will converge during the first stage. Thus to simplify further we introduce successive cancellation among the stages.

After the kth stage, define G to be the group of users that satisfy
$$P_b(i) \in [0, \epsilon] \cup [1 - \epsilon, 1] \qquad \forall i \in G \tag{13}$$
where ϵ is a small positive number. Denote $\overline{G}$ to be the complement of G. Make decisions that
$$b_i = \mathrm{sign}(P_b(i) - 0.5) \qquad \forall i \in G. \tag{14}$$

Buy canceling the IUI, the decorrelated system model for the users in $\overline{G}$ can be formulated as
$$W_{\overline{GG}}^{-1} R_{\overline{GG}}^{-1} y_{\overline{G}} - W_{\overline{GG}}^{-1} R_{\overline{GG}}^{-1} R_{\overline{G}G} W_{GG} b_G = b_{\overline{G}} + \tilde{n}_{\overline{G}}. \tag{15}$$

Here $R_{\overline{GG}}$ denotes the sub-block matrix of R that only contains the columns and rows corresponding to users in $\overline{G}$. $\tilde{n}_{\overline{G}}$ is the colored Gaussian noise of the sub-system with zero mean and covariance matrix $\sigma^2(W_{\overline{GG}} R_{\overline{GG}} W_{\overline{GG}})^{-1}$. Consequently, in the $(k+1)$st stage, we apply the PDA detection procedure only on the sub-system model.

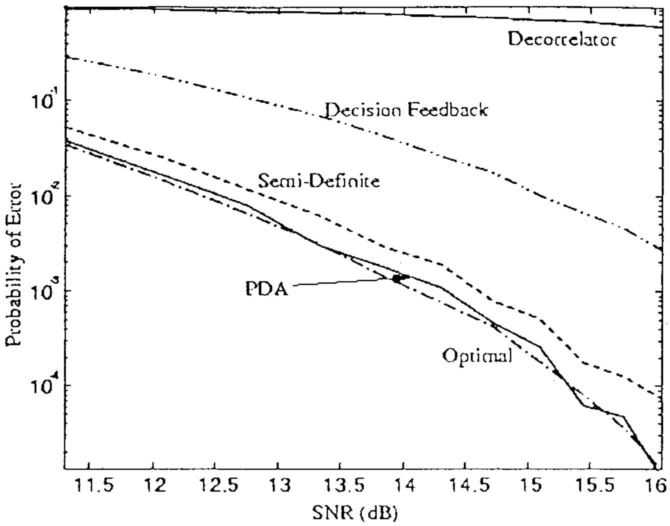

Fig. 1. 29-users, length-31 Gold codes as signature sequences, 100 000 Monte Carlo runs.

3) Performance: Bit-Flipping: It has been noted that when optimal and PDA solutions to (2) differ, they usually disagree in one element only. Thus, as an inexpensive way to improve the PDA detector, we add a "bit-flip" stage after PDA has converged. This is actually a one-step coordinate descent [7].

III. Computer Simulation Results

In this section, we use several computer simulation examples to show the performance and the computational cost of the PDA detector. Besides the proposed PDA detector, the decorrelating detector [1], the DFD [6], the semidefinite relaxation method [10], and the optimal ML detector [8] are compared in the examples. In the successive cancellation part of the PDA detector, we set $\epsilon = 10^{-2}/4\text{SNR}$ where SNR is the signal to noise ratio. For the semidefinite relaxation algorithm, the number of randomizations is set to 20.

In the first 29-user example, we use length-31 Gold codes as the signature sequences. The user signal amplitudes are randomly and independently generated by $W_{ii} \sim N(4.5, 4), \forall i$, and are limited within the range of $[2, 7]$ [$N(.)$ represents the Normal distribution]. Fig. 1 shows the performance comparison based on 100 000 Monte Carlo runs with importance sampling.

In the second example, we fix the SNR to be 12 dB. The signature sequences are randomly generated and the ratio between the spreading factor and the number of users is fixed at 1.2. Let the number of users vary from 3 to 60. Fig. 2 shows the worst case computational complexity measured in terms of the number of multiplications plus number of additions of the PDA detector and of the Semidefinite Relaxation method. It is known that the computational cost of the Semidefinite Relaxation method is $O(K^{3.5})$ [10]. Therefore, we claim that the computational cost for the PDA detector is significantly less than $O(K^{3.5})$. Simulation results show that the computational cost is in fact $O(K^3)$.

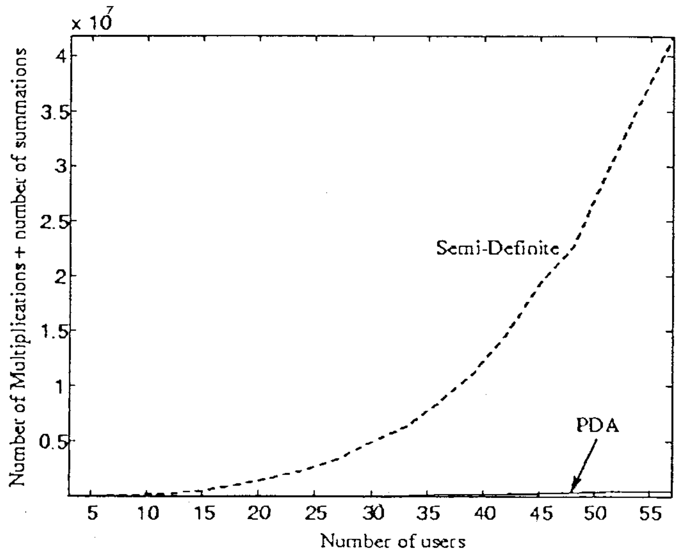

Fig. 2. Comparison on the worst case computational costs, random signature sequences, spreading factor = 1.2K, SNR = 12 dB.

IV. Conclusions

A new algorithm based on the idea of PDA is proposed for the multiuser detection in synchronous CDMA communications. Simulation results show that the PDA detector provides near-optimal performance, with the overall computational cost $O(K^3)$, where K is the number of users. We will extend the PDA idea to multiuser detection over fading channels, as well as multiuser detection for asynchronous CDMA in our future research.

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