Wireless Multicasting with Channel Uncertainty

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***Abstract*— We consider wireless multicasting where a source of common information is transmitted to a group of receivers over block fading channels. Communication between the transmitter and each of the receivers is implemented by specifying a minimum signal to noise plus interference ratio (SINR) threshold; if the threshold is met, the communication is successful at a corresponding rate, otherwise the communication fails. Such error controlled reception converts wireless channels into erasure channels, upon which forward error correction or retransmission is concatenated to achieve reliable communication. Assuming only channel distribution information at the transmitter, we derive the optimal SINR threshold given the transmit power constraint. We show, in the low transmit power regime, the optimal ratio between the SINR threshold and the transmit power is determined only by the channel distributions.**

I. Introduction

In the layered network architecture, one of the key functions of the data link layer is to transform the raw transmission facility into virtual error free logical links to the upper layers [1]. Communications over such logical links should be reliable in the sense that information from a transmitter should reach the receivers with a probability of error below a predetermined small constant.

Practical wireless data network often achieves reliable information delivery via a concatenated scheme combining error controlled reception with retransmission [2]. Information is transmitted in the form of packets. If the received signal to noise plus interference ratio (SINR) of a packet is above a predetermined threshold T, the packet is successfully received in the sense of small error probability. If a packet is not received successfully, however, it is dropped by the receiver without being forwarded to the upper layers. Such error controlled reception converts a wireless channel into an erasure channel. Conventionally, retransmission is used on top of error controlled reception to further guarantee that source packets can reach their receivers with high probability. If a packet is not received by the desired receiver, a retransmission of the same packet will be scheduled at a later time. When feedback is not difficult to obtain, retransmission is a cost effective way to achieve reliable information delivery due to its simplicity and its advantage of small latency.

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In wireless communication, if a transmitter sends information to a distant receiver, other nearby receivers may be able to obtain the information without extra cost on the transmit power [3]. Since wireless channel is a shared medium by its nature, and because the transmission energy is a precious resource, wireless systems usually encourage multicast transmission, which sends common information to benefit a group of receivers rather than one [4]. Unfortunately, in multicast communication, the retransmission mechanism becomes inefficient. If the number of receivers is large and the channels are lossy, the system can be dominated by retransmissions and consequently achieves a low multicast throughput [5]. One way to overcome such multicast inefficiency is to use forward error correction (FEC) instead of retransmission. Since the FEC coding is applied to the multicast erasure channel, i.e., in concatenation to the error controlled reception, the memory requirement is significantly less than the optimal information theoretic channel coding for the original wireless channel. Among FEC codes for erasure channels, fountain codes [6][5] form a class of attractive candidates. The basic idea of fountain code is to transmit packets constructed from random linear combinations of the source. As long as a receiver collected certain numbers of such random combinations, it will be able to decode the source with high probability [5]. Fountain code has several important properties. It is rate optimal since it is capacity achieving for erasure channels. It is rateless in the sense that the same code achieves the erasure channel capacity simultaneously for all erasure probabilities, hence it also achieves the common information capacity of a multicast erasure channel. With the help of fountain codes, the effective communication rate between a transceiver pair is approximately given by the multiplication of the successful communication rate[1] and the probability of communication success. The communication rate of a multicast system is simply given by the minimum effective

[1]This refers to the communication rate given that the communication is successful.

rate of the transceiver pairs.

In the concatenated communication schemes, either with retransmission or with FEC coding, the choice of the SINR threshold affects both the successful communication rate and the probability of communication success, which jointly determine the multicast rate. Optimization of the SINR threshold is termed *rate control* in this paper. We study wireless multicast communication with block channel fading, which models the joint effect of channel gain variation and the variation of the interference from other terminals. We assume the transmitter only knows channel distribution information and does not obtain any feedback about the channel states. Both concatenated transmission schemes using retransmission and FEC are considered. We show that, when the transmit power is low, the optimal SINR threshold is approximately linear in the transmit power. The ratio between the optimal SINR threshold and the transmit power is determined only by the channel distributions. It is not a function of the transmit power; it does not depend on the modulation scheme. We give a lower bound to the inefficiency of the concatenated schemes in the low power regime. We also show that the concatenated schemes are asymptotically optimal in the high power regime.

II. System Model

Consider the multicast system illustrated in Figure 1, where the source node S wants to transmit a common information to K receivers $D_1, \ldots, D_K$. Assume both the

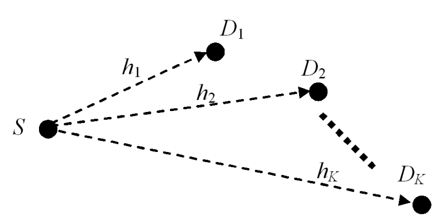

Fig. 1. An illustration of wireless multicast communication.

source and the receivers have single antenna. Time is divided into blocks of equal length. The channel gain from S to D_i is denoted by a_i, which experiences block fading. Let the transmit power be P. Let I_i be the noise plus interference power at receiver D_i. The received SINR at D_i is given by

$$\text{SINR}_i = \frac{|a_i|^2 P}{I_i} = |h_i|^2 P \quad (1)$$

We assume I_i remains constant within any block, but can vary among blocks. $|h_i|^2$ in (1) can be regarded as the normalized channel gain for receiver D_i in a particular block. Due to the possible intractability of the channel gain and the interference, it is often difficult to know $|h_i|^2$ precisely at the transmitter. However, we assume the stationary distribution of $|h_i|^2$, whose density function is denoted by $f_i(|h_i|^2)$, is known at the transmitter. We assume the receivers know the channel states.

Assume there is a SINR threshold T and a corresponding communication rate R. For any transceiver pair, in each block, if the received SINR is above T, the communication is successful in the sense of delivering R unit information from the transmitter to the receiver with probability of error below a predetermined small constant. If the received SINR is below T on the other hand, the communication fails. Such error controlled reception converts a wireless channel into an erasure channel. We term R the successful communication rate, and generally write $R(T)$ as a function of T. The exact expression of $R(T)$ depends on communication details such as the modulation and demodulation schemes. For communication between S and D_i, the probability of communication success is given by

$$\rho_i = Pr\left(|h_i|^2 P \geq T\right) = \int_{\frac{T}{P}}^{\infty} f_i(|h_i|^2) d|h_i|^2 \quad (2)$$

Note that $1 - \rho_i$ is the erasure probability or the outage probability of the corresponding erasure channel.

For the concatenated scheme with FEC coding, if the FEC code is both rateless and rate optimal, the effective communication rate of the erasure channel between S and D_i is given by

$$r_i = R(T)\rho_i \quad (3)$$

Since a multicast erasure channel is degraded, the maximum rate S can transmit common information to all the destinations is termed the multicast rate, which is given by

$$R_{\text{multi}} = \min_i r_i = \min_i R(T)\rho_i \quad (4)$$

For the concatenated scheme with retransmission, we assume no coding across multiple blocks. Assume the transmitter obtains feedbacks about communication success from the receivers at the end of each block. If the information block is not received at least once by each of the receivers, the same information will be retransmitted in the next block. Let n be the number of transmissions of an arbitrary information block. The probability that the $n \leq N$ is given by

$$Pr(n \leq N) = \prod_{i=1}^{K} [1 - (1 - \rho_i)^N] \quad (5)$$

Consequently, the multicast rate of the system is

$$R_{\text{multi}} = \frac{R(T)}{\sum_{N=0}^{\infty}(1 - \prod_{i=1}^{K}[1 - (1 - \rho_i)^N])} \quad (6)$$

Note that the right hand side of (6) is no larger than the right hand side of (4). Although (6) does not take a simple form as (4), given ρ_i, $i = 1, \ldots, K$, R_{multi} can be easily computed without involving infinite number of terms. For example,

$$R_{\text{multi}} = \frac{R(T)}{\frac{1}{\rho_1} + \frac{1}{\rho_2} - \frac{1}{1-(1-\rho_1)(1-\rho_2)}}, \qquad \text{if } K = 2 \quad (7)$$

If the channel gains are independently and identically distributed, we have

$$R_{\text{multi}} = \frac{R(T)}{\sum_{k=1}^{K}(-1)^{k+1}\begin{pmatrix} k \\ K \end{pmatrix}\frac{1}{1-(1-\rho)^k}}, \quad \text{if } \rho_i = \rho, \forall i \quad (8)$$

For both concatenated schemes, the rate control problem considered in this paper is defined as

$$\text{Given } P, \quad \text{Maximize } R_{\text{multi}}(T) \quad (9)$$

III. Rate Control and Its Optimality

Let us rewrite (2) as

$$\rho_i = Pr\left(|h_i|^2 P \geq T\right) = Pr\left(|h_i|^2 \geq \frac{T}{P}\right) \quad (10)$$

Since ρ_i is a function of $\frac{T}{P}$, in both concatenated schemes, the multicast communication rate can be written in the following form

$$R_{\text{multi}} = R(T)F\left(\frac{T}{P}\right) \quad (11)$$

Note that $1 - F\left(\frac{T}{P}\right)$ is a distribution function since, according to (4) and (6), $F(0) = 1$ and $F(\infty) = 0$.

A. Rate Control in The Low Power Regime

Since $R(0) = 0$, assume $R(T)$ is continuously differentiable, and hence can be written as

$$R(T) = \dot{R}T + o(T) \quad (12)$$

The following theorem indicates that the optimal SINR threshold is approximately linear in the transmit power in the low power regime.

Theorem 1: When $P \to 0$, the optimal SINR threshold that maximizes the multicast rate takes the following form

$$T = \alpha^* P + o(P) \quad (13)$$

where α^* is given by

$$\alpha^* = \arg\max_{\alpha} \alpha F(\alpha) \quad (14)$$

The proof of Theorem 1 is given in [7].

Although the value of $\dot{R}$ depends on communication details such as the modulation scheme and the block length, in the low power regime, the optimal ratio between T and P is determined only by the $F(.)$ function. It is not a function of P; it does not depend on $\dot{R}$.

Define the energy cost of the multicast system as the normalized transmit energy of delivering one unit common information to all the receivers, which is given by

$$E_N = \frac{P}{R_{\text{multi}}} \quad (15)$$

Let the bandwidth of the multicast system be B, similar to the definition introduced by Verdú in [8], define the spectral efficiency of the system as the multicast communication rate normalized by the system bandwidth, i.e.,

$$\mathcal{C} = \frac{R_{\text{multi}}}{B} \quad (16)$$

When comparing systems with the same bandwidth, we can simply define $\mathcal{C}_B = B\mathcal{C} = R_{\text{multi}}$ as the spectral efficiency that indicates how efficiently the bandwidth resource is used. The spectral efficiency and energy cost tradeoff is obtained by regarding $\mathcal{C}_B$ as a function of the logarithm of the energy cost, i.e.,

$$\mathcal{C}_B = \mathcal{C}_B(\log E_N) \quad (17)$$

If $R(T) \leq \dot{R}T$, the minimum energy cost is achieved when P approaches zero.

$$E_{N\min} = \lim_{P\to 0}\frac{P}{R_{\text{multi}}} \quad (18)$$

As shown in [8], the spectral efficiency and energy cost tradeoff curve is approximately linear in the low power regime. The slope, termed the wideband slope, is given by

$$\mathcal{S}_0 = \lim_{E_N \downarrow E_{N\min}} \frac{R_{\text{multi}}}{\log E_N - \log E_{N\min}} \quad (19)$$

The minimum energy costs and the wideband slopes of the two concatenated schemes are characterized by the following lemma.

Lemma 1: Let $R(T)$ be continuous and second order differentiable, i.e.,

$$R(T) = \dot{R}T + \frac{1}{2}\ddot{R}T^2 + o(T^2) \quad (20)$$

Assume $R(T) \leq \dot{R}T$. For the concatenated schemes, the minimum energy cost of the system is given by

$$E_{N\min} = \frac{1}{\dot{R}\alpha^* F(\alpha^*)} \quad (21)$$

The wideband slope is given by

$$\mathcal{S}_0 = \frac{2\dot{R}^2 F(\alpha^*)}{-\ddot{R}} \quad (22)$$

The proof of Lemma 1 is presented in [7].

Lemma 1 shows that letting $T = \alpha^* P$ achieves the optimal spectral efficiency and energy cost tradeoff for the concatenated schemes in the low power regime.

B. Optimality of the Concatenated Schemes

Compared with the information theoretic optimal channel coding, the concatenated schemes have the advantage of requiring significantly less memory and computation. It is natural to ask, if it *is* feasible to average out channel variation in the information theoretic sense, how much do we lose by using the concatenated schemes? This question is addressed by the following two lemmas in the lower power and the high power regimes, respectively.

The following lemma gives a lower bound to the suboptimality of the concatenated schemes in the low power regime.

Lemma 2: Given P, let $R_{\text{multi}}^{\text{opt}}$ be the common information capacity of the multicast channel. Let R_{multi} be the multicast rate of the concatenated scheme (either with FEC or with retransmission). We have

$$\lim_{P\to 0}\frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}}\geq\frac{\dot{R}\alpha^{*}F(\alpha^{*})}{\min_i E[|h_i|^2]} \tag{23}$$

The proof of Lemma 2 is given in [7].

Note that (23) provides an upper bound to the minimum energy cost of the concatenated schemes since

$$\lim_{P\to 0}\frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}}=\frac{E_{N\ \min}^{\text{opt}}}{E_{N\min}} \tag{24}$$

In the high power regime, we assume the communication is efficient in the sense that, given the probability of reception error requirement, for large T, $R(T)$ can be approximated by

$$R(T)=\log T+o(\log T) \tag{25}$$

Note that this is the case for many common signaling schemes such as the complex QAM, the cross constellation, etc. [9]. Since if the ambient noise is averaged out in the information theoretic sense $R(T)=\log(1+T)$ can also be approximated by $\log T$ for large T, the follow lemma gives the asymptotic optimality of the concatenated schemes in the high power regime.

Lemma 3: Assume $\lim_{T\to\infty}\frac{R(T)}{\log T}=1$. Given P, let $R_{\text{multi}}^{\text{opt}}$ be the common information capacity of the multicast channel. On one hand, if the SINR threshold in the concatenated scheme is given by $T=\alpha P$, for a fixed α, asymptotically, the multicast rate satisfies,

$$\lim_{P\to\infty}\frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}}=F(\alpha) \tag{26}$$

On the other hand, if the SINR threshold is chosen optimally to maximize R_{multi}, asymptotically, we have

$$\lim_{P\to\infty}\frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}}=1 \tag{27}$$

The proof of Lemma 3 is presented in [7].

According to Lemma 3, the story in the high power regime is quite different from the one in the low power regime. First, although the concatenated schemes can be significantly suboptimal in the low power regime, they are asymptotically optimal in the high power regime. Second, letting $T=\alpha^{*}P$ performs as good as the optimal rate control in the low power regime; unfortunately, in the high power regime, such simplification can bring significant rate loss.

IV. Numerical Results

In this section, we present some numerical comparisons to give an intuitive understanding about the results derived in Section III.

A. Inefficiency of The Concatenation in The Low Power Regime

Let us consider a multicast system with 10 receivers. Assume the channels between the transmitter and the receivers are i.i.d., and hence the multicast channel is degraded. If we average out channel variation in the information theoretic sense, given transmit power P, the multicast rate (or the common information capacity) is given by

$$R_{\text{multi}}^{\text{opt}}=E\left[\log\left(1+|h_1|^2P\right)\right] \tag{28}$$

Therefore, the minimum energy cost and the wideband slope of the system are obtained, similar to [8], as

$$\begin{aligned} E_{N\ \min}^{\text{opt}} &= \lim_{P\to 0}\frac{P}{E\left[\log\left(1+|h_1|^2P\right)\right]}=\frac{1}{E[|h_1|^2]} \\ \mathcal{S}_0^{\text{opt}} &= \frac{2\left(\dot{R}_{\text{multi}}^{\text{opt}}\right)^2}{-\ddot{R}_{\text{multi}}^{\text{opt}}}=\frac{2E[|h_1|^2]^2}{E[|h_1|^4]} \end{aligned} \tag{29}$$

For the concatenated schemes, to avoid mixing different suboptimalities of the communication details, we assume

$$R(T)=\log(1+T) \tag{30}$$

Consequently, the minimum energy cost and the wideband slope of the concatenated schemes can be obtained according to Lemma 1.

$$\begin{aligned} E_{N\min} &= \frac{1}{\alpha^{*}F(\alpha^{*})} \\ \mathcal{S}_0 &= 2F(\alpha^{*}) \end{aligned} \tag{31}$$

Suppose the channels are Rayleigh faded, i.e., $\forall i$,

$$f(|h_i|^2)=\exp(-|h_i|^2) \tag{32}$$

Consider the concatenated scheme with FEC[2]. The probability of communication success is given by

$$F(\alpha)=Pr(|h_i|^2\geq\alpha)=\exp(-\alpha) \tag{33}$$

Therefore, we obtain from (14) that

$$\alpha^{*}=\arg\max_{\alpha}\alpha\exp(-\alpha)=1 \tag{34}$$

The minimum energy cost and the wideband slope of the concatenated scheme with FEC are equal to, respectively,

$$\begin{aligned} E_{N\min} &= \frac{1}{\alpha^{*}F(\alpha^{*})}=e \\ \mathcal{S}_0 &= 2F(\alpha^{*})=\frac{2}{e} \end{aligned} \tag{35}$$

[2]We assume fountain code is used.

These parameters of the optimal scheme are given by

$$\begin{aligned} E_{N\ \min}^{\text{opt}} &= \frac{1}{E[|h_1|^2]} = 1 \\ \mathcal{S}_0^{\text{opt}} &= \frac{2E[|h_1|^2]^2}{E[|h_1|^4]} = 1 \end{aligned} \quad (36)$$

Figure 2 shows the spectral efficiency and energy cost tradeoff curves of the information theoretic optimal scheme and the concatenated scheme with FEC. We can see that not averaging out channel variation can introduce significant suboptimality in the low power regime.

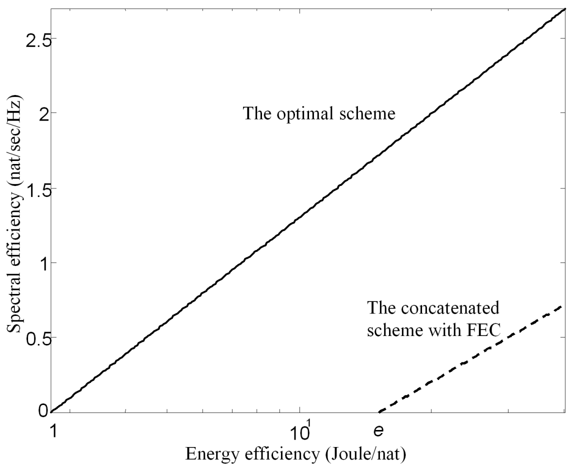

Fig. 2. Comparisons on the spectral efficiency and energy cost tradeoff of the optimal scheme and the concatenated scheme with FEC. 10 receiver multicast system with i.i.d. Rayleigh fading channels.

B. Asymptotic Behavior

Define $\frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}}$ as the normalized multicast rate. Figure 3 illustrates the normalized multicast rates of the concatenated schemes as functions of the transmit power. The FEC_{simp} and $\text{Retransmission}_{\text{simp}}$ curves are the normalized rates of the simplified versions, using rate control $T = \alpha^* P$, corresponding to the concatenated schemes with FEC and retransmission, respectively. We can clearly see that letting $T = \alpha^* P$ is not a good choice in the moderate and high power regimes. Although Lemma 2 promises the asymptotic optimality of the concatenated schemes, with a moderate transmit power, the suboptimality of the concatenated schemes can still be significant.

Define $R_{\text{multi}}^{\text{simp}}$ as the multicast rate of the simplified version. Since $R(T) = \log(1+T)$ and the channels are i.i.d., (23) holds with equality. Therefore, according to Lemmas 2 and 3, for the concatenated scheme with FEC, we have

$$\begin{aligned} \lim_{P\to 0} \frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}} &= \alpha^* F(\alpha^*) = \frac{1}{e} \approx 0.37 \\ \lim_{P\to\infty} \frac{R_{\text{multi}}^{\text{simp}}}{R_{\text{multi}}^{\text{opt}}} &= F(\alpha^*) = \frac{1}{e} \approx 0.37 \end{aligned} \quad (37)$$

For the concatenated scheme with retransmission, we have

$$\begin{aligned} \lim_{P\to 0} \frac{R_{\text{multi}}}{R_{\text{multi}}^{\text{opt}}} &= \alpha^* F(\alpha^*) \approx 0.15 \\ \lim_{P\to\infty} \frac{R_{\text{multi}}^{\text{simp}}}{R_{\text{multi}}^{\text{opt}}} &= F(\alpha^*) \approx 0.18 \end{aligned} \quad (38)$$

These asymptotic behaviors are approximately verified in Figure 3.

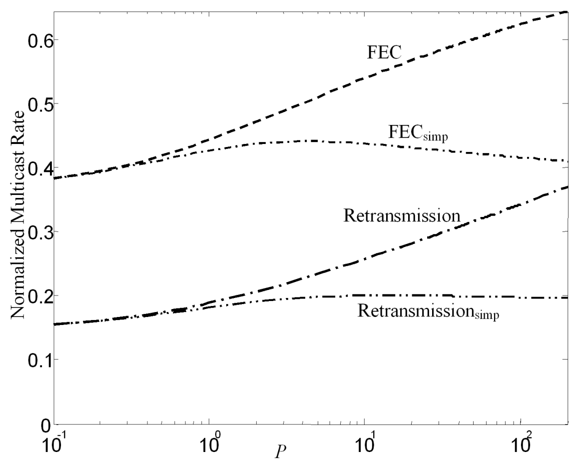

Fig. 3. Comparisons on normalized multicast rates of the concatenated schemes. 10 receiver multicast system with i.i.d. Rayleigh fading channels. FEC_{simp} and $\text{Retransmission}_{\text{simp}}$ are the corresponding concatenated schemes using rate control $T = \alpha^* P$.

C. The Impact of Channel Uncertainty

If the block length is long enough so that (30) holds true, then the inefficiency of the concatenated scheme with FEC comes only from not averaging out channel variation. It is easy seen that such inefficiency depends on the channel distribution. Intuitively, if the channel states are perfectly known, the multicast rates of the concatenated schemes should be close to the common information capacity. In order to understand the impact of the channel uncertainly to the normalized multicast rates of the concatenated schemes, in this section, instead of assuming Rayleigh fading, we assume the channel gains are i.i.d. and follow Nakagami-m distribution with $E[|h_i|^2] = 1$. In other words,

$$f(|h_i|^2) = m^m \frac{|h_i|^{2(m-1)}}{\Gamma(m)} \exp(-m|h_i|^2) \quad (39)$$

where

$$\Gamma(m) = \int_0^\infty x^{m-1} \exp(-x) dx \quad (40)$$

is the Gamma function. The reason we consider Nakagami fading is that, since $m = \frac{E[|h_i|^2]^2}{\text{var}[|h_i|^2]}$, the parameter m gives a measure of the relative uncertainty of the channel. For example, when $m = 1$, the channel is Rayleigh-faded, where not averaging out channel variation introduces significant rate loss as we saw in Figure 2. When $m \to \infty$ on the other hand, since $|h_i|^2$ will be close to 1 with high probability, it is expected that both concatenated schemes should be close to optimal even in the low power regime. The effect of large m can be achieved via the use of multiple antennas. If the transmitter has single antenna while each of the receivers have m receive antennas, assume the channel gain between any antenna pairs follow independent Rayleigh fading, after receiver beamforming, the effective channel gain, $|h_i|$, is χ^2 distributed with $2m$ degrees of freedom. The density function of $\frac{|h_i|^2}{E[|h_i|^2]}$ is then given by (39). Similar effect can also be achieved via the use of multiple transmit antennas.

For the information theoretic optimal scheme, since the multicast channel is degraded, the minimum energy cost of the multicast system is again given by

$$E_{N_{\min}}^{\text{opt}} = \lim_{P \to 0} \frac{P}{E\left[\log\left(1 + |h_1|^2 P\right)\right]} = \frac{1}{E[|h_1|^2]} = 1 \quad (41)$$

For the concatenated schemes, the probability of communication success is given by

$$\rho(\alpha) = Pr(|h_i|^2 \geq \alpha) = \sum_{i=0}^{m-1} m^i \frac{\alpha^i}{i!} \exp(-m\alpha) \quad (42)$$

Therefore, for the concatenated scheme with FEC, we have

$$F(\alpha) = \rho(\alpha) \quad (43)$$

For the concatenated scheme with retransmission, we have

$$F(\alpha) = \frac{1}{\sum_{k=1}^{K} (-1)^{k+1} \begin{pmatrix} K \\ k \end{pmatrix} \frac{1}{1-(1-\rho(\alpha))^k}} \quad (44)$$

Based on the fact that (23) in Lemma 2 holds with equality, the minimum energy costs of the two concatenated schemes are computed and illustrated in Figure 4. With $m = 4$ the concatenated scheme with FEC doubles the minimum energy cost of the optimal scheme, while the retransmission scheme doubles the energy cost one more time due to its multicast inefficiency.

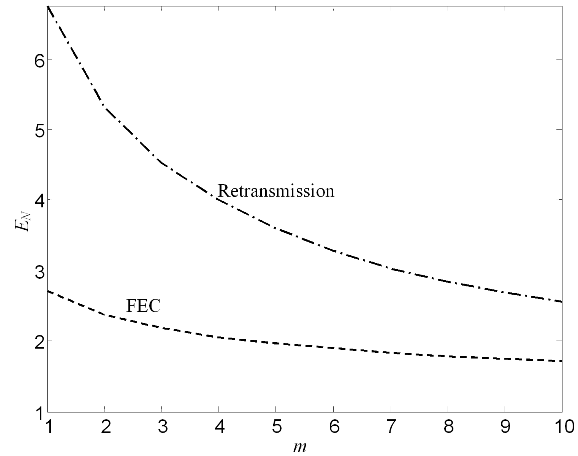

Fig. 4. Comparisons on the minimum energy costs of the concatenated schemes. 10 receiver multicast system with i.i.d. Nakagami-m fading channels.

D. General Discussions

Since the information theoretic optimal scheme is not always feasible (or even known) in practical systems, the concatenated schemes can be attractive alternatives in the sense that they require significantly less memory and computation. Because a multicast erasure channel is always degraded, the achievable communication rates of the concatenated schemes are theoretically tractable. The inefficiency of the concatenated schemes come from two parts. First, the error controlled reception quantizes the channel gain into binary values in the sense that it considers SINR to be either T or 0. Such quantization limits the system's capability of averaging out channel variation. Second, the retransmission brings further inefficiency to multicast systems, since when retransmitting an information block, the shared wireless channel is used to benefit only part of the receivers.

Disregard of the inefficiencies, the concatenated scheme with retransmission is widely used in wireless packet network systems. When feedback is not difficult to obtain, such concatenated scheme is easy to implement and has the key advantage of small transmission latency. Even though the retransmission method is inefficient for multicast communications, the inefficiency may not be as serious as one might expect. For the systems considered in Section IV, the i.i.d channel fading and the large number of multicast receivers extremely unfavors the retransmission method. However, with a proper rate control, even with 10 receivers, the retransmission mechanism only loses less than half (approximately) of the multicast rate on top of the concatenated scheme with FEC, as seen in Figures 3 and 4.

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