On Rate Control of Wireless Multicasting

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Abstract This paper studies wireless multicast communication, where a source of common information is transmitted to a group of receivers over fading channels. Communication between the transmitter and each of the receivers is implemented by specifying a minimum signal to noise ratio (SNR) threshold; if the threshold is met, the communication is successful at a corresponding rate, otherwise the communication fails. The determination of SNR threshold that maximizes the effective multicast communication rate is termed *rate control.* By modeling the channel from the transmitter to each of the receivers as an erasure channel and assuming only channel distribution information at the transmitter, the optimal SNR threshold under a given transmit power is derived. It is shown that, in the low transmit power regime, the optimal ratio between the SNR threshold and the transmit power is determined only by the channel distributions.

Key words multicast, rate control

1 Introduction

In the layered network architecture, one of the key functions of the data link layer is to transform the raw transmission facility into a virtual error free logical link to the upper layers [1]. In wireless communication, reliable information delivery can be achieved in various ways. For example, from an information theoretic point of view, reliable communication is achievable using channel coding. However, capacity achieving channel codes for wireless channel with additive noise often contain high precision output symbols. Although using channel code to average out ambient noise is a relatively easy task, when channel experiences fading, averaging out channel variation may require channel code being long enough to assume ergodic fading process. If channel fading is slow, such long channel coding can be infeasible due to the excessive memory and computation demand [2][3]. Alternatively, practical wireless systems usually achieve reliable information delivery via a concatenated scheme combining error controlled reception with retransmission [4]. Information is transmitted in the form of packets. If the received signal to noise ratio (SNR) of a packet is above a predetermined threshold T and the packet passes a cyclic redundancy check, the packet is successfully received in the sense that its probability of error is small enough to be considered reliable. If a packet is not received successfully, however, it is dropped by the receiver without being forwarded to the upper layers. Such error controlled reception converts a wireless channel to an erasure channel. In conventional data networks, retransmission is used on top of error controlled reception to further guarantee that source packets can reach their receivers with high probability. If a packet is not received by the desired receiver, a retransmission of the same packet will be scheduled at a later time.

In wireless communication, if a transmitter sends information to a distant receiver, other nearby receivers often obtain the information without extra cost on the transmit power [5]. Since wireless channel is a shared medium by its nature, and because the transmission energy is a treasured resource, wireless systems usually encourage multicast transmission, which sends common information to benefit a group of receivers rather than one [5][6]. Unfortunately, in multicast communication, the retransmission mechanism becomes inefficient. If the number of receivers is large and the channels are lossy, the system will be dominated by retransmissions and consequently achieves a low multicast communication rate [7].

One way to overcome such multicast inefficiency is to use forward error correction (FEC) instead of retransmission. Such FEC coding is applied to the multicast erasure channel, i.e., in concatenation to the error controlled reception, and hence requires significantly less memory than the optimal information theoretic channel coding for the original wireless multicast channel. Among FEC codes for erasure channels, fountain codes [8][9][10] form a class of attractive candidates. The basic idea of fountain code is to transmit packets constructed from random linear combinations of the source. As long as a receiver collected certain numbers of such random combinations, it will be able to decode the source with

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high probability [7]. Fountain code has several important properties. It is rate optimal since it is capacity achieving for erasure channels. It is rateless in the sense that the same code achieves the erasure channel capacity simultaneously for all erasure probabilities, hence it also achieves the common information capacity of a multicast erasure channel. With the help of fountain codes, the effective communication rate between a transceiver pair is the multiplication of the successful communication rate[1] and the probability of communication success. The multicast communication rate is simply given by the minimum effective rate of the transceiver pairs. Consequently, rate control problem arises since the successful communication rate and the success probability are not independent parameters.

This paper studies wireless multicast communication with block channel fading. We assume the transmitter only knows the channel distribution information, and does not obtain feedback from the receivers. We consider the concatenated transmission scheme that combines error controlled reception with FEC. Assume there is a pre-determined SNR threshold T, such that, for any transceiver pair, if the received SNR in a block is above T, the communication is successful at a corresponding rate R; if the received SNR is below T, the communication fails. Given a fixed transmit power P, we study the optimization of T that maximizes the multicast communication rate. We show that, when the transmit power is low, the optimal SNR threshold is linear in the transmit power. The ratio between the optimal SNR threshold and the transmit power is determined only by the channel distributions. It is not a function of the transmit power; it does not depend on the modulation scheme. Under the assumption of optimal rate control, we then analyze the performance of the concatenated scheme in the low power regime in terms of spectral efficiency and energy efficiency tradeoff [11]. Our results show that, the suboptimality of the concatenated scheme can be significant, if the optimal information theoretic channel coding is indeed feasible.

2 System Model

Consider the multicast system illustrated in Figure 1, where the source node S wants to transmit a common information to N receivers $D_1, \ldots, D_N$. Assume both the source and the receivers have single antenna. Time is divided into blocks of equal length. The channel gain from S to D_i is denoted by h_i, which experiences block fading with a stationary density function of $f_i(|h_i|^2)$. We assume the transmitter only knows channel distribution information. There is no feedback from the receivers to the transmitter.

Let the transmit power be fixed at P. The received SNR at D_i is given by

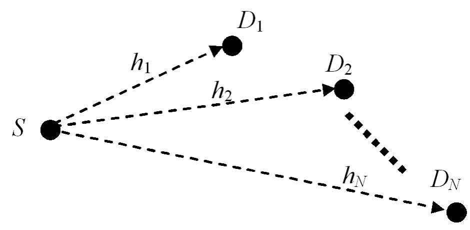

Figure 1: An illustration of wireless multicast communication.

$$\mathrm{SNR}_i = \frac{|h_i|^2 P}{N_0} \tag{1}$$

where N_0 is the one side noise spectral density.

Assume there is a SNR threshold T. For any transceiver pair, in each block, if the SNR is above T, the communication is successful at a communication rate R; if the SNR is below T, the communication rate is zero. Such error controlled reception converts a wireless channel into an erasure channel. We term R the successful communication rate, and generally write $R(T)$ as a function of T. The exact expression of $R(T)$ depends on communication details such as the modulation and demodulation schemes.

The effective communication rate of the erasure channel between S and D_i is given by

$$\begin{aligned} r_i &= R(T) Pr\left(\frac{|h_i|^2 P}{N_0} \geq T\right) \\ &= R(T) \int_{\frac{TN_0}{P}}^{\infty} f_i(|h_i|^2) d|h_i|^2 \end{aligned} \tag{2}$$

where $Pr\left(\frac{|h_i|^2 P}{N_0} \geq T\right)$ is the erasure probability, or the outage probability, associated to the channel from S to D_i.

Since a multicast erasure channel is degraded, we define the multicast communication rate as the common information rate of the multicast erasure channel. Assume rateless FEC coding, the multicast communication rate is given by

$$R_{\text{multi}} = \min_i r_i = \min_i R(T) Pr\left(\frac{|h_i|^2 P}{N_0} \geq T\right) \tag{3}$$

The rate control problem considered in this paper is defined as

$$\text{Given } P, \quad \text{Maximize } R_{\text{multi}}(T) \tag{4}$$

3 Main Results

Let us rewrite the multicast communication rate as follows

$$\begin{aligned} R_{\text{multi}} &= R(T) \min_i Pr\left(\frac{1}{|h_i|^2} \leq \frac{P}{TN_0}\right) \\ &= R(T) F\left(\frac{P}{TN_0}\right) \end{aligned} \tag{5}$$

[1] This refers to the communication rate given that the communication is successful.

Here $F\left(\frac{P}{TN_0}\right)$ is a distribution function since $F(0) = 0$ and $F(\infty) = 1$.

Assume both $R(.)$ and $F(.)$ are continuous and differentiable. To maximize (5), T must satisfy the following equality.

$$\dot{R}(T)F\left(\frac{P}{TN_0}\right) = R(T)f\left(\frac{P}{TN_0}\right)\frac{P}{T^2N_0} \tag{6}$$

where $f(x) = \frac{dF(x)}{dx}$ is the density function corresponding to $F(.)$.

We have the following theorem.

Theorem 1 *If $R(T)$ is linear in T, then the optimal T that maximizes the multicast rate takes the form $T = \frac{P}{\alpha N_0}$, with α satisfying the following equality.*

$$F(\alpha) = \alpha f(\alpha) \tag{7}$$

Proof: Since $R(0) = 0$, if $R(T)$ is linear in T, we must have

$$R(T) = \dot{R}T \tag{8}$$

Substitute (8) into (6). The optimal T should satisfy

$$F\left(\frac{P}{TN_0}\right) = \frac{P}{TN_0}f\left(\frac{P}{TN_0}\right) \tag{9}$$

Since (9) is only a function of $\frac{P}{TN_0}$, the result of the theorem then follows. $\diamondsuit$

Note that when P is close to zero, T should be close to zero as well. Consequently, $R(T)$ can be approximated by its first order Taylor expansion, which is approximately linear in T. Although the value of $\dot{R}$ in (8) depends on communication details such as the modulation scheme, in the low power regime, the optimal ratio between T and P is determined only by the $F(.)$ function via (7). It is not a function of P; it does not depend on $\dot{R}$.

As introduced by Verdú in [11], when a system operates in the low power regime, its performance can be characterized by the spectral efficiency and energy efficiency tradeoff curve. We refer to [11] for the detailed definitions of the tradeoff curve and the explanation of the low power regime analysis. Since the spectral efficiency is approximately linear in the logarithm of the energy efficiency [11], the tradeoff curve is determined by two key parameters: the minimum transmit energy per bit and the wideband slope.

The normalized transmit energy per bit of the multicast system is defined as follows.

$$\frac{E_b}{N_0} = \frac{P}{N_0R_{\text{multi}}} \tag{10}$$

The minimum transmit energy per bit is given by

$$\frac{E_b}{N_0}_{\text{min}} = \lim_{P\to 0}\frac{P}{N_0R_{\text{multi}}} \tag{11}$$

The wideband slope is defined by[2]

$$\mathcal{S}_0 = \lim_{\frac{E_b}{N_0}\downarrow\frac{E_b}{N_0}_{\text{min}}}\frac{R_{\text{multi}}}{\log\frac{E_b}{N_0} - \log\frac{E_b}{N_0}_{\text{min}}} \tag{12}$$

Suppose $R(T)$ is continuous and second order differentiable.

$$R(T) = \dot{R}(0)T + \frac{1}{2}\ddot{R}(0)T^2 + o(T^2) \tag{13}$$

Given P, let $T = \frac{P}{\alpha N_0}$ with α being determined by (7). We have the following results in the low power regime.

Corollary 1 *The minimum transmit energy per bit of the multicast system is given by*

$$\frac{E_b}{N_0}_{\text{min}} = \frac{\alpha}{\dot{R}(0)F(\alpha)} = \frac{1}{\dot{R}(0)f(\alpha)} \tag{14}$$

The wideband slope of the multicast system is given by

$$\mathcal{S}_0 = \frac{2\dot{R}(0)^2F(\alpha)}{-\ddot{R}(0)} \tag{15}$$

Proof: According to [11], we have

$$\mathcal{S}_0 = \frac{2\dot{R}_{\text{multi}}(0)^2}{-\ddot{R}_{\text{multi}}(0)} = \frac{2\dot{R}(0)^2F(\alpha)^2}{-\ddot{R}(0)F(\alpha)} = \frac{2\dot{R}(0)^2F(\alpha)}{-\ddot{R}(0)} \tag{16}$$

$\diamondsuit$

4 Low Power Regime Comparison

Compared with the information theoretic optimal channel coding, the concatenated scheme has the advantage of requiring significantly less memory and computation. Therefore, it is a good alternative when the information theoretic optimal scheme is infeasible. It is natural to ask, if it *is* feasible to average out channel variation optimally, how much do we lose by using the concatenated scheme? In this section, we present a performance comparison of the concatenated scheme and the optimal scheme in the low power regime.

We assume each block is long enough so that the ambient noise can be averaged out in the information theoretic sense. In other words, in the concatenated scheme, we assume

$$R(T) \approx \log(1 + T) \tag{17}$$

Note that under this assumption, if the channels are static, the concatenated scheme is optimal.

Now, suppose the channels are Rayleigh faded, and hence the wireless multicast channel is degraded. Without the loss of generality, assume receiver D_1 has the weakest channel, whose channel gain $|h_1|^2$ yields the following density function.

$$f_1(|h_1|^2) = \frac{1}{2}e^{-\frac{|h_1|^2}{2}} \tag{18}$$

[2]Since we are comparing systems with the same bandwidth, the wideband slope is normalized by the system bandwidth.

For the optimal scheme, the multicast rate is given by

$$R_{\text{multi}}^{\text{opt}} = E\left[\log\left(1 + \frac{|h_1|^2 P}{N_0}\right)\right] \tag{19}$$

Therefore, the corresponding minimum transmit energy per bit and the wideband slope are given respectively by

$$\frac{E_b}{N_0}_{\min}^{\text{opt}} = \frac{1}{E[|h_1|^2]} = \frac{1}{2} \tag{20}$$

$$\mathcal{S}_0^{\text{opt}} = \frac{2E[|h_1|^2]^2}{E[|h_1|^4]} = 1 \tag{21}$$

For the concatenated scheme, we obtain $F(\alpha)$ from (2) as

$$F(\alpha) = \int_{\frac{1}{\alpha}}^{\infty} f_1(|h_1|^2) d|h_1|^2 = e^{-\frac{1}{2\alpha}} \tag{22}$$

Hence the optimal ratio between T and $\frac{P}{N_0}$ can be computed from Equality (7) as

$$\alpha = \frac{1}{2} \tag{23}$$

Consequently, the minimum transmit energy per bit and the wideband slope of the concatenated scheme are given respectively by

$$\frac{E_b}{N_0}_{\min} = \frac{1}{\dot{R}(0) f(\alpha)} = \frac{e}{2} \tag{24}$$

$$\mathcal{S}_0 = \frac{2\dot{R}(0)^2 F(\alpha)}{-\ddot{R}(0)} = \frac{2}{e} \tag{25}$$

Figure 2 illustrates the spectral efficiency and energy efficiency tradeoff curves of the optimal scheme and the concatenation scheme in the low power regime. We can see that, when it is feasible to average out the channel variation optimally, the suboptimality of the concatenated scheme can be significant in the low power regime.

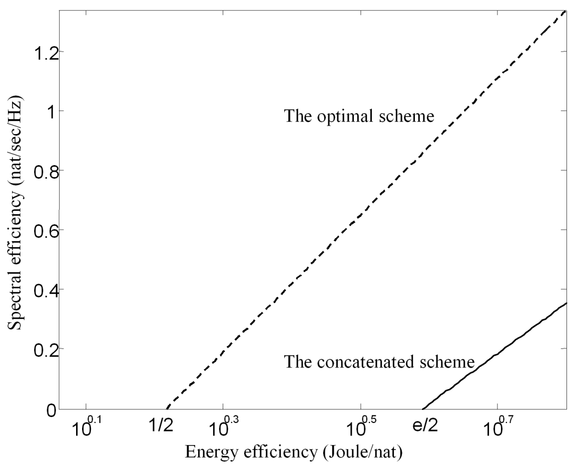

Figure 2: Comparison on spectral efficiency and energy efficiency tradeoffs of the optimal scheme and the concatenated scheme.

5 Conclusion

This paper studied multicast communication over wireless block fading channels. We considered the rate control problem of the concatenated scheme that combines error controlled reception with forward error correction. We showed that in the low power regime, the optimal signal to noise ratio threshold is proportional to the transmit power. The optimal ratio between the signal to noise ratio threshold and the transmit power is determined only by the channel distributions. It is not a function of the transmit power; it does not depend on the modulation scheme. Although the concatenated multicast scheme is a simple alternative when the information theoretic optimal channel coding is infeasible, the suboptimality of the concatenated scheme can be significant in the low power regime if it *is* feasible to average out channel variation optimally.

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