# Optimal Grouping and User Ordering for Sequential Group Detection in Synchronous CDMA

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*Abstract*— **The sequential group detection technique [6] is a generalization of the decision feedback detector: in the latter, users are successively demodulated and cancelled one-by-one, while in the former this basic operation is performed simultaneously on groups of users. The computational complexity of a Group Decision Feedback Detector (GDFD) is exponential in the largest size of the groups [6]; thus instead of using the partition of users as design parameters, choosing the "maximum group size" is more reasonable in practice. Given the maximum group size, a grouping algorithm is proposed. It is shown that the proposed grouping algorithm maximizes the Asymptotic Symmetric Energy (ASE) of the multiuser detection system. Furthermore, based on a set of lower bounds on Asymptotic Group Symmetric Energy (AGSE) of the GDFD, it is shown that the proposed grouping algorithm, in fact, maximizes the AGSE lower bound for every group of users. Together with a fast computational method based on branch-and-bound, the theoretical analysis of the grouping algorithm enables the offline estimation of the computational cost and the performance of GDFD. Simulation results are presented to verify the theoretical results.**

## I. Introduction

In synchronous Code Division Multi-Access (CDMA) communication systems, the near-far problem caused by interuser interference has been widely studied. When the source signal is binary- or integer-valued, the conventional linear decorrelator often fails to produce reliable decisions for the CDMA channel. The computation of the optimal decision, however, is generally NP-hard and thus is exponential in the number of users [2]. Several new algorithms have been proposed to provide reliable solutions with relatively low computational cost. Among the sub-optimal algorithms, the decision-driven detection methods, including decision feedback [5] [9], group detection [6], and multistage detection [3] [4], are popular. Although the Decision Feedback Detector (DFD) is simple and performs well, there are situations when a marginal increase in computation can provide significant improvement in performance [10].

The main drawback of DFD is that detections are made userwise; the decision on the strong user is obtained by treating the weak users as noise. However, when user chip sequences are correlated, this noise becomes biased, and thus is naturally harmful to the userwise detection. The idea of sequential group detection was first introduced by Varanasi in [6] and can be viewed as a group version of the decision feedback detection. GDFD first divides users into several groups. The users with relatively high correlations are assigned to the same group, and the correlations between users in different groups are relatively low. Similar to DFD, GDFD makes decisions sequentially based on successive cancelation. However, instead of making decisions userwise, GDFD makes decisions groupwise, i.e., the decisions on users in the same group (the correlated users) are made simultaneously. The computational expense for a GDFD is approximately exponential in the largest group size, and this is expected to be small if the largest group size is small.

In [6], the sizes of the groups are design parameters. However, in practice, given a user signal set, it is not easy for one to find the correlated users and assign them to groups. Since the largest group size is closely related to the overall computational cost, in this paper, we consider the largest group size as the only design parameter. A grouping and ordering algorithm is proposed to find the optimal size and users for each group. Theoretical results are given to show the optimality in terms of the ASE. Together with a fast computational method modified from [10], the proposed GDFD method provides a unifying and efficient way to improve the DFD with small extra computation.

The rest of the paper is organized as follows. In section II, we review the problem model and the theoretical results on the performance measure given in [6]. In section III, given the largest group size, a grouping and ordering algorithm is proposed to maximize the ASE of the system. Proof of optimality is given in the appendix. A fast computational method is proposed for the GDFD and a theoretical upper bound on computational cost is derived. Simulation results on a small example as well as on a system of 100 users are presented in section IV. Conclusions are provided in section V.

## II. Problem Formulation and Performance Measure of GDFD

A discrete-time equivalent model for the matched-filter outputs at the receiver of a CDMA channel is given by the $K$-length vector [2]

$$y = Hb + n \tag{1}$$

where $b \in \{-1, +1\}^K$ denotes the $K$-length vector of bits transmitted by the $K$ active users. Here $H = W^{\frac{1}{2}} R W^{\frac{1}{2}}$ is a nonnegative definite signature waveform correlation matrix, $R$ is the symmetric normalized correlation matrix with unit diagonal elements, $W$ is a diagonal matrix whose $k$-th

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diagonal element, $w_k$, is the received signal energy per bit of the $k$-th user, and $n$ is a real-valued zero-mean Gaussian random vector with a covariance matrix $\sigma^2 H$.

When all the user signals are equally probable, the optimal solution of (1) is the output of a Maximum Likelihood (ML) detector [2]

$$\phi_{ML} : \hat{b} = \arg \min_{b \in \{-1,+1\}^K} \left( b^T H b - 2y^T b \right) \tag{2}$$

The ML detector has the property that it minimizes, among all detectors, the probability that not all users' decisions are correct. Usually, $\phi_{ML}$ is considered NP-hard and exponentially complex to implement.

The idea of successive cancelation is that a correct decision on the strong users will improve the performance for weak users. In order to avoid an exponentially complex search among all users, it is intuitive to divide users into several groups, and to make decisions sequentially and groupwise. This was first introduced by Varanasi in [6]. Here we will present the GDFD in an alternative way since the results are to be used in later sections of the paper.

Suppose users are partitioned into an ordered set of $P$ groups, $G_0$, ..., $G_{P-1}$. The number of users in group $G_i$ is denoted by $|G_i|$. The decision on group $\{G_0\}$ is made by

$$\hat{b}_{G_0} = \arg \min_{b_{G_0} \in \{-1,+1\}^{|G_0|}} \left[ \min_{b_{\bar{G}_0}} \left( b^T H b - 2y^T b \right) \right] \tag{3}$$

where $b_{G_0}$ and $\hat{b}_{G_0}$ denote, respectively, the parts of vectors $b$ and $\hat{b}$ that correspond to users in group $G_0$, and $\bar{G}_0$ denotes the complement of $G_0$, i.e., the union of $G_1$, ..., $G_{P-1}$. The decisions of (3) are then used to subtract the multiple-access interference due to users in $G_0$ from the remaining decision statistics $y_{\bar{G}_0}$. The detector for the next group $G_1$ is designed under the assumption that the multiple-access interference cancelation is perfect. This process of interference cancelation and group detection is carried out sequentially for users in groups $G_2$, ..., $G_{P-1}$, with the group detector for group $G_i$ taking advantage of the decisions made by group detectors for $G_0$, ..., $G_{i-1}$. Denote the channel model for the user expurgated channel that only has users in groups $G_i$, ..., $G_{P-1}$ by

$$y^{(i)} = H^{(i)} b^{(i)} + n^{(i)} \tag{4}$$

The decisions on group $G_i$ can be represented as

$$\hat{b}_{G_i} = \arg \min_{b^{(i)}_{G_i} \in \{-1,+1\}^{|G_i|}} \left[ \min_{b^{(i)}_{\bar{G}_i}} \left( {b^{(i)}}^T H^{(i)} b^{(i)} - 2{y^{(i)}}^T b^{(i)} \right) \right] \tag{5}$$

The group detection procedure is illustrated in Figure 1.

In multi-user detection, the Asymptotic Symmetric Energy (ASE) is an important performance measure. Define the probability that not all users are detected correctly as $P(\sigma, \phi)$, then the ASE for the detector $\phi$ [9] is given by

$$\eta(\phi) = sup \left\{ e \geq 0; \lim_{\sigma \to 0} \frac{P(\sigma, \phi)}{Q\left(\frac{\sqrt{e}}{\sigma}\right)} < \infty \right\} \tag{6}$$

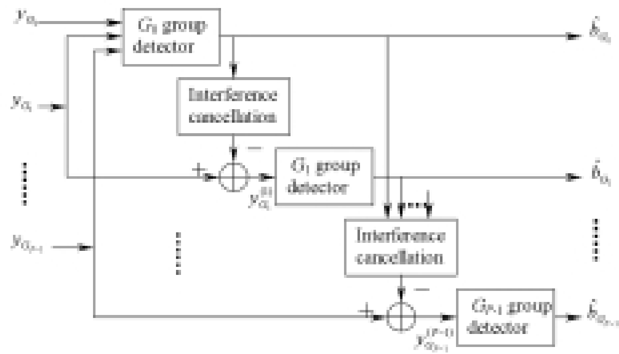


Fig. 1. Detection Procedure of GDFD

where $\sigma^2$ is the additive noise variance (see (1)), and $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$. The ASE for the optimal detector $\phi_{ML}$ is given by

$$\eta(\phi_{ML}) = d^2_{min} = \min_{e \in \{-1,0,1\}^K \setminus \{0\}^K} e^T H e \tag{7}$$

where "\" is set subtraction and $d_{min}$ is known as the minimum distance of matrix $H$ [8]. Similarly, we can define the Asymptotic Group Symmetric Energy (AGSE) for each user group. For a group detector, define the probability that not all users in group $\{G_i\}$ are detected correctly as $P_{G_i}(\sigma, \phi)$, and correspondingly we have

$$\eta_{G_i}(\phi) = sup \left\{ e \geq 0; \lim_{\sigma \to 0} \frac{P_{G_i}(\sigma, \phi)}{Q\left(\frac{\sqrt{e}}{\sigma}\right)} < \infty \right\} \tag{8}$$

as the AGSE for group $\{G_i\}$. Although an exact performance analysis of GDFD is intractable [6], one can obtain upper and lower bounds for the AGSE of all groups. In the above description of the GDFD, define $J^{(i)} = \left[ {H^{(i)}}^{-1} \right]$, and denote $J^{(i)}_{G_i G_i}$ to be the sub-matrix of $J^{(i)}$ that only contains the columns and rows corresponding to users in $G_i$. Define $d_{G_i,min}$ to be the minimum distance of matrix $\left( J^{(i)}_{G_i G_i} \right)^{-1}$, i.e.,

$$d^2_{G_i,min} = \min_{e \in \{-1,0,1\}^{|G_i|} \setminus \{0\}^{|G_i|}} e^T \left( J^{(i)}_{G_i G_i} \right)^{-1} e \tag{9}$$

Then the AGSE for group $G_i$ can be bounded by

$$\min(d^2_{G_0,min}, ..., d^2_{G_i,min}) \leq \eta_{G_i}(\phi) \leq d^2_{G_i,min} \tag{10}$$

A similar result can be found in [6]. The upper bound in (10) is reached when all decisions on the users in group $G_1$ through group $G_{i-1}$ are correct.

## III. Optimal Grouping and Ordering for GDFD

It is known that the performance of the decision-driven multi-user detector is significantly affected by the order of the users. In GDFD, different partitioning of the users and different detection orderings of the groups will result in different performances. Since finding the group decision (5) is generally NP hard, the computational cost for (5) is exponential in the group size $|G_i|$. Hence the overall computation for GDFD can be considered exponential in the maximum group size, which is defined by $|G|_{max} = \max(|G_0|, ..., |G_{P-1}|)$. In this section, we develop a grouping and ordering algorithm that maximizes the ASE of the GDFD given $|G|_{max}$ as a design parameter.

Denote the Cholesky decomposition of $H$ by $L^T L = H$, where $L$ is a lower triangular matrix. Multiply both sides of (1) by $(L^{-1})^T$ to obtain,

$$\tilde{y} = (L^{-1})^T y = Lb + \tilde{n} \tag{11}$$

Where $E[\tilde{n}\tilde{n}^T] = \sigma^2 I$. Partition the matrices and the vectors according to $G_0$ and $\bar{G}_0$ to obtain

$$\begin{bmatrix} \tilde{y}_{G_0} \\ \tilde{y}_{\bar{G}_0} \end{bmatrix} = \begin{bmatrix} L_{G_0G_0} & 0 \\ L_{\bar{G}_0G_0} & L_{\bar{G}_0\bar{G}_0} \end{bmatrix} \begin{bmatrix} b_{G_0} \\ b_{\bar{G}_0} \end{bmatrix} + \begin{bmatrix} \tilde{n}_{G_0} \\ \tilde{n}_{\bar{G}_0} \end{bmatrix} \tag{12}$$

The decision on group $G_0$ in (3) can be written as

$$\hat{b}_{G_0} = \arg \min_{b_{G_0} \in \{-1,+1\}^{|G_0|}} \|L_{G_0G_0} b_{G_0} - \tilde{y}_{G_0}\|_2^2 \tag{13}$$

Therefore, the AGSE of group $G_0$ is determined by the minimum distance of matrix $L^T_{G_0G_0} L_{G_0G_0}$. Since $H = L^T L$, we have

$$\begin{aligned} L^T_{G_0G_0} L_{G_0G_0} &= \left[(H^{-1})_{G_0G_0}\right]^{-1} = \left[J^{(0)}_{G_0G_0}\right]^{-1} \\ \eta_{G_0}(\phi_{GDFD}) &= d^2_{G_0,min} \end{aligned} \tag{14}$$

A similar result can be obtained for group $G_i$. In the description of GDFD in section II, if we let $H^{(i)} = {L^{(i)}}^T L^{(i)}$, then ${L^{(i)}}^T_{G_iG_i} {L^{(i)}}_{G_iG_i} = \left(J^{(i)}_{G_iG_i}\right)^{-1}$. Notice that ${L^{(i)}}_{G_iG_i} = L_{G_iG_i}$, we have

$$L^T_{G_iG_i} L_{G_iG_i} = \left(J^{(i)}_{G_iG_i}\right)^{-1} \tag{15}$$

The above result shows that $d_{G_i,min}$ is determined by the diagonal block-matrix $L_{G_iG_i}$ of $L$. Now, given all the decisions on group $G_0$ to group $G_{i-1}$ are correct, denote the probability that not all the users in group $G_i$ are detected correctly by $P_e(G_i|G_0, ..., G_{i-1}) \approx Q\left(\frac{d_{G_i,min}}{\sigma}\right)$. The probability that not all the $K$ users are detected correctly can be represented as

$$P(\sigma, \phi) = 1 - \prod_{i=0}^{P-1} \left[1 - Q\left(\frac{d_{G_i,min}}{\sigma}\right)\right] \tag{16}$$

Therefore, the ASE of GDFD is given by

$$\eta(\phi_{GDFD}) = \min(d^2_{G_0,min}, ..., d^2_{G_{P-1},min}) \tag{17}$$

Recall that the computational cost for a GDFD is exponential in the largest group size $|G|_{max}$. If $|G|_{max}$ is given as a design parameter, the problem is then to find an optimal partition and detection order that maximizes $\min(d^2_{G_0,min}, ..., d^2_{G_{P-1},min})$.

**Grouping and Order Algorithm :** Find the optimal grouping and detection order via the following steps.

*Step 1:* Partition the $K$ users into two groups $\{G_0\}$ and $\{\bar{G}_0\}$ with $|G_0| \le |G|_{max}$. Among these partitions ($\{G_0\}$ and $|G_0|$ are not fixed), select the one that maximizes $d_{G_0,min}$ (which is the minimum distance of matrix $\left[J^{(0)}_{G_0G_0}\right]^{-1}$).

*Step 2:* Partition the remaining $K - |G_0|$ users into two groups $G_1$ and $\bar{G}_1$ with $|G_1| \le |G|_{max}$. Among these partitions, select the one that maximizes $d_{G_1,min}$ (the minimum distance of matrix $\left[J^{(1)}_{G_1G_1}\right]^{-1}$).

*Step 3:* Continue this process until all the users are assigned to groups.

**Example 1 :** The algorithm is illustrated by the following 4-user example. Suppose the $H$ matrix is given by

$$H = \begin{bmatrix} 4.30 & 1.00 & 0.60 & 0.30 \\ 1.00 & 3.00 & 1.70 & 0.50 \\ 0.60 & 1.70 & 2.20 & 0.70 \\ 0.30 & 0.50 & 0.70 & 1.90 \end{bmatrix} \tag{18}$$

Assume that the desired maximum group size is $|G|_{max} = 2$. In step 1 of the algorithm, the possible choices for group $G_0$ and the resulting $d^2_{G_0,min}$ are shown in Table I. Therefore, the best choice for group $G_0$ is {user 0}. Then,

| User(s) | 0 | 1 | 2 | 3 | 0,1 |
|---|---|---|---|---|---|
| $d^2_{G_0,min}$ | 3.96 | 1.62 | 1.14 | 1.67 | 1.69 |
| User (s) | 0,2 | 0,3 | 1,2 | 1,3 | 2,3 |
| $d^2_{G_0,min}$ | 1.14 | 1.68 | 1.74 | 1.62 | 1.24 |

TABLE I

Choices for group $G_0$ and the corresponding $d^2_{G_0,min}$

for the user expurgated channel, we have

$$H^{(1)} = \begin{bmatrix} 3.00 & 1.70 & 0.50 \\ 1.70 & 2.20 & 0.70 \\ 0.50 & 0.70 & 1.90 \end{bmatrix} \tag{19}$$

The possible choices for group $G_1$ and the resulting $d_{G_1,min}$ are shown in Table II. We can see that the best choice for group $G_1$ is {user 1, user 2}. Naturally {user 3} will be the last group. The resulting ASE for this partitioning and ordering is $\eta = 1.78$.

| User(s) | 1 | 2 | 3 | 1,2 | 1,3 | 2,3 |
|---|---|---|---|---|---|---|
| $d^2_{G_1,min}$ | 1.69 | 1.14 | 1.68 | 1.78 | 1.68 | 1.24 |

TABLE II

Choices for group $G_1$ and the corresponding $d_{G_1,min}$

The above example has 4 users and $|G|_{max} = 2$. One may have posited that partitioning users into 2 groups with

2 users in each group is a good choice. For example, assign groups as $\{user0, user3\}$ and $\{user1, user2\}$. We get $\eta = 1.68 < 1.78$. Furthermore, instead of having one 2-user group, we now have two 2-user groups. This will also result in additional computation in detection (see the computational analysis at the end of this section).

**Proposition 1 :** The proposed grouping and ordering algorithm maximizes the ASE in (17).

See Appendix of [11] for the proof.

The proposed grouping and ordering algorithm is also optimal in the following sense.

**Proposition 2 :** The proposed grouping and ordering algorithm maximizes the performance lower bound in (10) for every group. In other words, suppose $G$ is the grouping and ordering result obtained from the proposed algorithm, and $G_k$ is one of the groups in $G$. Further suppose there is another group and detection sequence $\hat{G}$ with $\hat{G}_l$ being one of the groups in $\hat{G}$, and $\hat{G}_l = G_k$. Then the following result holds,

$$\min(d^2_{G_1,min}, ..., d^2_{G_k,min}) \geq \min(d^2_{\hat{G}_1,min}, ..., d^2_{\hat{G}_l,min}) \tag{20}$$

See Appendix of [11] for the proof.

In addition to the above 2 propositions, we can derive a fast computational method for GDFD, which is a modified version of the method proposed in [10].

Similar to (12), suppose that the decisions on groups $\{G_1\}$, ..., $\{G_{i-1}\}$ have already been made. From (5) in the user expurgated channel, partition the matrices by $G_i$ and $\bar{G}_i$, where $\bar{G}_i$ is the complement of $G_i$ in the user expurgated channel. We obtain the white noise model,

$$\begin{bmatrix} \tilde{y}^{(i)}_{G_i} \\ \tilde{y}^{(i)}_{\bar{G}_i} \end{bmatrix} = \begin{bmatrix} L^{(i)}_{G_iG_i} & 0 \\ L^{(i)}_{\bar{G}_iG_i} & L^{(i)}_{\bar{G}_i\bar{G}_i} \end{bmatrix} \begin{bmatrix} b^{(i)}_{G_i} \\ b^{(i)}_{\bar{G}_i} \end{bmatrix} + \begin{bmatrix} \tilde{n}^{(i)}_{G_i} \\ \tilde{n}^{(i)}_{\bar{G}_i} \end{bmatrix} \tag{21}$$

and, using the fact that $L^{(i)}_{G_iG_i} = L_{G_iG_i}$, the decision on group $G_i$ is made by

$$\hat{b}_{G_i} = \arg \min_{b^{(i)}_{G_i} \in \{-1,+1\}^{|G_i|}} \left\| L_{G_iG_i} b^{(i)}_{G_i} - \tilde{y}^{(i)}_{G_i} \right\|^2_2 \tag{22}$$

We propose the following steps for the group detection.

**Computational Method for GDFD :** Suppose the GDFD has $P$ groups, $G_0$, ..., $G_{P-1}$

*1)* Initialize $\tilde{y}^{(1)} = (L^{-1})^T y$, $L^{(1)} = L$. Let $i = 1$;

*2)* Partition $L^{(i)}$ as shown in (21) according to group $G_i$ and its complement $\bar{G}_i$. Find the decision on group $G_i$ by

$$\hat{b}_{G_i} = \arg \min_{b_{G_i} \in \{-1,+1\}^{|G_i|}} \left\| L_{G_iG_i} b_{G_i} - \tilde{y}^{(i)}_{G_i} \right\|^2_2 \tag{23}$$

*3)* Compute $\tilde{y}^{(i+1)}$ by

$$\tilde{y}^{(i+1)} = \tilde{y}^{(i)}_{\bar{G}_i} - L^{(i)}_{\bar{G}_iG_i} \hat{b}_{G_i} \tag{24}$$

Let

$$L^{(i+1)} = L^{(i)}_{\bar{G}_i\bar{G}_i} \tag{25}$$

*4)* Let $i = i + 1$. If $i < P$, go to step 2; otherwise, stop the computation.

The computational cost for step 1 is $\frac{K(K+1)}{2}$ multiplications and $\frac{K(K-1)}{2}$ additions. Assume the computational cost for step 2 can be bounded by

$$\text{“}\times\text{”} \leq M(|G_i|) \quad , \quad \text{“}+\text{”} \leq S(|G_i|) \tag{26}$$

where “$\times$” denotes the number of multiplications and “$+$” denotes the number of additions. In step 3, since $b$ can only take known discrete values, $Lb$ can be precomputed and stored. Thus, only $|G_i| \sum_{k=i+1}^{P-1} |G_k|$ additions are needed. Therefore, the overall computational cost is bounded by

$$\begin{aligned} \text{“}\times\text{”} &\leq \frac{K(K+1)}{2} + \sum_{k=0}^{P-1} [M(|G_k|)] \\ \text{“}+\text{”} &\leq \frac{K(K-1)}{2} + \sum_{k=0}^{P-1} \left[ S(|G_k|) + |G_k| \sum_{j=k+1}^{P-1} |G_j| \right] \end{aligned} \tag{27}$$

Furthermore, we recommend to use the depth-first branch-and-bound-based algorithm proposed in [11] for step 2 in the GDFD procedure.

## IV. Simulation Results

**Example 1 - continued :** In the previous 4-user example, $\eta(\phi_{GDFD}) = 1.78$. The ASE for optimal decorrelator-based DFD and the ML detector can be obtained from [9] as $\eta(\phi_{DFD}) = 1.69$ and $\eta(\phi_{ML}) = 1.8$. The simulation results are shown in Figure 2, which are consistent with the theoretical analysis.

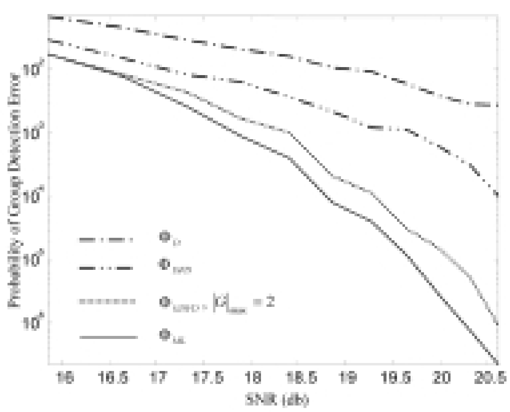


Fig. 2. Performance comparison (4 users, 10000 Monte-Carlo runs.)

**Example 2 :** In this 8-user example, we study the situation when a strict computational limit exists. The system transition matrix $H$ is randomly generated as

$$H = \begin{bmatrix} 2.0 & 0.9 & -2.0 & -0.3 & -0.3 & 0.3 & 0.8 & 1.7 \\ 0.9 & 3.7 & -0.4 & -1.1 & 0.4 & -0.4 & 0.3 & 1.4 \\ -2.0 & -0.4 & 3.3 & 1.0 & 1.1 & 0.3 & -0.3 & -1.3 \\ -0.3 & -1.1 & 1.0 & 3.0 & -1.1 & -0.3 & 0.3 & 1.2 \\ -0.3 & 0.4 & 1.1 & -1.0 & 3.4 & -0.3 & 0.3 & -0.4 \\ 0.3 & -0.4 & 0.3 & -0.3 & -0.3 & 2.7 & 0.9 & -1.2 \\ 0.8 & 0.3 & -0.3 & 0.3 & 0.3 & 0.9 & 2.6 & -0.4 \\ 1.7 & 1.4 & -1.3 & 1.2 & -0.4 & -1.2 & -0.4 & 4.5 \end{bmatrix} \tag{28}$$

Suppose the design parameter is given as $|G|_{max} = 3$. We obtained the optimal grouping and detection order as $\{\text{users } 1, 4, 3\}, \{\text{users } 7, 5\}, \{\text{user } 6\}, \{\text{users } 0, 2\}$. The computational upper bound for the number of multiplications using the GDFD is 54 multiplications and 59 additions. Now let the computational upper limit for the number of multiplications vary from 30 to 90. The performance of the $\phi_{GDFD}$ compared with the "any-time" sub-optimal algorithms (represented as $\phi_{BB}$) proposed in [10] is shown in Figure 3.

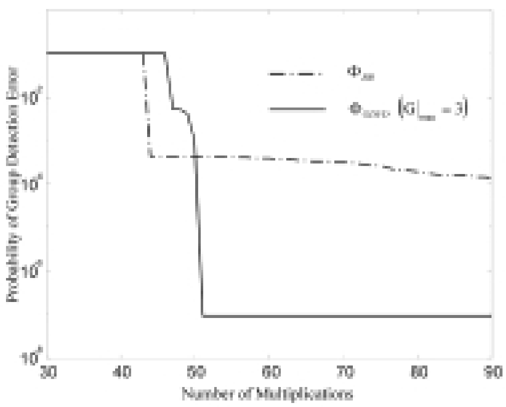


Fig. 3. Performance comparison of $\phi_{GDFD}$ and $\phi_{BB}$ with computational constraints. (8 users, 10000 Monte-Carlo runs, $SNR = 21.14db$.)

As shown in [10], $\phi_{BB}$ finds the first feasible solution with a low computational cost, and improves its performance when the upper limit of computational cost increases. However, in this example, the ASE for $\phi_{BB-1}$ is $\eta(\phi_{BB-1}) = \eta(\phi_{DDFD}) = 0.89$, and $\eta(\phi_{BB-7}) = 0.89$, $\eta(\phi_{BB-8}) = 1.14$. Therefore, the performance of $\phi_{BB}$ improves slowly. GDFD, although costs more in finding the first feasible solution, is evidently more efficient in this case.

**Example 3 :** In this final example, we randomly generated one thousand 10-user systems. Let the largest group size vary from 2 to 6. Figure 4 shows the average group size among all the randomly generated systems. It appears that the average group size is small ($\approx \frac{|G|_{max}}{2}$), thereby demonstrating that the GDFD can provide substantial improvement in performance with little additional computational cost.

## V. Conclusion

An optimal grouping and ordering algorithm for Group Decision Feedback Detector is proposed. Together with a fast computational method based on the idea of branch and bound, the proposed algorithm provides a systematic way of improving the Decision Feedback Detector, especially when correlation exists among the users. Simulation results show that GDFD with the optimal grouping and ordering algorithm provides a significant improvement over D-DFD, while the increase in computational cost is marginal and even negative in some cases. The proposed method can be easily extended to finite-alphabet signals instead of binary

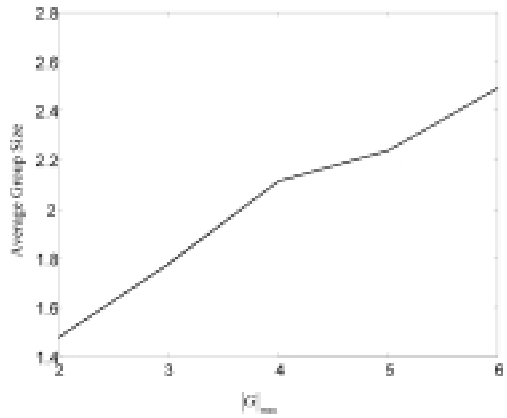


Fig. 4. Average group size for 10-user system. (Spreading factor = 12, 1000 Monte-Carlo runs)

ones. Our future research work will focus on extending the GDFD to asynchronous CDMA systems.

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