Optimal User Ordering and Time Labeling for Ideal Decision Feedback Detection in Asynchronous CDMA

Jie Luo, *Member, IEEE*, Krishna R. Pattipati, *Fellow, IEEE*, Peter K. Willett, *Fellow, IEEE*, and Fumihiro Hasegawa, *Member, IEEE*

***Abstract*—A strategy of user ordering and time labeling for a decision feedback (DF) detector in asynchronous code-division multiple-access communications is proposed and is proved to be optimal for the ideal DF detector. The proposed algorithm requires $O(K^4)$ offline operations, where K is the number of users. Although error propagation complicates the analysis of the actual DF detector, computer simulations show that, with the proposed user ordering and time labeling, the performance of an actual DF detector overlays the theoretical bound in most cases.**

***Index Terms*—Asynchronous, code-division multiple access (CDMA), multiuser detection, optimal detection sequence.**

I. Introduction

SINCE OPTIMAL multiuser detection of code-division multiple-access (CDMA) communications is generally NP-hard [1], suboptimal algorithms that provide reliable performances with polynomial complexity have been widely studied for more than 15 years. Among them, the linear detectors and decision-driven detectors are particularly popular [1], and the decision feedback (DF) detector [2] is one of the most efficient of the latter category. Compared with the conventional matched filter and the decorrelator, the DF detector provides significantly better accuracy with only $O(K^2)$ complexity [3].

Due to the successive cancellation structure of a DF detector, user ordering plays an important role [1]. In synchronous CDMA, the optimal ordering that maximizes the symmetric energy (SE), which characterizes the asymptotic group detection error, of the decorrelating DF detector was found in [3] and requires an $O(K^3)$ offline computation. However, when users are asynchronous, both user ordering and *time labeling* will affect performance [2].

In asynchronous CDMA, it is commonly accepted that users should be detected either in decreasing order of their signal powers or in chronological order of their arrival times. In synchronous CDMA [3], there are $K!$ different user orders, and ordering users according to decreasing signal power is not necessarily optimal when user correlations are considered; this remains so in the asynchronous case. Furthermore, although it may at first appear that ordering in terms of arrival times is at least a fixed strategy, one can actually consider any user as the first-arriving user by fixing a fictitious initial bit to be zero.

In this letter, we study user ordering and time labeling for the DF detector. Although our final goal is to minimize its probability of error, even the asymptotic performance of a DF detector is, due to error propagation, hard to estimate. In [2], assuming no error propagation, the theoretical asymptotic performance of the ideal DF detector is given. Building on this, we find a user ordering and time labeling that maximizes the SE of the ideal DF detector. We further show in computer simulations that, with the proposed user ordering and time labeling, the asymptotic performance of an actual DF detector is indistinguishable from the theoretical performance bound. The overall computation for the optimal user ordering and time labeling is shown to be $O(K^4)$, and is, of course, considered as offline computational load, since it is required only once for a given user configuration.

II. Asynchronous CDMA and the DF Detector

The asynchronous CDMA system can be described in the z domain by [1]

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{W}\boldsymbol{b}(z) + \boldsymbol{v}(z) \tag{1}$$

where $\boldsymbol{y}(z)$ is a $K \times 1$ column vector of the received signal (which is the output of K matched filters at the receiver); $\boldsymbol{R}(z)$ is the $K \times K$ signature correlation matrix; $\boldsymbol{W}$ is a diagonal matrix whose ith diagonal component w_i is the square root of the signal power of the ith user; $\boldsymbol{b}(z)$ is the binary user signal vector;[1] and $\boldsymbol{v}(z)$ is a colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}(z)$, where σ^2 is the power of the white noise before the matched filter. Assume that the relative delays between user signals with the same time index are within $(-T, T)$, i.e., signals with the same time index overlap with each other. The correlation matrix $\boldsymbol{R}(z)$ can be represented and can be factorized as [1], [2]

$$\begin{aligned}\boldsymbol{R}(z) &= \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1} \\ &= \left(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z\right)\left(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}\right) \\ &= \boldsymbol{F}\left(z^{-1}\right)^T \boldsymbol{F}(z)\end{aligned} \tag{2}$$

where $\boldsymbol{F}(z)$ is a causal and stable minimum-phase matrix filter. In (2), $\boldsymbol{R}[0]$ is a symmetric matrix with unity diagonal components and whose off-diagonal components represent the corre-

[1] In the time-domain representation, we denote $b(n)$ to be the binary signal vector for the nth time frame and denote $b_i(n)$ to be the binary signal of user i in time frame n. The assignment of the nth or $(n+1)$st bit of each user to $b(n)$ is the time labeling issue.

Paper approved by X. Wang, the Editor for Modulation Detection and Equalization of the IEEE Communications Society. Manuscript received January 16, 2002; revised June 12, 2002. This work was supported in part by the University of Connecticut Research Foundation #444636, in part by the Office of Naval Research under Contract N00014-98-1-0465, 00014-00-1-0101, and in part by NUWC under Contract N66604-01-1-1125. This paper was presented in part at the IEEE International Conference on Acoustics, Speech, and Signal Processing, Orlando, FL, May 2002.

J. Luo is with the Institute for Systems Research, University of Maryland, College Park, MD 20742 USA.

K. R. Pattipati and P. K. Willett are with with the Electrical and Computer Engineering Department, University of Connecticut, Storrs, CT 06269 USA (e-mail: krishna@engr.uconn.edu).

F. Hasegawa is with the Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G3, Canada.

Digital Object Identifier 10.1109/TCOMM.2003.819239

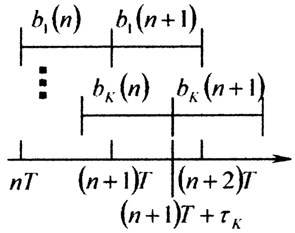

Fig. 1. Bit epochs for asynchronous CDMA (T is the symbol duration, τ_i is the time delay for user i).

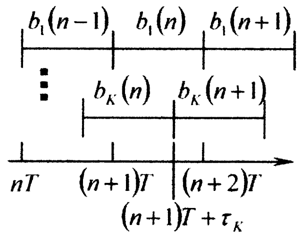

Fig. 2. Bit epochs for an equivalent system by changing the time labeling of user 1 in Fig. 1 ($b_1(n-1)$ in this figure is physically $b_1(n)$ in Fig. 1).

lation between user signatures at the same time index; and $\boldsymbol{R}[1]$ is a singular matrix whose components represent the signature correlations relating to successive time frames. $\boldsymbol{F}[0]$ is a lower triangular matrix. $\boldsymbol{F}[1]$ is a singular matrix, and it becomes strict upper triangular when users are ordered chronologically [2].

The asynchronous CDMA system is illustrated in Fig. 1 and a detailed description of the system model can be found in [1].

It is easy to see that the time labeling of the system is not unique. In Fig. 1, suppose we change the time label for user 1: an equivalent bit epoch can be obtained as in Fig. 2.

Applying the anticausal feed-forward filter $\left(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z\right)^{-1}$ to both sides of (1), we obtain the white noise model in both the z domain and the time domain, respectively [2]

$$\begin{aligned}\tilde{\boldsymbol{y}}(z) &= \left(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}\right)\boldsymbol{W}\boldsymbol{b}(z) + \tilde{\boldsymbol{v}}(z)\\ \tilde{\boldsymbol{y}}(n) &= \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(n) + \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(n-1) + \tilde{\boldsymbol{v}}(n)\end{aligned} \tag{3}$$

where $\tilde{\boldsymbol{y}}(z) = \left(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z\right)^{-1}\boldsymbol{y}(z)$ and $\tilde{\boldsymbol{v}}$ is a zero-mean white Gaussian noise vector.

The decorrelating DF detector for asynchronous CDMA [2] makes decisions sequentially and utilizes past decisions in addition to channel outputs, where $\hat{b}_i(n)$ is given by

$$\operatorname{sign}\left(\tilde{y}_i(n) - \sum_{j=1}^{i-1} F[0]_{ij}\hat{b}_j(n) - \sum_{j=1}^{K} F[1]_{ij}\hat{b}_j(n-1)\right). \tag{4}$$

The DF detector assumes that the decisions for user bit $b_i(n-1)$ are made prior to the decision of $b_j(n)$, $\forall\, i,j$. Hence, the performance is affected by both user ordering and time labeling.

In (4), assuming that the past decisions are correct, the asymptotic effective energy (AEE), which characterizes the asymptotic probability of detection error, of user i in time index n can be represented by $E_i = (F[0]_{ii}w_i)^2$. The SE, which characterizes the asymptotic probability of group detection error at time index n, can be found via [3]

$$E = \min_{i=1,2,\ldots,K} (F[0]_{ii}w_i)^2. \tag{5}$$

III. Optimal User Ordering and Time Labeling

Rewrite the system model (3) as

$$\begin{aligned}&\boldsymbol{F}[0]^T\left[\tilde{\boldsymbol{y}}(n) - \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(n-1)\right]\\ &\qquad = \boldsymbol{F}[0]^T\boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(n) + \boldsymbol{F}[0]^T\tilde{\boldsymbol{v}}(n).\end{aligned} \tag{6}$$

Given the idealized assumption that the past decisions are correct, i.e., $\boldsymbol{b}(n-1)$ is known when detecting $\boldsymbol{b}(n)$, the above system model is equivalent to a synchronous CDMA model, in which $\boldsymbol{R}_{\mathrm{AEC}} = \boldsymbol{F}[0]^T\boldsymbol{F}[0]$, termed the asymptotic effective correlation (AEC), is the equivalent signature correlation matrix. The DF detector with the idealized assumption is termed "ideal" DF detector [2] since it does not suffer from the problem of error propagation. We begin with the following.

Proposition 1: Given the time labeling, suppose $\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P}$ is the signature correlation matrix of the same system, but with a different user order ($\boldsymbol{P}$ is an arbitrary permutation matrix). The AEC matrix of the permuted system satisfies $\tilde{\boldsymbol{R}}_{\mathrm{AEC}} = \boldsymbol{P}^T\boldsymbol{R}_{\mathrm{AEC}}\boldsymbol{P}$, i.e., the equivalent synchronous system is invariant to user permutations.

Proof: Since factorization (2) is unique, given the requirement that $\boldsymbol{F}(z)$ be causal, stable, and minimum phase, instead of proving the proposition directly, we show that if $\tilde{\boldsymbol{F}}[0]$ and $\tilde{\boldsymbol{F}}[1]$ satisfy

$$\begin{aligned}\tilde{\boldsymbol{F}}[0]^T\tilde{\boldsymbol{F}}[0] &= \boldsymbol{P}^T\boldsymbol{F}[0]^T\boldsymbol{F}[0]\boldsymbol{P}\\ \tilde{\boldsymbol{R}}[1] &= \tilde{\boldsymbol{F}}[0]^T\tilde{\boldsymbol{F}}[1]\end{aligned} \tag{7}$$

then $\tilde{\boldsymbol{R}}(z) = \tilde{\boldsymbol{F}}\left(z^{-1}\right)^T\tilde{\boldsymbol{F}}(z)$ is the factorization of $\tilde{\boldsymbol{R}}(z)$.

In fact, from $\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P}$, we have $\tilde{\boldsymbol{R}}[0] = \boldsymbol{P}^T\boldsymbol{R}[0]\boldsymbol{P}$ and $\tilde{\boldsymbol{R}}[1] = \boldsymbol{P}^T\boldsymbol{R}[1]\boldsymbol{P}$. Since $\boldsymbol{R}(z) = \boldsymbol{F}\left(z^{-1}\right)^T\boldsymbol{F}(z)$

$$\begin{aligned}\boldsymbol{R}[0] &= \boldsymbol{F}[0]^T\boldsymbol{F}[0] + \boldsymbol{F}[1]^T\boldsymbol{F}[1]\\ \boldsymbol{R}[1] &= \boldsymbol{F}[0]^T\boldsymbol{F}[1].\end{aligned} \tag{8}$$

According to the assumption, we have

$$\tilde{\boldsymbol{F}}[1] = \tilde{\boldsymbol{F}}[0]^{-T}\tilde{\boldsymbol{R}}[1]. \tag{9}$$

Therefore

$$\begin{aligned}\tilde{\boldsymbol{F}}[1]^T\tilde{\boldsymbol{F}}[1] &= \tilde{\boldsymbol{R}}[1]^T\tilde{\boldsymbol{F}}[0]^{-1}\tilde{\boldsymbol{F}}[0]^{-T}\tilde{\boldsymbol{R}}[1]\\ &= \tilde{\boldsymbol{R}}[1]^T\boldsymbol{P}^T\boldsymbol{F}[0]^{-1}\boldsymbol{F}[0]^{-T}\boldsymbol{P}\tilde{\boldsymbol{R}}[1]\\ &= \boldsymbol{P}^T\boldsymbol{F}[1]^T\boldsymbol{F}[1]\boldsymbol{P}\end{aligned} \tag{10}$$

which gives

$$\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P} = \tilde{\boldsymbol{F}}\left(z^{-1}\right)^T\tilde{\boldsymbol{F}}(z). \tag{11}$$

Furthermore, it is easy to verify that, if $\boldsymbol{F}(z)$ is causal, stable, and minimum phase, $\tilde{\boldsymbol{F}}(z)$ is also causal, stable, and minimum phase. □

Since the equivalent synchronous system is invariant to user permutations, the SE of the ideal DF detector can be maximized by applying the user ordering technique in [3, Th. 1] to $\boldsymbol{W}\boldsymbol{R}_{\mathrm{AEC}}\boldsymbol{W}$. That is, select the first user of the new order (denote this user's index as i_1) as one that has the highest AEE among all users if each one of them were to be detected by a decorrelator. For $k = 2,\ldots,K$ select the kth user of the new order (denote this user's index as i_k) as the user that has the highest AEE among the remaining $K-k+1$ users, when each of them is detected by a decorrelator for the user-expurgated channel consisting of just those remaining users. Then the optimal user order that maximizes SE is $\{i_1, i_2, \ldots, i_K\}$. A proof of the optimality can be easily derived from [6, Prop. 1].

Evidently, we can perform user ordering for all possible time labelings and choose the one that maximizes the SE. However, obtaining $\boldsymbol{F}[0]$ and $\boldsymbol{F}[1]$ from $\boldsymbol{R}[0]$ and $\boldsymbol{R}[1]$ requires an iterative procedure [4], which is computationally expensive. Since $\boldsymbol{R}[0]$ and $\boldsymbol{R}[1]$ for different time labelings are different, we certainly do not want to apply the iterative procedure to all time labelings to find the best.

Fortunately, this is unnecessary. Given a time labeling, we first order users according to their times of arrival (see Fig. 1). Apparently, the chronological user ordering vector uniquely represents the corresponding time labeling. For example, the time labeling in Fig. 1 can be represented by a vector $T_1 = [1, \ldots, K]$, and the time labeling in Fig. 2 is represented by a vector $T_2 = [2, \ldots, K, 1]$. To change time labeling from T_1 to T_2, we only need to change the time index definition of user 1, i.e., $\forall\, n$, redefine $b_1(n+1)$ in time labeling T_1 as $b_1(n)$ in time labeling T_2.[2] Consequently, we denote the conversion from time labeling T_1 to T_2 by

$$T_1 \xrightarrow{\{\text{user } 1\}} T_2. \tag{12}$$

Note that, for a valid time labeling, $b_i(n)$ must overlap with $b_j(n)\ \forall\, i, j$ and for all n. We have the following.

Proposition 2: Suppose there is a time labeling T_G, where $T_1 \xrightarrow{G} T_G$ converts T_1 to T_G, i.e., $\forall$ user $i \in G$, redefine $b_i(k+1)$ in T_1 as $b_i(k)$ in time label T_G. Then G can be separated into two sets, G_1, G_2, where $G_1 = \{\text{user } 1\}$ and G_2 is the rest of the users in G, i.e., $G_2 = G \setminus G_1$. The operation $T_1 \xrightarrow{G} T_G$ can also be written as

$$T_1 \xrightarrow{G_1} T_2 \text{ followed by } T_2 \xrightarrow{G_2} T_G. \tag{13}$$

Proof: Suppose user $i \in G$ and user $1 \notin G$. Since the users are ordered chronologically in T_1, the delay of user signal i, termed as τ_i, is greater than the delay of user signal 1, termed as τ_1. In other words, the relative delay between user signal i and user signal 1 is $\tau_i - \tau_1 \geq 0$. Now, $T_1 \xrightarrow{G} T_G$ requires a redefinition of $b_i(k+1)$ in T_1 as $b_i(k)$ in time label T_G. The relative delay between user signal i and user signal 1 in T_G becomes $\tau_i + T - \tau_1 > T$. This means that $b_i(k)$ and $b_1(k)$ in time label T_G do not overlap, which is not valid according to the assumption of the system model. Therefore, user $1 \in G$ must be true.

Since $T_1 \xrightarrow{\{\text{user } 1\}} T_2$, T_2, and T_G are valid time labelings, we can represent $T_1 \xrightarrow{G} T_G$ by (13). □

From *Proposition 2*, it is easy to see that the only valid time labelings are circular permutations, that is, $T_1 = [1, \ldots, K]$, $T_2 = [2, \ldots, K, 1]$, ..., $T_K = [K, 1, \ldots, K-1]$. Even better, we have the following.

[2]An equivalent conversion can be expressed as $\forall\ j \neq 1$, $\forall\ n$, redefining $b_j(n-1)$ in time labeling T_1 as $b_j(n)$ in time labeling T_2. However, without loss of generality, we only consider a single-direction conversion in this paper, i.e., $\forall\ (i, n)$, the redefinition of $b_i(n-1)$ in T_1 as $b_i(n)$ in T_2 is prohibited.

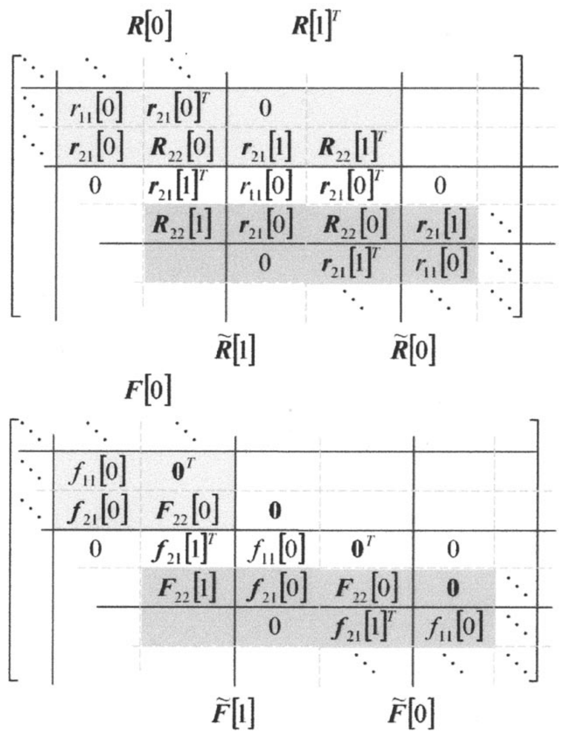

Fig. 3. Relation between $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$, $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ matrices of different time labelings.

Proposition 3: Partition $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$, $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ corresponding to time labeling T_1 on their second diagonal components (from upper-left corner) as

$$\boldsymbol{R}[0] = \begin{bmatrix} r_{11}[0] & \boldsymbol{r}_{21}[0]^T \\ r_{21}[0] & \boldsymbol{R}_{22}[0] \end{bmatrix}, \quad \boldsymbol{R}[1] = \begin{bmatrix} 0 & \boldsymbol{r}_{21}[1]^T \\ \mathbf{0} & \boldsymbol{R}_{22}[1] \end{bmatrix}$$
$$\boldsymbol{F}[0] = \begin{bmatrix} f_{11}[0] & \mathbf{0}^T \\ \boldsymbol{f}_{21}[0] & \boldsymbol{F}_{22}[0] \end{bmatrix}, \quad \boldsymbol{F}[1] = \begin{bmatrix} 0 & \boldsymbol{f}_{21}[1]^T \\ \mathbf{0} & \boldsymbol{F}_{22}[1] \end{bmatrix}.$$

Then, the matrices corresponding to time labeling T_2 become, respectively, $\tilde{\boldsymbol{R}}[0]$, $\tilde{\boldsymbol{R}}[1]$, $\tilde{\boldsymbol{F}}[0]$, and $\tilde{\boldsymbol{F}}[1]$

$$\tilde{\boldsymbol{R}}[0] = \begin{bmatrix} \boldsymbol{R}_{22}[0] & \boldsymbol{r}_{21}[1] \\ \boldsymbol{r}_{21}[1]^T & r_{11}[0] \end{bmatrix}$$
$$\tilde{\boldsymbol{R}}[1] = \begin{bmatrix} \boldsymbol{R}_{22}[1] & \boldsymbol{r}_{21}[0] \\ \mathbf{0}^T & 0 \end{bmatrix}$$
$$\tilde{\boldsymbol{F}}[0] = \begin{bmatrix} \boldsymbol{F}_{22}[0] & \mathbf{0} \\ \boldsymbol{f}_{21}[1]^T & f_{11}[0] \end{bmatrix}$$
$$\tilde{\boldsymbol{F}}[1] = \begin{bmatrix} \boldsymbol{F}_{22}[1] & \boldsymbol{f}_{21}[0] \\ \mathbf{0}^T & 0 \end{bmatrix}. \tag{14}$$

Proof: Suppose we view a K-user M-frame asynchronous CDMA system as a KM-user synchronous CDMA system. Assuming M is large and ignoring the marginal effects, the signature correlation matrix $\boldsymbol{R}$ and the Cholesky decomposition matrix $\boldsymbol{F}^T\boldsymbol{F} = \boldsymbol{R}$ of the synchronous system are illustrated in Fig. 3.

By partitioning $\boldsymbol{R}$ and $\boldsymbol{F}$ according to the time frame definition of T_1 (shown by black solid lines in Fig. 3), the diagonal block matrices are equal to $\boldsymbol{R}[0]$ and $\boldsymbol{F}[0]$, respectively, while the first sub-off-diagonal block matrices are $\boldsymbol{R}[1]$ and $\boldsymbol{F}[1]$, respectively (shown by the light-grey blocks in Fig. 3). Note that, due to the circular-permutation relationship between T_1 and T_2, conversion of time labeling from T_1 to T_2 only changes the time frame definition; hence, if we partition $\boldsymbol{R}$ and $\boldsymbol{F}$ according to the time frame definition of T_2 (dashed grey lines in Fig. 3), the resulting diagonal block matrices must equal $\tilde{\boldsymbol{R}}[0]$ and $\tilde{\boldsymbol{F}}[0]$, respectively, and the first sub-off-diagonal block matrices are $\tilde{\boldsymbol{R}}[1]$ and $\tilde{\boldsymbol{F}}[1]$ (the dark-grey blocks in Fig. 3), respectively. This can be easily extended to all time labelings. □

It may also be observed from *Proposition 3* that the SEs of the ideal DF detectors using chronological user ordering are identical. Furthermore, the performances of the actual DF detectors with chronological user ordering are identical as well, since time labeling does not change the detection order of the physical signals.

With the above results, the user ordering and time labeling algorithm proceeds as follows.

User Ordering and Time Labeling Procedure

1) Suppose the $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$, $\boldsymbol{F}[0]$, and $\boldsymbol{F}[1]$ matrices set for an arbitrary time labeling and user ordering is given. Without changing the time labeling, order users according to their times of arrival and obtain the corresponding $\boldsymbol{F}[0]$ and $\boldsymbol{F}[1]$ matrices via *Proposition 1* and (2).
2) Via *Proposition 2,* obtain the other $K-1$ time labelings with the corresponding chronological user order.
3) Via *Proposition 3*, obtain the corresponding $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ matrices for the other $K-1$ time labelings.
4) Compute $\boldsymbol{R}_{\text{AEC}}$ for all K different time labelings.
5) Apply the user ordering proposed in [3, Th. 1] to $\boldsymbol{W}\boldsymbol{R}_{\text{AEC}}\boldsymbol{W}$ for the K time labelings to obtain the optimal user order and the corresponding SE of the ideal DF detector for the K time labelings.
6) Choose the time labeling and user ordering pair that maximizes the ideal SE.

IV. Computer Simulations

In this section, we use computer simulations to show the effect of the proposed user ordering and time labeling on the performance of an actual DF detector. In generating the signature correlation matrix, we use the system model introduced in [5]. The time delays of user signals are random and uniformly distributed within a symbol duration. The square roots of user signal powers, $w_i \; \forall \, i$, are random and uniformly distributed in [2, 7]. Fig. 4 shows a 29-user example. We compare the performances of the DF detector with chronological user ordering, the DF detector with decreasing power user ordering (with a randomly generated time labeling), the DF detector and the ideal DF detector with optimal user ordering and time labeling. Since the ideal DF detector assumes no error propagation, the performance of the ideal DF detector with optimal user ordering and time labeling serves as a theoretical lower bound to the performance of an actual DF detector. Apparently, this lower bound is not necessarily reachable.

In synchronous CDMA, ordering users according to decreasing signal power should be near optimal. However, this appears not to be so in asynchronous CDMA. As shown

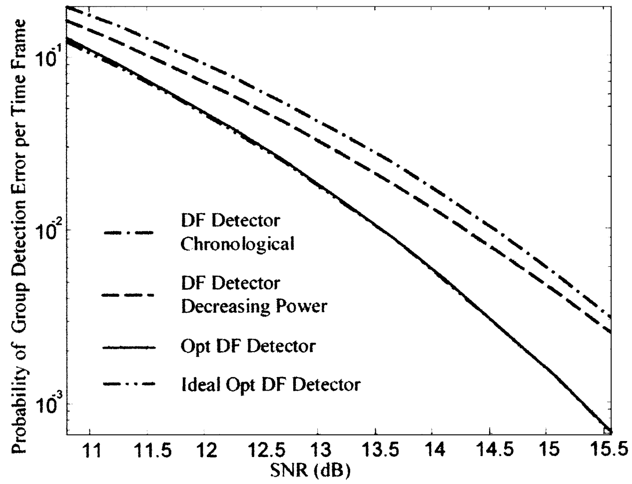

Fig. 4. Performance comparison of different detectors. 29 users, 31-length binary random signature sequences, 1 000 000 Monte–Carlo runs.

in Fig. 4, the performance of the DF detector with users ordered according to decreasing power can be almost as bad as chronological user ordering. Further, the performance differences between the practical DF detector and the ideal DF detector, both with optimal user ordering and time labeling, are indistinguishable at high signal-to-noise ratios.

V. Conclusion

The time labeling and user ordering that jointly optimize the asymptotic performance of an ideal decision feedback detector in asynchronous CDMA are given. Simulation results show that the ordering provided is not just asymptotically optimal (i.e., in terms of symmetric energy), but is also practically significant. The proposed ordering can be performed offline with a computational complexity of $O(K^4)$. The performance improvement can be substantial when compared with a chronological or a received signal power user ordering, the natural and simplest first choices. The ideas can easily be extended to group decision feedback detection [6] in asynchronous CDMA.

References

[1] S. Verdu, *Multiuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.
[2] A. Duel-Hallen, “A family of multiuser decision-feedback detectors for asynchronous code-division multiple-access channels,” *IEEE Trans. Commun.*, vol. 43, pp. 421–433, Feb.-Apr. 1995.
[3] M. Varanasi, “Decision feedback multiuser detection: A systematic approach,” *IEEE Trans. Inform. Theory*, vol. 45, pp. 219–240, Jan. 1999.
[4] P. Alexander and L. Rasmussen, “On the windowed Cholesky factorization of the time-varying asynchronous CDMA channel,” *IEEE Trans. Commun.*, vol. 46, pp. 735–737, June 1998.
[5] M. Pursley, “Performance evaluation for phase-coded spread-spectrum multiple-access communication—Part I: System analysis,” *IEEE Trans. Commun.*, vol. COM-25, pp. 795–799, Aug. 1977.
[6] J. Luo, K. Pattipati, P. Willett, and G. Levchuk, “Optimal grouping algorithm for a group decision feedback detector in synchronous CDMA communications,” *IEEE Trans. Commun.*, vol. 51, pp. 341–346, Mar. 2003.