Fast Optimal and Suboptimal Any-Time Algorithms for CWMA Multiuser Detection

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Abstract — **A fast optimal algorithm based on the branch and bound method, coupled with an iterative lower bound update, is proposed for the joint detection of binary symbols of K users in a synchronous correlated waveform multiple-access (CWMA) channel with Gaussian noise. The proposed method significantly decreases the average computational cost and the Decision Feedback Detector (DFD) [2] is proved to be the first order approximation to the optimal algorithm. Furthermore, a suboptimal algorithm using the Probabilistic Data Association (PDA) is also proposed. It is shown that the suboptimal algorithm gives a near-optimal performance at a computational cost of $O(K^3)$.**

I. Problem Formulation and The Optimal Algorithm

A discrete-time equivalent model for the matched-filter outputs at the receiver of a CWMA channel is given by the K-length vector [1]

$$y = Hb + n \tag{1}$$

where $E[nn^T] = \sigma^2 H$ is the colored Gaussian noise. Suppose $H = L^T L$ is the Cholesky decomposition of H. Define $\tilde{y} = L^{-1^T} y$, $D = Lb$, and denote the kth component of D and $\tilde{y}$ by D_k and $\tilde{y}_k$, respectively. The optimal solution of (1) can be written as

$$\phi_{ML} : \hat{b} = \arg \min_{b \in \{-1,+1\}^K} \sum_{k=1}^{K} (D_k - \tilde{y}_k)^2 \tag{2}$$

Since L is a lower triangular matrix, D_k depends only on $(b_1, b_2, ..., b_k)$. When the decisions for the first k users are fixed, the term $\xi_k = \sum_{i=1}^{k} (D_i - \tilde{y}_i)^2$ can serve as a lower bound of (2). This lower bound is in fact an unconstrained MMSE solution and is achievable when the binary constraints on $(b_{k+1}, ..., b_K)$ are disregarded. In the proposed optimal algorithm, we sort users according to the DFD ordering algorithm proposed in theorem 1 of [2], then apply the branch and bound method with depth-first search. The first feasible solution obtained is proved to be the solution of DFD method. The computation for finding the first feasible solution requires $\frac{K(K+3)}{2}$ multiplications and $K(K+1)$ additions.

II. The PDA Detector

PDA idea is one of the most successful methods for target tracking. PDA treats undecided binary parameters as binary random variables. In CWMA, we first rewrite the model as

$$H^{-1} y = b + \tilde{n} = b_i e_i + \sum_{j \neq i} b_j e_j + \tilde{n} = b_i e_i + v_i \tag{3}$$

where e_i represents the vector whose ith component is 1 and other components are 0. v_i is the equivalent noise to user i. One form of the proposed multistage PDA detector is

(1) $\forall i$, initialize $P_b(i) = 0.5$. Set stage counter $k = 1$.

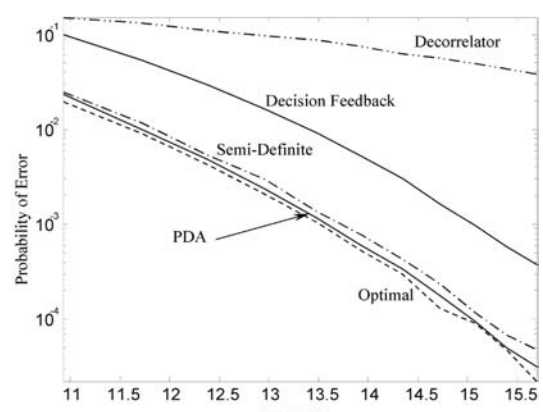
(2) $\forall i$, based on the current value of $P_b(j)$, $(j \neq i)$, update $P_b(i) = P(b_i = 1|y)$.

(3) Perform step 2 until $P_b(i)$ converges for all users. Make a decision on user signal i using $b_i = +1$ if $P_b(i) \geq 0.5$, $b_i = -1$ if $P_b(i) < 0.5$.

Further simplifications on the computation are obtained by using Gaussian approximation to v_i, using Sherman-Morrison-Woodbury formula to update the inverses of the covariance matrices, using the successive cancelation idea to reduce the problem size, and using coordinate descent to remove single bit errors. The overall computational cost is $O(K^3)$.

III. Simulation Results

The following figure shows a 13-user, 15-length random signature example (100000 Monte-Carlo runs). The PDA detector gives a significant better performance than the Decorrelator and the DFD, and its computational cost is significantly lower than that of the optimal detector and that of the semi-definite relaxation algorithm.



References

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[2] M. K. Varanasi, "Decision feedback multiuser detection: a systematic approach," *IEEE Trans. Inform. Theory*, vol. 45, pp. 219–240, Jan. 1999.

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