OPTIMAL USER ORDERING AND TIME LABELING FOR DECISION FEEDBACK DETECTION IN ASYNCHRONOUS CDMA

J. Luo, K. Pattipati, P. Willett, F. Hasegawa

ECE Dept., Univ. of Connecticut, Storrs, CT06269
Email:{jieluo,krishna,willett,hiro}@engr.uconn.edu

ABSTRACT

A strategy for user ordering and time labeling for a decision feedback (DF) detector in asynchronous Code-Division Multiple Access (CDMA) communications is discussed. Ordering and labeling would at first appear to be of a complexity exponential in K, the number of users. Surprisingly, optimal sequencing requires only $O(K^4)$ operations, and is needed only once per packet: it is thus a cheap way to obtain an often marked improvement in performance, compared to power-ordering and chronological labeling.

1. INTRODUCTION

Optimal multiuser detection of CDMA communications is generally NP hard [1], and hence suboptimal algorithms that provide reliable performances with polynomial complexity have been sought. Among them, the linear and decision-driven detectors are particularly popular, and the DF detector [2] is one of the most efficient of these latter. Compared with the conventional matched filter and the decorrelator, the DF detector provides much better performance with complexity only $O(K^2)$ [3].

Since a DF structure detects users sequentially and then "subtracts them off", user ordering plays an important role [1]. In synchronous CDMA, the optimal ordering that maximizes the symmetric energy (SE) of the decorrelating DF detector is found in [3]. However, when users are asynchronous, not only user ordering but also time labeling will affect performance [2].

In asynchronous CDMA, it is commonly accepted that users should be detected either in decreasing order of their power or chronologically in their arrival time. Now, in synchronous CDMA [3] there are $K!$ different user orders, and ordering users according to decreasing signal power is not necessarily optimal when user correlation is considered; this remains so in the asynchronous case. Further, although it may at first appear that an ordering in terms of arrival time is at least fixed, this actually is not the case [2]: one can actually consider any user as the first-arriving user by fixing a fictitious initial bit to be zero.

Here we study user ordering and time labeling for the DF detector. Although our final goal is to minimize its probability of error, even the asymptotic performance of a DF detector is, due to error propagation, hard to estimate. In [2], assuming no error propagation, the theoretical asymptotic performance of the ideal DF detector is given. Building on this, we find a user ordering and time labeling that maximizes the SE of the ideal DF detector. We further show in computer simulations that, with the proposed user ordering and time labeling, the asymptotic performance of an actual DF detector is indistinguishable from that of the idealized DF detector. Computation is also a concern: the overall computation for the optimal user ordering and time labeling will be seen to be $O(K^4)$, and is of course required only once for a given user configuration.

2. ASYNCHRONOUS CDMA AND THE DECISION FEEDBACK DETECTOR

The asynchronous CDMA system can be described in the z domain by [1]

$$\mathbf{y}(z) = \mathbf{R}(z)\mathbf{W}\mathbf{b}(z) + \mathbf{v}(z) \quad (1)$$

where $\boldsymbol{y}$ is a $K \times 1$ column vector of the received signal (which is the output of K matched-filters at the receiver); $\boldsymbol{R}$ is the $K \times K$ signature correlation matrix; $\boldsymbol{W}$ is a diagonal matrix whose i^{th} diagonal component w_i is the square root of the signal power of the i^{th} user; $\boldsymbol{b}$ is the binary user signal vector[1]; and $\mathbf{v}$ is a

This work is supported by University of Connecticut Research Foundation #444636, the Office of Naval Research under contract #N00014-98-1-0465, #00014-00-1-0101, and NUWC under contract N66604-01-1-1125

[1] In the time domain representation, we denote $\mathbf{b}(n)$ to be the binary signal vector for the n^{th} time frame and denote $b_i(n)$ to be the binary signal of user i in time frame n. The assignment of the n^{th} or $(n+1)^{st}$ bit of each user to $\mathbf{b}(n)$ is the time labeling issue.

colored Gaussian noise with zero mean and covariance $\sigma^2\boldsymbol{R}(z)$, where σ^2 is the power of the white noise before the matched-filter. The correlation matrix $\boldsymbol{R}(z)$ can be represented as [1]

$$\boldsymbol{R}(z) = \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1]z^{-1} \tag{2}$$

Here $\boldsymbol{R}[0]$ is a symmetric matrix with unity diagonal components and whose off-diagonal components represent the correlation between user signatures at the same time index; and $\boldsymbol{R}[1]$ is a singular matrix whose components represent the signature correlations relating to successive time frames. Denote the component on the i^{th} row and j^{th} column of $\boldsymbol{R}[1]$ by $R[1]_{ij}$: since user signal i in time frame n cannot simultaneously be correlated with that of j in time frame $n-1$ and in time frame $n+1$, we have $R[1]_{ij}R[1]_{ji} = 0$.

The asynchronous CDMA system is illustrated in Figure 1 and a detailed description of the system model can be found in [1]. It is easy to see that the time label-

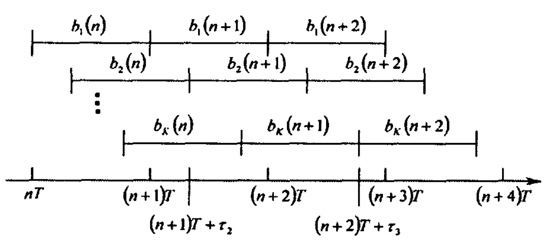

Fig. 1. Bit epochs for Asynchronous CDMA (T is the symbol duration, τ_i is the time delay for user i).

ing of the system is not unique. In Figure 1, suppose we change the time label for user 1: an equivalent bit epoch can be obtained as in Figure 2.

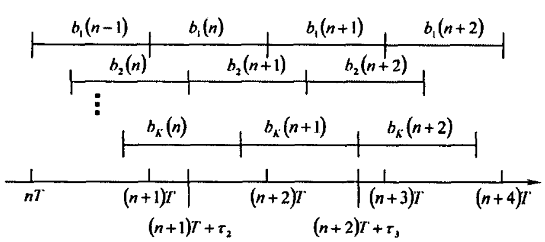

Fig. 2. Bit epochs for an equivalent system by changing the time label of user 1 in Figure 1.

Now, given a time labeling, and after ordering users according to their times of arrival, $\boldsymbol{R}[1]$ in (2) becomes an upper triangular matrix with zero diagonal components. It is found in [2] that the correlation matrix $\boldsymbol{R}(z)$ can be factored as

$$\boldsymbol{R}(z) = (\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)(\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1}) \tag{3}$$

where $\boldsymbol{F}[0]$ is a lower triangular matrix, and $\boldsymbol{F}[1]$ is singular.

Applying the anticausal feed forward filter $(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)^{-1}$ to both sides of (1), we obtain the white noise model [2]

$$\tilde{\boldsymbol{y}}(z) = (\boldsymbol{F}[0] + \boldsymbol{F}[1]z^{-1})\,\boldsymbol{W}\boldsymbol{b}(z) + \tilde{\boldsymbol{v}}(z) \tag{4}$$

where $\tilde{\boldsymbol{y}}(z) = (\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)^{-1}\boldsymbol{y}(z)$ and $\tilde{\boldsymbol{v}}$ is a white Gaussian noise vector with zero mean and covariance matrix $\sigma^2\mathbf{I}$. The corresponding time domain representation of the white noise model is

$$\tilde{\boldsymbol{y}}(n) = \boldsymbol{F}[0]\,\boldsymbol{W}\boldsymbol{b}(n) + \boldsymbol{F}[1]\,\boldsymbol{W}\boldsymbol{b}(n-1) + \tilde{\boldsymbol{v}}(n) \tag{5}$$

The DF detector for asynchronous CDMA [2] makes decisions sequentially and utilizes past decisions in addition to channel outputs, where $\hat{b}_i(n)$ is

$$\operatorname{sign}\left(\tilde{y}_i(n) - \sum_{j=1}^{i-1} F[0]_{ij}\hat{b}_j(n) - \sum_{j=1}^{K} F[1]_{ij}\hat{b}_j(n-1)\right) \tag{6}$$

The DF detector assumes that the decisions for user bit $b_i(n-1)$ are made prior to the decision of $b_j(n)$, $\forall i, j$, and hence both ordering and labeling are of concern.

In (6), assuming that the past decisions are correct, the ideal SE of the DF detector at time index n can be found via [3]

$$\eta = \min_{i=1,2,\ldots,K} (F[0]_{ii}w_i)^2 \tag{7}$$

3. OPTIMAL USER ORDERING AND TIME LABELING

Rewrite the system model (5) as,

$$\begin{aligned} &\boldsymbol{F}[0]^T\left(\tilde{\boldsymbol{y}}(n) - \boldsymbol{F}[1]\,\boldsymbol{W}\boldsymbol{b}(n-1)\right) \\ &= \boldsymbol{F}[0]^T\boldsymbol{F}[0]\,\boldsymbol{W}\boldsymbol{b}(n) + \boldsymbol{F}[0]^T\tilde{\boldsymbol{v}}(n) \end{aligned} \tag{8}$$

Given the idealized assumption that the past decisions are correct, the above system model is equivalent to a synchronous CDMA model, in which $R_{AEC} = \boldsymbol{F}[0]^T\boldsymbol{F}[0]$, termed the asymptotic effective correlation, is the equivalent signature correlation matrix. We begin with:

Proposition 1: Given the time labeling, suppose $\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P}$ is the signature correlation matrix of the same system but with a different user order ($\boldsymbol{P}$ is an arbitrary permutation matrix). The AEC matrix of the permuted system satisfies $\tilde{\boldsymbol{R}}_{AEC} = \boldsymbol{P}^T\boldsymbol{R}_{AEC}\boldsymbol{P}$. The SE is maximized by applying the user ordering technique in Theorem 1 of [3] to $\boldsymbol{W}\boldsymbol{R}_{AEC}\boldsymbol{W}$.

Proof: Define a sequence of matrices $\boldsymbol{R}^{(n)}$, where

$$\boldsymbol{R}^{(n)} = \boldsymbol{R}[0] - \boldsymbol{R}[1]^T\left\{\boldsymbol{R}^{(n-1)}\right\}^{-1}\boldsymbol{R}[1] \tag{9}$$

and $\boldsymbol{R}^{(0)} = \boldsymbol{R}[0]$. From the iterative procedure proposed in [4], it follows that, in the above procedure, $\boldsymbol{R}^{(n)} \to \boldsymbol{R}_{AEC}$ when $n \to \infty$. Since $\tilde{\boldsymbol{R}}(z) = \boldsymbol{P}^T\boldsymbol{R}(z)\boldsymbol{P}$, we have $\tilde{\boldsymbol{R}}[0] = \boldsymbol{P}^T\boldsymbol{R}[0]\boldsymbol{P}$ and $\tilde{\boldsymbol{R}}[1] = \boldsymbol{P}^T\boldsymbol{R}[1]\boldsymbol{P}$. By defining the corresponding iterative procedure for the permuted system, we can see that

$$\tilde{\boldsymbol{R}}^{(n)} = \boldsymbol{P}^T\boldsymbol{R}^{(n)}\boldsymbol{P} \tag{10}$$

Therefore, $\tilde{\boldsymbol{R}}_{AEC} = \boldsymbol{P}^T\boldsymbol{R}_{AEC}\boldsymbol{P}$. The proof follows from Theorem 1 of [3]. $\diamond$

Via Proposition 1, we can do user ordering for all possible time labelings and choose that which maximizes the SE. Unfortunately, the matrices $\boldsymbol{R}[0]$ and $\boldsymbol{R}[1]$ for different time labelings are different: we certainly do not want to apply the expensive iterative procedure (9) to all time labelings to find the best.

Happily, this is unnecessary. Given a time labeling, we first order users according to their time of arrival (see Figure 1). We can then assign a corresponding time label. For example, the time label in figure 1 can be represented by vector $T_1 = [1, \ldots, K]$, and the time label in Figure 2 is represented by vector $T_2 = [2, \ldots, K, 1]$. To change time label from T_1 to T_2, we only need to change the time index definition of user 1. Consequently, we denote the conversion from time label T_1 to T_2 by

$$T_1 \xrightarrow{\{user1\}} T_2 \tag{11}$$

Notice that for a valid time labeling, $b_i(n)$ must overlap with $b_j(n)$ $\forall i, j$ and for any n. We have:

Proposition 2: Suppose there is a time labeling T_G, where $T_1 \xrightarrow{G} T_G$ that convert T_1 to T_G. Then this G can be separated into two sets, G_1, G_2, where $G_1 = \{user\ 1\}$. The operation $T_1 \xrightarrow{G} T_G$ can also be separated to

$$T_1 \xrightarrow{G_1} T_2 \quad \text{then} \quad T_2 \xrightarrow{G_2} T_G \tag{12}$$

Proof: Suppose *user* $i \in G$ and *user* $1 \notin G$. Redefine $b_i(k+1)$ in time label T_1 to $b_i(k)$ in T_G. Now $b_i(k)$ and $b_1(k)$ in time label T_G do not overlap, and this is not valid. Therefore, *user* $1 \in G$ must be true. $\diamond$

It is clear now that the only valid time labelings are $T_1 = [1, \ldots, K]$, $T_2 = [2, \ldots, K, 1]$, $\ldots$, $T_K = [K, 1, \ldots, K-1]$. Even better, we have:

Proposition 3: Partition $\boldsymbol{R}[0]$, $\boldsymbol{R}[1]$, $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ on their second diagonal components (from upper-left corner) as

$$\boldsymbol{R}[0] = \begin{bmatrix} r_{11}[0] & \boldsymbol{r}_{21}[0]^T \\ \boldsymbol{r}_{21}[0] & \boldsymbol{R}_{22}[0] \end{bmatrix}, \quad \boldsymbol{R}[1] = \begin{bmatrix} 0 & \boldsymbol{r}_{12}[1] \\ \boldsymbol{0} & \boldsymbol{R}_{22}[1] \end{bmatrix}$$
$$\boldsymbol{F}[0] = \begin{bmatrix} f_{11}[0] & \boldsymbol{0} \\ \boldsymbol{f}_{21}[0] & \boldsymbol{F}_{22}[0] \end{bmatrix}, \quad \boldsymbol{F}[1] = \begin{bmatrix} 0 & \boldsymbol{f}_{12}[1] \\ \boldsymbol{0} & \boldsymbol{F}_{22}[1] \end{bmatrix}$$

Then, the matrices corresponding to time labeling T_2 become respectively $\tilde{\boldsymbol{R}}[0], \tilde{\boldsymbol{R}}[1], \tilde{\boldsymbol{F}}[0]$ and $\tilde{\boldsymbol{F}}[1]$:

$$\begin{bmatrix} \boldsymbol{R}_{22}[0] & \boldsymbol{r}_{12}[1]^T \\ \boldsymbol{r}_{12}[1] & r_{11}[0] \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{R}_{22}[1] & \boldsymbol{r}_{21}[0] \\ \boldsymbol{0} & 0 \end{bmatrix}$$
$$\begin{bmatrix} \boldsymbol{F}_{22}[0] & \boldsymbol{0} \\ \boldsymbol{f}_{12}[1] & f_{11}[0] \end{bmatrix}, \quad \begin{bmatrix} \boldsymbol{F}_{22}[1] & \boldsymbol{f}_{21}[0] \\ \boldsymbol{0} & 0 \end{bmatrix}$$

Proof: We can view the asynchronous CDMA system as a KM-user synchronous CDMA system (as introduced in [1]). The overall signature correlation matrix $\boldsymbol{R}$ is illustrated in Figure 3.

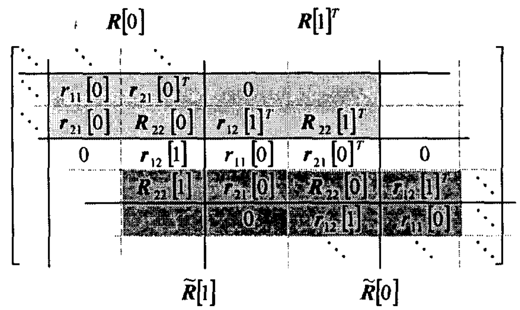

Fig. 3. Relation between signature correlation matrices of different time labelings.

By partitioning the $\boldsymbol{R}$ matrix according to the time frame definition of T_1 (shown by black solid lines in Figure 3), the diagonal block matrices are equal to $\boldsymbol{R}[0]$, while the first off-diagonal block matrices are $\boldsymbol{R}[1]$ and $\boldsymbol{R}[1]^T$ (shown by the light-grey blocks in Figure 3). Conversion of time labeling from T_1 to T_2 only changes the time frame definition; hence, if we partition the $\boldsymbol{R}$ matrix according to the time frame definition of T_2 (dashed grey lines in Figure 3), the resulting diagonal block matrices must be equal to $\tilde{\boldsymbol{R}}[0]$ and the first off-diagonal block matrices are $\tilde{\boldsymbol{R}}[1]$ and $\tilde{\boldsymbol{R}}[1]^T$ (the dark-grey blocks in Figure 3). This can be easily extended to all time labelings. $\diamond$

User Ordering and Time Labeling Procedure:

(1) Choose an arbitrary time labeling. Order users according to their times of arrival.

(2) Apply the iterative procedure proposed in [4] to obtain $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$.

(3) Via Proposition 2, get the other $K-1$ time labelings with the same physical user order.

(4) Via Proposition 3, obtain the corresponding $\boldsymbol{F}[0]$, $\boldsymbol{F}[1]$ matrices for the $K-1$ time labelings.

(5) Compute $\boldsymbol{R}_{AEC}$ for all K different time labelings.

(6) Apply the user ordering proposed in Theorem 1 of [3] to $\boldsymbol{W}\boldsymbol{R}_{AEC}\boldsymbol{W}$ for the K time labelings, obtain the optimal user order and the corresponding SE of the ideal DF detector for the K time labelings.

(7) Choose the time labeling and user ordering pair that maximizes the ideal SE.

Since the iterative procedure needs to be applied once in the DF detector for asynchronous CDMA, the extra computation to obtain the optimal user order and time labeling is $O(K^4)$ per packet.

4. COMPUTER SIMULATIONS

In this section, we use computer simulations to show that, with the proposed user ordering and time labeling, the performance of an actual DF detector is usually indistinguishable from the performance of the ideal DF detector. In generating the signature correlation matrix, we use the system model introduced in [5]. The time delays of user signals are random and uniformly-distributed within a symbol duration. The square roots of user signal powers are randomly and independently generated by $w_i \sim N(4.5, 4)$, $\forall i$, and are limited within the range of $[2, 7]$. Figure 4 shows a 29-user example. We compare the performances of the Decorrelator, the DF detector with chronological user ordering, the DF detector with decreasing power user ordering, the DF detector and the ideal DF detector with optimal user ordering and time labeling. The SE of a DF detector with chronological user ordering is identical for all time labelings (see Proposition 3).

In synchronous CDMA, ordering users according to decreasing signal power should be near-optimal. However, this appears not to be so in asynchronous CDMA: as shown in Figure 4, the performance of the DF detector with users ordered according to decreasing power can be almost as bad as chronological user ordering. Further, the performance differences between the practical DF detector and the ideal DF detector, both with optimal user ordering and time labeling, are indistinguishable at high signal-to-noise ratios.

5. CONCLUSION

The time labeling and user ordering that jointly optimize the asymptotic performance of an ideal decision feedback detector in asynchronous CDMA are given. Simulation shows that ordering provided is not just asymptotically (i.e., in terms of symmetric energy), but also practically correct. The performance improvement can be significant as compared to chronological or received power user-orderings, the natural and simplest first choices – interestingly, and unlike the synchronous CDMA case, ordering according to received power appears not to be a dependable approach. The ideas can easily been extended to group decision feedback detection [6] in asynchronous CDMA.

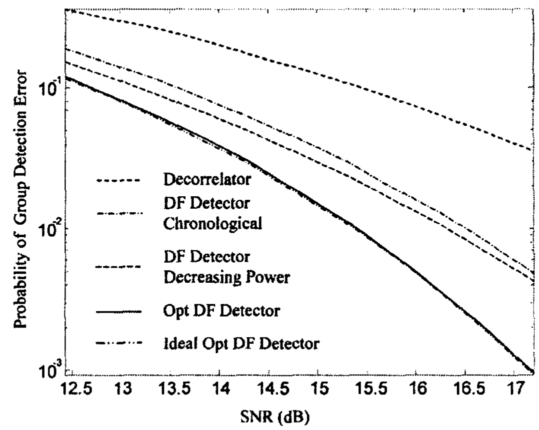

Fig. 4. Performance Comparison of different detectors. 29 Users, 31 Gold-Code as signature sequences, 1000000 Monte-Carlo runs.

6. REFERENCES

[1] S. Verdu, "Multiuser Detection", Cambridge University Press, New York, 1998.

[2] A. Duel-Hallen, "A Family of Multiuser Decision-Feedback Detectors for Asynchronous Code-Division Multiple-Access Channels", IEEE Trans. on Comm., Vol. 43, pp. 421–433, Feb./Mar./Apr. 1995.

[3] M. Varanasi, "Decision Feedback Multiuser Detection: A Systematic Approach", IEEE Trans. on Info. Theory, Vol. 45, pp. 219-240, Jan. 1999.

[4] P. Alexander, L. Rasmussen, "On the Windowed Cholesky Factorization of the Time-Varying Asynchronous CDMA Channel", IEEE Trans. on Comm., Vol. 46., pp. 735-737, June 1998.

[5] M. Pursley, "Performance Evaluation for Phase-Coded Spread-Spectrum Multiple-Access Communication-Part I: System Analysis", IEEE Trans. on Comm., Vol. COM-25, pp. 795-799, Aug. 1977.

[6] J. Luo, K. Pattipati, P. Willett, "Optimal Grouping and User Ordering for Sequential Group Detection in Synchronous CDMA", IEEE GLOBECOM 2001, Texas, Nov. 2001.