Generalized Channel Coding Theorems for Random Multiple Access Communication

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***Abstract*—This paper extends the channel coding theorems of [1][2] to time-slotted random multiple access communication systems with a generalized problem formulation. Assume that users choose their channel codes arbitrarily in each time slot. When the codeword length can be taken to infinity, fundamental performance limitation of the system is characterized using an achievable region defined in the space of channel code index vector each specifies the channel codes of all users. The receiver decodes the message if the code index vector happens to locate inside the achievable region and reports a collision if it falls outside the region. A generalized system error performance measure is defined as the maximum of weighted probabilities of different types of communication error events. Upper bounds on the generalized error performance measure are derived under the assumption of a finite codeword length. It is shown that "interfering users" can be introduced to model not only the impact of interference from remote transmitters, but also the impact of channel uncertainty in random access communication.**[1]

I. Introduction

Users in a distributed wireless network often have bursty short messages that must be disseminated in a timely manner over time-varying and non-stationary channels. Full user coordination such as joint channel coding in these cases can be infeasible or expensive in the sense of excessive overhead. Communication parameters, such as whether a transmitter will send a packet or not, can even be unknown to the corresponding receiver. Although coding redundancy is still needed to improve communication reliability, channel coding investigation for distributed and random communication systems requires a problem formulation that is quite different from the classical ones.

In [1], we proposed a new channel coding model for time-slotted random multiple access communication systems. We focused on communication and coding within one time-slot or one packet. Each transmitter is equipped with a randomly generated codebook that supports multiple communication rate options [1]. Communication rate of each transmitter is determined arbitrarily, with the rate information being shared neither among the transmitters nor with the receiver. An achievable rate region was defined in the following sense. As the codeword length is taken to infinity, if the communication rate vector, which contains the rates of all users, happens to locate inside the rate region, the receiver will decode the messages with a diminishing error probability, while if the rate vector falls outside the region, the receiver will report a packet collision with a probability approaching one. We showed that the achievable rate region coincides with the Shannon information rate region of the multiple access channel without a convex hull operation [1]. The asymptotic result was then extended in [2] to a rate and error probability tradeoff bound under the assumption of a finite codeword length.

Compared with a classical channel coding model, the system models of [1][2] extended the definition of "communication error" from its classical meaning of erroneous message decoding to the new meaning of failing to give the expected outcome, whose definition should be specified in the physical layer module. Such an extension enabled the relaxation of joint channel coding constraint and consequently established a bridge toward the development of rigorous coding theorems for distributed communication systems. Note that in the channel coding models of [1][2], a transmitter is provided with multiple transmission options that can correspond to different input distributions, which consequently imply different communication parameters. When the parameters of interest are not limited to the communication rate, it is easy to argue that using a rate region to characterize the system performance may no longer be appropriate. A distributed communication system has multiple types of communication error events, e.g., decoding error and collision miss detection error [2]. Using one probability variable to represent error performance in the tradeoff bound, as given in [2], cannot give the full tradeoff picture that may be needed for system design. Furthermore, system models of [1][2] assume that state of the wireless communication channel is known at the receiver. Such an assumption is reasonable in a classical system where message transmission is carried out continuously over a long time interval. With bursty short transmissions in a random access system, however, obtaining a precise estimate of the time-varying channel state at the receiver can be difficult.

In this paper, we extend the channel coding theory of [1][2] to address the concerns raised above. As in [1][2], we consider a time-slotted random multiple access system and focus on communication within one time slot. Each transmitter is equipped with a randomly generated codebook containing a finite number of codeword classes. Each codeword class is defined as a code which, if chosen, implies the values of a set of communication parameters that include but are not limited to the communication rate. Channel code and its implied communication parameters are determined arbitrarily and individually by the transmitters within the available choices. We consider an elementary decoder that is only interested in decoding the message of one user but can choose to decode other user messages if necessary. As shown in [1], performance of an

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elementary decoder can be used to derive the performance of a general receiver that is interested in decoding the messages of multiple users. As the codeword length (which is also the length of a time slot) is taken to infinity, we characterize the fundamental performance limitation of the system using an achievable region defined in the space of the code index vector, which specifies the coding choices of all users, in a sense similar to [1]. In the case of finite codeword length, we define a generalized error performance measure that assigns different exponential weights to different types of error events. Assume that the system chooses an operation region (defined in the paper) which is a subset of the achievable region. Performance bounds on the tradeoff between the operation region and the generalized error performance measure are derived. We also show that a class of "interfering users" can be introduced in the problem formulation to model not only the impact of remote interfering transmitters, but also the impact of channel uncertainty at the receiver. Furthermore, in the channel coding models introduced in [1][2] and in this paper, the receiver needs to consider both tasks of message decoding and collision detection. When the receiver is not interested in decoding the messages of all users, we show that the collision detection task requires a thoughtful specification on the expected system outcome (or equivalently the communication error events) corresponding to each of the code index vectors. Discussions on this issue and on channel code estimation are presented.

II. Problem Formulation

Consider a $(K+M)$-user random multiple access system over a symbol-synchronous discrete-time memoryless channel, where the users are indexed from 1 to $K+M$. Time is slotted with each slot equaling N symbol durations, which is also the length of a packet or a codeword. We assume that channel coding is applied only within each time slot. The channel is characterized by a conditional distribution $P_{Y|X_1,\cdots,X_{K+M}}$ where, for $k \in \{1,\cdots,K+M\}$, $X_k \in \mathcal{X}$ is the channel input symbol of user k with $\mathcal{X}$ being the finite input alphabet, and $Y \in \mathcal{Y}$ is the channel output symbol with $\mathcal{Y}$ being the finite output alphabet. Assume that at the beginning of a time slot, each user, say user k, chooses an arbitrary[2] channel code, specified by a code index parameter $g_k \in \mathcal{G}_k$, from a finite set $\mathcal{G}_k = \{g_{k1},\cdots,g_{k|\mathcal{G}_k|}\}$ with cardinality $|\mathcal{G}_k|$. The code index parameter is shared neither among the users nor with the receiver. Assume that g_k determines a communication rate parameter $r_k(g_k)$ for user k in nats per symbol. The user then encodes Nr_k data nats, denoted by a message w_k, into a packet (codeword) of N symbols, using a random coding scheme specified in the following. For all $k \in \{1,\cdots,K+M\}$, we assume that user k is equipped with a codebook library $\mathcal{L}_k = \{\mathcal{C}_{k\theta_k} : \theta_k \in \Theta_k\}$ in which codebooks are indexed by a set Θ_k. Each codebook has $|\mathcal{G}_k|$ classes of codewords, and each codeword class is termed a code. The i^{th} $(i \in \{1,\cdots,|\mathcal{G}_k|\})$ code has $\lfloor e^{Nr_{ki}} \rfloor$ codewords with the same length of N symbols, where r_{ki} is the communication rate corresponding to code g_{ki}. Note that in this coding scheme, each codeword in the codebook is mapped to a message and code index pair (w_k, g_k). Let $\mathcal{C}_{k\theta_k}(w_k, g_k)_j$ be the j^{th} symbol of the codeword corresponding to message and code index pair (w_k, g_k) in codebook $\mathcal{C}_{k\theta_k}$. User k first selects the codebook by generating θ_k according to a distribution ϑ_k such that random variables $X_{(w_k,g_k),j} : \theta_k \to \mathcal{C}_{k\theta_k}(w_k, g_k)_j$ are i.i.d. according to an input distribution $P_{X|g_k}$. The codebook $\mathcal{C}_{k\theta_k}$ is then used to map (w_k, g_k) into a codeword, denoted by $\boldsymbol{x}_{(w_k,g_k)}$.

We use a bold font vector variable to denote the corresponding variables of all users. For example, $\boldsymbol{w}$ and $\boldsymbol{g}$ denote the messages and the code indices of all users. $\boldsymbol{P}_{\boldsymbol{X}|\boldsymbol{g}}$ denote the input distributions of all users, etc. Given a vector variable $\boldsymbol{g}$, we use g_k to denote its element corresponding to user k. We use g_{ki} to denote a particular value in the alphabet of g_k. Let $\mathcal{S} \subset \{1,\cdots,K+M\}$ be a user subset, and $\bar{\mathcal{S}}$ be its complement. We use $\boldsymbol{g}_{\mathcal{S}}$ to denote the vector that is extracted from $\boldsymbol{g}$ with only elements corresponding to users in $\mathcal{S}$.

We categorize users with indices $\{1,\cdots,K\}$ as "regular users" and other users as "interfering users". For each regular user $k \in \{1,\cdots,K\}$, we assume that the receiver knows the randomly selected codebook $\mathcal{C}_{k\theta_k}$. Codebook information can be conveyed by sharing the random codebook generation algorithm with the receiver [1][2]. For each interfering user $k \in \{K+1,\cdots,K+M\}$, we assume that the receiver knows the set of input distributions $\{P_{X|g_k} | g_k \in G_k\}$, but not the codebook $\mathcal{C}_{k\theta_k}$. In other words, messages of the interfering users are not decodable at the receiver. There are two reasons why we include interfering users in the system model. First, for reasons such as decoding complexity constraint, the receiver may not have the capability to fully process the codebook information of all users. Regarding some of the users as interfering users still allows the receiver to take advantage of their input distribution information to improve coding performance. Second, interfering user can be used to model channel uncertainty at the receiver. For example, if the compound channel has $|\mathcal{G}|$ possible realizations, one can introduce an interfering user whose code index takes $|\mathcal{G}|$ possible values each corresponding to a channel realization. When the interfering user chooses a specific code index, which is unknown to the receiver, the conditional channel distribution is set to match the corresponding channel realization.

We consider an elementary decoder that is only interested in decoding the message of User 1, although the decoder can choose to decode the messages of some other regular users if such joint decoding is beneficial. As explained in [1], performance bound of an elementary decoder can be used to derive the corresponding bound of a general decoder that is interested in decoding messages from multiple users. Note that whether the message of User 1 can be decoded reliably or not may depend on the channel codes of all users. We assume that, before packet transmission, the receiver pre-determines an "operation region" $\mathcal{R}$, which is a set of code index vectors. Determination of the operation region $\mathcal{R}$ depends on the performance objective of the receiver. Let $\boldsymbol{g}$ be the actual code index vector with the corresponding rate vector being $\boldsymbol{r}$. We assume that the receiver *intends* to decode the message of User 1 if $\boldsymbol{g} \in \mathcal{R}$. The receiver *intends* to report a collision for User 1 if $\boldsymbol{g} \notin \mathcal{R}$. Note that $\boldsymbol{g}$ is unknown at the receiver. In each time slot, upon receiving the channel output symbols $\boldsymbol{y}$, the receiver estimates the code index vector, denoted by $\hat{\boldsymbol{g}}$. The receiver outputs the estimated message and code index of User 1, denoted by $(\hat{w}_1, \hat{g}_1)$, if $\hat{\boldsymbol{g}} \in \mathcal{R}$ and a pre-determined decoding error probability requirement is satisfied. Otherwise,

[2]Here "arbitrary" means the parameter is determined randomly with its statistical information possibly unavailable at the physical layer.

the receiver reports a collision for User 1.

Given the operation region $\mathcal{R}$, and conditioned on $\boldsymbol{g}$ and $\boldsymbol{w}$ being the actual code index and message vectors, communication error probability as a function of $\boldsymbol{g}$ is defined as follows.

$$P_e(\boldsymbol{g}) = \left\{ \begin{array}{l} \max_{\boldsymbol{w}} Pr\left\{(\hat{w}_1, \hat{g}_1) \neq (w_1, g_1) | (\boldsymbol{w}, \boldsymbol{g})\right\}, \forall \boldsymbol{g} \in \mathcal{R} \\ \max_{\boldsymbol{w}} 1 - Pr\left\{\text{“collision” or} \right. \\ \qquad \left. (\hat{w}_1, \hat{g}_1) = (w_1, g_1) | (\boldsymbol{w}, \boldsymbol{g})\right\} \quad \forall \boldsymbol{g} \notin \mathcal{R} \end{array} \right. . \tag{1}$$

Note that, according to (1) when $\boldsymbol{g} \notin \mathcal{R}$, although we expect that the receiver should output a collision, we do not regard correct message decoding as a communication error. Such a definition is chosen based on the assumption that the primary objective of the decoder is to guarantee the reliability of its message output. In other words, whether code indices of the other users are correctly detected or not is of no interest to the elementary decoder. We will maintain this communication error definition in Sections II, III when deriving the basic coding results, and then discuss its extension in Section IV.

We define the system error probability as $P_{es} = \max_{\boldsymbol{g}} P_e(\boldsymbol{g})$. Furthermore, let $0 < \alpha(\boldsymbol{g}) \leq 1$ be an arbitrary function of $\boldsymbol{g}$, we define “generalized error performance” of the system as

$$\text{GEP}(\alpha) = \max_{\boldsymbol{g}} P_e(\boldsymbol{g})^{\alpha(\boldsymbol{g})}. \tag{2}$$

A generalized error performance measure allows the system to assign different exponential weights to different types of communication error events (corresponding to different code index vectors). We can also get $\text{GEP}(\alpha) = P_{es}$ by choosing $\alpha(\boldsymbol{g}) \equiv 1$.

III. Basic Channel Coding Theorems

Given a random multiple access system described in Section II. Let us fix the coding parameters[3] that are not functions of the codeword length. We say that an operation region is achievable if there exits a set of decoding algorithms whose system error probability converges to zero as the codeword length is taken to infinity, i.e., $\lim_{N\to\infty} P_{es} = 0$. The following theorem gives an achievable region for the random multiple access system with an elementary decoder.

Theorem 1: Consider a $(K+M)$-user random multiple access system described in Section II. Let $\boldsymbol{r}$ be the communication rate vector corresponding to code index vector $\boldsymbol{g}$, and $r_k(g_k)$ be the element of $\boldsymbol{r}$ corresponding to user k. The following region defined in the space of $\boldsymbol{g}$ is achievable.

$$\mathcal{R} = \left\{ \boldsymbol{g} \left| \begin{array}{l} \forall \mathcal{S} \subseteq \{1, \dots, K\}, 1 \in \mathcal{S}, \exists \tilde{\mathcal{S}} \subseteq \mathcal{S}, 1 \in \tilde{\mathcal{S}}, \\ \text{such that, } \sum_{k \in \tilde{\mathcal{S}}} r_k(g_k) \\ \quad < I_{\boldsymbol{g}}(\boldsymbol{X}_{k \in \tilde{\mathcal{S}}}; Y | \boldsymbol{X}_{k \in \{1, \cdots, K\} \setminus \mathcal{S}}) \end{array} \right. \right\}, \tag{3}$$

where the mutual information $I_{\boldsymbol{g}}(\boldsymbol{X}_{k \in \tilde{\mathcal{S}}}; Y | \boldsymbol{X}_{k \in \{1, \cdots, K\} \setminus \mathcal{S}})$ is computed using input distribution $\boldsymbol{P}_{\boldsymbol{X}|\boldsymbol{g}}$. ■

Theorem 1 can be proved by following the proof of [1, Theorem 3] with only minor revisions. Note that, although the interfering users do not show up explicitly in the expression of $\mathcal{R}$, their code indices do affect the conditional channel distribution and the mutual information terms in (3).

[3] Such as rate functions, alphabets of code indices and input distributions.

Because both collision report and correct message decoding are included in the set of expected outcomes for $\boldsymbol{g} \notin \mathcal{R}$, the following theorem follows immediately from the achievable region definition.

Theorem 2: For the random multiple access system considered in Theorem 1, any subset of an achievable region is also achievable.

Next, we will consider the case when the codeword length is finite. As shown in [1], depending on the actual code index vector, which is unknown to the receiver, the receiver may need to jointly decode the messages of multiple users in order to recover the message of User 1. Therefore, we will first need to analyze the performance of a “$(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder” that targets at decoding the messages of a particular user subset specified by $\mathcal{D} \subseteq \{1, \cdots, K\}$. Let $\mathcal{R}_{\mathcal{D}}$ be the operation region of the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder. When the code index vector $\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}$ is inside the operation region, the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder intends to decode the messages of all users *and only the users* in $\mathcal{D}$. When the code index vector $\boldsymbol{g} \notin \mathcal{R}_{\mathcal{D}}$ is outside the operation region, the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder intends to report collision for all users in $\mathcal{D}$. Let $\boldsymbol{g}$ be the actual code index vector. Let $(\hat{\boldsymbol{w}}_{\mathcal{D}}, \hat{\boldsymbol{g}}_{\mathcal{D}})$ be the decoding output. Error probability of the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder is defined as

$$P_{e\mathcal{D}}(\boldsymbol{g}) = \left\{ \begin{array}{l} \max_{\boldsymbol{w}} Pr\left\{(\hat{\boldsymbol{w}}_{\mathcal{D}}, \hat{\boldsymbol{g}}_{\mathcal{D}}) \neq (\boldsymbol{w}_{\mathcal{D}}, \boldsymbol{g}_{\mathcal{D}}) | (\boldsymbol{w}, \boldsymbol{g})\right\}, \\ \qquad \forall \boldsymbol{g} \in \mathcal{R}_{\mathcal{D}} \\ \max_{\boldsymbol{w}} 1 - Pr\left\{\text{“collision” or} \right. \\ \left. (\hat{\boldsymbol{w}}_{\mathcal{D}}, \hat{\boldsymbol{g}}_{\mathcal{D}}) = (\boldsymbol{w}_{\mathcal{D}}, \boldsymbol{g}_{\mathcal{D}}) | (\boldsymbol{w}, \boldsymbol{g})\right\} \quad \forall \boldsymbol{g} \notin \mathcal{R}_{\mathcal{D}} \end{array} \right. . \tag{4}$$

Similar to the definition in (1), when $\boldsymbol{g} \notin \mathcal{R}_{\mathcal{D}}$, we do not regard correct message decoding as an error event, even though we expect that the receiver should report a collision.

Given $0 < \alpha(\boldsymbol{g}) \leq 1$ as an arbitrary function of $\boldsymbol{g}$, the generalized error performance is defined by

$$\text{GEP}_{\mathcal{D}}(\alpha) = \max_{\boldsymbol{g}} P_{e\mathcal{D}}(\boldsymbol{g})^{\alpha(\boldsymbol{g})}. \tag{5}$$

The following theorem gives an upper bound on the achievable generalized error performance of a $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder.

Theorem 3: Consider a $(K+M)$-user random multiple access system described in Section II. There exists a decoding algorithm for the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder, such that

$$\begin{aligned} \text{GEP}_{\mathcal{D}}(\alpha) \leq \max \Bigg\{ & \max_{\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}} \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} \neq \emptyset}} \Bigg[\sum_{\substack{\tilde{\boldsymbol{g}} \in \mathcal{R}_{\mathcal{D}}, \\ \tilde{\boldsymbol{g}}_{\mathcal{S}} = \boldsymbol{g}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}) E_{m\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \tilde{\boldsymbol{g}})\} \\ & + \max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \boldsymbol{g}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}') E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\} \Bigg], \\ & \max_{\tilde{\boldsymbol{g}} \notin \mathcal{R}_{\mathcal{D}}} \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} \neq \emptyset}} \sum_{\substack{\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}') E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\} \Bigg\}. \end{aligned} \tag{6}$$

$E_{m\mathcal{D}}(\mathcal{S},\boldsymbol{g},\tilde{\boldsymbol{g}})$ and $E_{i\mathcal{D}}(\mathcal{S},\boldsymbol{g},\boldsymbol{g}')$ in the above equation are given by,

$$E_{m\mathcal{D}}(\mathcal{S},\boldsymbol{g},\tilde{\boldsymbol{g}}) = \max_{0<\rho\le 1} -\rho \sum_{k\in\mathcal{D}\setminus\mathcal{S}} \tilde{r}_k(\tilde{g}_k) + \max_{0<s\le 1} -\log \sum_{Y}\sum_{\boldsymbol{X}_{\mathcal{S}}}\prod_{k\in\mathcal{S}\cap\mathcal{D}} P_{X|g_k}(X_k) \times \left(\sum_{\boldsymbol{X}_{\mathcal{D}\setminus\mathcal{S}}}\prod_{k\in\mathcal{D}\setminus\mathcal{S}} P_{X|g_k}(X_k)P(Y|\boldsymbol{X}_{\mathcal{D}},\boldsymbol{g}_{\bar{\mathcal{D}}})^{1-s}\right) \times \left(\sum_{\boldsymbol{X}_{\mathcal{D}\setminus\mathcal{S}}}\prod_{k\in\mathcal{D}\setminus\mathcal{S}} P_{X|\tilde{g}_k}(X_k)P(Y|\boldsymbol{X}_{\mathcal{D}},\tilde{\boldsymbol{g}}_{\bar{\mathcal{D}}})^{\frac{s}{\rho}}\right)^{\rho},$$

$$E_{i\mathcal{D}}(\mathcal{S},\boldsymbol{g},\boldsymbol{g}') = \max_{0<\rho\le 1}\max_{0<s\le 1-\rho} \frac{\alpha(\boldsymbol{g})}{s[\alpha(\boldsymbol{g}')-\alpha(\boldsymbol{g})]+\alpha(\boldsymbol{g})} \times \left\{ -\rho\sum_{k\in\mathcal{D}\setminus\mathcal{S}} r_k(g_k) - \log\sum_{Y}\sum_{\boldsymbol{X}_{\mathcal{S}}}\prod_{k\in\mathcal{S}\cap\mathcal{D}} P_{X|g_k}(X_k) \times \left(\sum_{\boldsymbol{X}_{\mathcal{D}\setminus\mathcal{S}}}\prod_{k\in\mathcal{D}\setminus\mathcal{S}} P_{X|g_k}(X_k)P(Y|\boldsymbol{X}_{\mathcal{D}},\boldsymbol{g}_{\bar{\mathcal{D}}})^{\frac{s}{s+\rho}}\right)^{s+\rho} \times \left(\sum_{\boldsymbol{X}_{\mathcal{D}\setminus\mathcal{S}}}\prod_{k\in\mathcal{D}\setminus\mathcal{S}} P_{X|g'_k}(X_k)P(Y|\boldsymbol{X}_{\mathcal{D}},\boldsymbol{g}'_{\bar{\mathcal{D}}})\right)^{1-s} \right\}, \quad (7)$$

where $r_k(g_k)$, $\tilde{r}_k(\tilde{g}_k)$ are the communications rates corresponding respectively to g_k and $\tilde{g}_k$, and $P(Y|\boldsymbol{X}_{\mathcal{D}},\boldsymbol{g}_{\bar{\mathcal{D}}})$ is defined as

$$P(Y|\boldsymbol{X}_{\mathcal{D}},\boldsymbol{g}_{\bar{\mathcal{D}}}) = \sum_{\boldsymbol{X}_{\bar{\mathcal{D}}}}\prod_{k\in\bar{\mathcal{D}}} P_{X|g_k}(X_k)P_{Y|\boldsymbol{X}}(Y|\boldsymbol{X}). \quad (8)$$

■

The proof of Theorem 3 is given in [3, Appendix A].

Let us now come back to the system with an elementary decoder that is only interested in decoding the message of User 1 but can choose to decode the messages of other regular users if necessary. Assume that the elementary decoder is composed of many $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoders, each corresponds to a user subset $\mathcal{D}\subseteq\{1,\cdots,K\}$ with $1\in\mathcal{D}$ and an operation region $\mathcal{R}_{\mathcal{D}}$. After receiving the channel output symbols, the elementary decoder first carries out all the $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoding operations. If at least one $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoder outputs an estimated message and code index pair, and the estimation outputs (i.e., not including the collision reports) of all the $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoders agree with each other, then the receiver outputs the corresponding estimate $(\hat{w}_1,\hat{g}_1)$ for User 1. Otherwise, the receiver reports a collision for User 1.

Let $\mathcal{R}$ be the operation region of the elementary decoder. Since the decoder intends to decode the message of User 1 if $\boldsymbol{g}\in\mathcal{R}$, we must have $\mathcal{R}\subseteq\bigcup_{\mathcal{D}:\mathcal{D}\subseteq\{1,\cdots,K\},1\in\mathcal{D}}\mathcal{R}_{\mathcal{D}}$. On the other hand, for a given $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoder, since we do not regard correct message decoding as a communication error event for $\boldsymbol{g}\notin\mathcal{R}_{\mathcal{D}}$, shrinking the operation region of a $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoder will not hurt its generalized error performance. Consequently, it does not cause any performance degradation to assume that the operation regions of the $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoders form a partitioning of $\mathcal{R}$. In other words,

$$\mathcal{R} = \bigcup_{\mathcal{D}:\mathcal{D}\subseteq\{1,\cdots,K\},1\in\mathcal{D}} \mathcal{R}_{\mathcal{D}}, \qquad \mathcal{R}_{\mathcal{D}'}\cap\mathcal{R}_{\mathcal{D}}=\emptyset, \quad \forall\mathcal{D},\mathcal{D}'\subseteq\{1,\cdots,K\},\mathcal{D}'\neq\mathcal{D},1\in\mathcal{D},\mathcal{D}'. \quad (9)$$

The following theorem gives an upper bound on the achievable generalized error performance of the elementary decoder.

Theorem 4: Consider a $(K+M)$-user random multiple access system described in Section II. Assume that the receiver chooses an operation region $\mathcal{R}$. Let σ denote a partitioning of the operation region $\mathcal{R}$ satisfying (9). There exists a decoding algorithm such that the generalized error performance of the elementary decoder with $0<\alpha(\boldsymbol{g})\le 1$ is upper-bounded by,

$$\text{GEP}(\alpha) \le \min_{\sigma} \sum_{\mathcal{D}:\mathcal{D}\subseteq\{1,\cdots,K\},1\in\mathcal{D}} \text{GEP}_{\mathcal{D}}(\alpha), \quad (10)$$

where $\text{GEP}_{\mathcal{D}}(\alpha)$ is the generalized error performance of the $(\mathcal{D},\mathcal{R}_{\mathcal{D}})$-decoder, which can be further bounded by (6). ■

Theorem 4 is implied by Theorem 3.

IV. Collision Detection and Operation Margin

In the communication error definition specified in Section II, given an operation region $\mathcal{R}$, we do not regard correct message decoding as an error event for $\boldsymbol{g}\notin\mathcal{R}$. Consequently, even if the receiver decodes the message of User 1, it still cannot conclude with high probability that the actual code index vector is inside the operation region. In this section, we present extended coding theorems to support a stricter requirement on collision detection, as it is an important function for communication adaptation in the upper layer.

Let us assume that, in addition to choosing the operation region $\mathcal{R}$, the receiver chooses another region $\widehat{\mathcal{R}}$, termed the "operation margin", that is non-overlapping with the operation region, i.e., $\mathcal{R}\cap\widehat{\mathcal{R}}=\emptyset$. The elementary decoder intends to decode the message of User 1 for $\boldsymbol{g}\in\mathcal{R}$, and to report a collision for $\boldsymbol{g}\notin\mathcal{R}\cup\widehat{\mathcal{R}}$. While for $\boldsymbol{g}\in\widehat{\mathcal{R}}$, both correct message decoding and collision report are accepted as expected outcomes. The purpose of introducing the operation margin is to create a buffer zone between the operation region $\mathcal{R}$, where correct message decoding should be enforced, and the region $\overline{\mathcal{R}\cup\widehat{\mathcal{R}}}$, where collision report should be enforced. Providing the receiver with the option of moving some of the code index vectors into the operation margin $\widehat{\mathcal{R}}$ can help to avoid the ill-posed collision detection problem illustrated in [3]. Note that the revised system model is an extension to the one considered in Sections II and III since the latter can be viewed as choosing $\widehat{\mathcal{R}}$ as the compliment of $\mathcal{R}$, i.e., $\widehat{\mathcal{R}}=\bar{\mathcal{R}}$.

Given $\mathcal{R}$ and $\widehat{\mathcal{R}}$, communication error probability as a function of $\boldsymbol{g}$ is given by

$$P_e(\boldsymbol{g}) = \begin{cases} \max_{\boldsymbol{w}} Pr\left\{(\hat{w}_1,\hat{g}_1)\neq(w_1,g_1)|(\boldsymbol{w},\boldsymbol{g})\right\}, \forall\boldsymbol{g}\in\mathcal{R} \\ \max_{\boldsymbol{w}} 1-Pr\left\{\text{“collision” or } (\hat{w}_1,\hat{g}_1)=(w_1,g_1)|(\boldsymbol{w},\boldsymbol{g})\right\}, \quad \forall\boldsymbol{g}\in\widehat{\mathcal{R}} \\ \max_{\boldsymbol{w}} 1-Pr\left\{\text{“collision”}\right\} \quad \forall\boldsymbol{g}\notin\mathcal{R}\cup\widehat{\mathcal{R}} \end{cases}. \quad (11)$$

Define the system error probability and the generalized error performance measure as in (2). Let us fix the communication parameters that are not functions of the codeword length. We say an operation region and operation margin pair $(\mathcal{R}, \widehat{\mathcal{R}})$ is achievable if there exits a set of decoding algorithms whose system error probability converges to zero as the codeword length is taken to infinity. The following theorem is an extension of Theorem 1 for the revised system model.

Theorem 5: Consider a $(K+M)$-user random multiple access system with the revised system model described in Section IV. Let the operation region $\mathcal{R}$ be given by (3). Any operation region and operation margin pair $(\mathcal{R}, \widehat{\mathcal{R}})$ with an arbitrary choice of $\widehat{\mathcal{R}}$ is achievable. ■

Theorem 5 can be proved by following the same proof of Theorem 1.

Similar to Theorem 2, the following theorem is implied directly by the achievable region definition.

Theorem 6: For the random multiple access system considered in Theorem 5, if an operation region and operation margin pair $(\mathcal{R}, \widehat{\mathcal{R}})$ is achievable, then any other operation region and operation margin pair $(\mathcal{R}_1, \widehat{\mathcal{R}}_1)$ that satisfies $\mathcal{R}_1 \subseteq \mathcal{R}$ and $\mathcal{R}_1 \cup \widehat{\mathcal{R}}_1 \supseteq \mathcal{R} \cup \widehat{\mathcal{R}}$ is also achievable. ■

When the codeword length is finite, given the operation region $\mathcal{R}$ and the operation margin $\widehat{\mathcal{R}}$ of the elementary decoder, we again decompose the decoder into a set of "$(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoders" for all $\mathcal{D} \subseteq \{1, \cdots, K\}$ with $1 \in \mathcal{D}$. For each $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder, we denote its operation region by $\mathcal{R}_{\mathcal{D}}$ and set its operation margin as $\widehat{\mathcal{R}}_{\mathcal{D}} = (\mathcal{R} \cup \widehat{\mathcal{R}}) \setminus \mathcal{R}_{\mathcal{D}}$. Decoding procedure of the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder is the same as described in Section III, with the communication error probability being defined as,

$$P_{e\mathcal{D}}(\boldsymbol{g}) = \begin{cases} \max_{\boldsymbol{w}} Pr\left\{(\hat{\boldsymbol{w}}_{\mathcal{D}}, \hat{\boldsymbol{g}}_{\mathcal{D}}) \neq (\boldsymbol{w}_{\mathcal{D}}, \boldsymbol{g}_{\mathcal{D}}) | (\boldsymbol{w}, \boldsymbol{g})\right\}, & \forall \boldsymbol{g} \in \mathcal{R}_{\mathcal{D}} \\ \max_{\boldsymbol{w}} 1 - Pr\left\{\text{“collision” or } (\hat{\boldsymbol{w}}_{\mathcal{D}}, \hat{\boldsymbol{g}}_{\mathcal{D}}) = (\boldsymbol{w}_{\mathcal{D}}, \boldsymbol{g}_{\mathcal{D}}) | (\boldsymbol{w}, \boldsymbol{g})\right\} & \forall \boldsymbol{g} \in \widehat{\mathcal{R}}_{\mathcal{D}} \\ \max_{\boldsymbol{w}} 1 - Pr\left\{\text{“collision”}\right\} & \forall \boldsymbol{g} \notin \mathcal{R}_{\mathcal{D}} \cup \widehat{\mathcal{R}}_{\mathcal{D}} \end{cases} \quad (12)$$

The following theorem gives an upper bound on the achievable generalized error performance of a $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder.

Theorem 7: Consider a $(K+M)$-user random multiple access system with a $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder whose operation region and operation margin are denoted by $\mathcal{R}_{\mathcal{D}}$ and $\widehat{\mathcal{R}}_{\mathcal{D}}$ respectively. There exists a decoding algorithm to achieve the following generalized error performance bound.

$$\begin{aligned} \text{GEP}_{\mathcal{D}}(\alpha) \leq \max \Bigg\{ & \max_{\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}} \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} \neq \emptyset}} \Bigg[\sum_{\substack{\tilde{\boldsymbol{g}} \in \mathcal{R}_{\mathcal{D}}, \\ \tilde{\boldsymbol{g}}_{\mathcal{S}} = \boldsymbol{g}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}) E_{m\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \tilde{\boldsymbol{g}})\} \\ & + \max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \boldsymbol{g}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}') E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\} \Bigg] \\ & + \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} = \emptyset}} \Bigg[\max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}} \cup \widehat{\mathcal{R}}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \boldsymbol{g}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}') \tilde{E}_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\} \Bigg], \\ & \max_{\tilde{\boldsymbol{g}} \notin \mathcal{R}_{\mathcal{D}}} \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} \neq \emptyset}} \sum_{\substack{\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \exp\left\{-N\alpha(\boldsymbol{g}') E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\right\}, \\ & \max_{\tilde{\boldsymbol{g}} \notin \mathcal{R}_{\mathcal{D}} \cup \widehat{\mathcal{R}}_{\mathcal{D}}} \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} \neq \emptyset}} \sum_{\substack{\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}') E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\} \\ & + \sum_{\substack{\mathcal{S} \subset \{1, \cdots, K+M\} \\ \mathcal{D} \setminus \mathcal{S} = \emptyset}} \sum_{\substack{\boldsymbol{g} \in \mathcal{R}_{\mathcal{D}}, \\ \boldsymbol{g}_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \max_{\substack{\boldsymbol{g}' \notin \mathcal{R}_{\mathcal{D}} \cup \widehat{\mathcal{R}}_{\mathcal{D}}, \\ \boldsymbol{g}'_{\mathcal{S}} = \tilde{\boldsymbol{g}}_{\mathcal{S}}}} \exp\{-N\alpha(\boldsymbol{g}') \tilde{E}_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')\} \Bigg\} \quad (13) \end{aligned}$$

$E_{m\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \tilde{\boldsymbol{g}})$ and $E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')$ in the above equation are given by (7). $\tilde{E}_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')$ is given by

$$\begin{aligned} \tilde{E}_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}') = & \max_{0<s\leq 1} \frac{\alpha(\boldsymbol{g})}{s[\alpha(\boldsymbol{g}') - \alpha(\boldsymbol{g})] + \alpha(\boldsymbol{g})} \\ & \times \Bigg\{ -\log \sum_{Y} \sum_{\boldsymbol{X}_{\mathcal{D}}} \prod_{k\in\mathcal{D}} P_{X|g_k}(X_k) P(Y|\boldsymbol{X}_{\mathcal{D}}, \boldsymbol{g}_{\bar{\mathcal{D}}})^s \\ & \times P(Y|\boldsymbol{X}_{\mathcal{D}}, \boldsymbol{g}'_{\bar{\mathcal{D}}})^{1-s} \Bigg\}, \quad (14) \end{aligned}$$

where $r_k(g_k)$ is the communications rates corresponding g_k, and $P(Y|\boldsymbol{X}_{\mathcal{D}}, \boldsymbol{g}_{\bar{\mathcal{D}}})$ is defined as in (8). ■

The proof of Theorem 7 is given in [3, Appendix B].

With Theorem 7, a performance bound of the elementary decoder can be derived in a way similar to that in Section III. Let the operation region and the operation margin of the elementary decoder be given by $\mathcal{R}$ and $\widehat{\mathcal{R}}$. Assume that the elementary decoder is composed of many $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoders, each corresponds to a user subset $\mathcal{D} \subseteq \{1, \cdots, K\}$ with $1 \in \mathcal{D}$. Given the operation region $\mathcal{R}_{\mathcal{D}}$ of an $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoder, we set its operation margin as $\widehat{\mathcal{R}}_{\mathcal{D}} = (\mathcal{R} \cup \widehat{\mathcal{R}}) \setminus \mathcal{R}_{\mathcal{D}}$. By following the same decoding algorithm and the same discussion as presented in Section III, we can see that it does not cause any performance degradation to let the operation regions of the $(\mathcal{D}, \mathcal{R}_{\mathcal{D}})$-decoders form a partitioning of $\mathcal{R}$. Consequently, an upper bound on the achievable generalized error performance of the elementary decoder can be obtained, as stated in the following theorem.

Theorem 8: Consider a $(K+M)$-user random multiple access system. Assume that the receiver with an elementary decoder chooses an operation region $\mathcal{R}$ and an operation margin $\widehat{\mathcal{R}}$ with $\mathcal{R} \cap \widehat{\mathcal{R}} = \emptyset$. Let σ be a partitioning of the operation region $\mathcal{R}$ satisfying (9). There exists a decoding algorithm such that the generalized error performance of the elementary decoder with $0 < \alpha(\boldsymbol{g}) \leq 1$ is upper-bounded by (10) with $\text{GEP}_{\mathcal{D}}(\alpha)$ being further bounded by (13). ■

Theorem 8 is implied by Theorem 7.

Note that the generalized error performance bounds provided in Theorems 4 and 8 are implicit since the optimal partitioning scheme σ that maximizes the right hand side of (10) is not specified. To find the optimal partition, one needs to compute every single term on the right hand side of (10), (6) and (13) for all code index vectors and all user subsets. Because each term in the definitions of $E_{m\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \tilde{\boldsymbol{g}})$, $E_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')$ and $\tilde{E}_{i\mathcal{D}}(\mathcal{S}, \boldsymbol{g}, \boldsymbol{g}')$ involves the combinations of one user subset and two code index vectors, the computational complexity of finding the optimal partitioning scheme is therefore in the order of $O\left(2^K\left(\prod_{k=1}^{K+M}|\mathcal{G}_k|\right)^2\right)$.

V. Coding Complexity and Channel Code Estimation

Because random access communication often deals with packets (and therefore codewords) that are short in length, coding complexity concern in the new coding model is quite different from the classical ones. According to the decoding algorithms presented in [3], upon receiving the channel output symbols, the receiver needs to compute the likelihood values of all codewords corresponding to all code index vectors in the operation region. The complexity of such a decoding algorithm is in the order of $O\left(\sum_{\boldsymbol{g}\in\mathcal{R}}\exp\left(N\left(\sum_{k=1}^{K} r_k(g_k)\right)\right)\right)$. It is important to note that the number of code index vectors in the operation region can be huge. First, a random access receiver does not necessarily know which users will be active in the area. By taking potential transmitters into decoding consideration, the number of users in the channel coding model can be much larger than the number of active transmitters. Second, random access coding needs to equip a transmitter with multiple transmission options. If the system should be prepared for a wide range of communication environments, then the set of channel codes of each user can have a large cardinality. A simple way to avoid calculating the likelihood values of too many codewords in channel decoding is to first let the receiver estimate the code index vector using only the channel input and output distribution information, and then process only the codewords corresponding to the estimated channel codes. Even if the channel code estimation is not precise, meaning that the receiver may output a set of possible code index vectors, the outcome can still help to significantly reduce the number of codewords that should be further processed by the receiver. In addition to complexity reduction, channel code estimation is also useful for other system functions such as communication adaptation in the upper layer.

Let us assume that the receiver partition the space of code index vectors into L regions, denoted by $C_1, \cdots, C_L$. Let $\boldsymbol{g} \in C_i$ be the actual code index vector. Given the channel output $\boldsymbol{y}$ and the distribution information of the codebooks, the receiver wants to detect the region to which the code index vector belongs. Let the maximum likelihood estimate of the code index vector be $\hat{\boldsymbol{g}}$. In other words, $\hat{\boldsymbol{g}} = \text{argmax}_{\tilde{\boldsymbol{g}}} P(\boldsymbol{y}|\tilde{\boldsymbol{g}})$. The region detection is successful if $\hat{\boldsymbol{g}} \in C_i$. The following theorem gives an upper bound to the detection error probability as a function of $\boldsymbol{g}$.

Theorem 9: Consider a $(K+M)$-user random multiple access system with the code index region detection described above. Let $\boldsymbol{g}$ be the actual code index vector, which belongs to region C_i. The probability that the maximum likelihood estimate $\hat{\boldsymbol{g}}$ does not belong to C_i is upper-bounded by

$$Pr\{\hat{\boldsymbol{g}} \notin C_i\} \leq \max_{\tilde{\boldsymbol{g}}\notin C_i} \exp\left(-NE_c(\boldsymbol{g}, \tilde{\boldsymbol{g}})\right), \quad (15)$$

where $E_c(\boldsymbol{g}, \tilde{\boldsymbol{g}}) = \max_{0<s\leq 1} -\log\sum_Y P(Y|\boldsymbol{g})^s P(Y|\tilde{\boldsymbol{g}})^{(1-s)}$. ■

The proof of Theorem 9 is given in [3, Appendix C].

Assume that a receiver first detect the region to which the code index vector belongs, and then search decoding output among codewords corresponding to code index vectors within the detected region. Performance bound of such a receiver can be easily derived by combining the results of Theorems 3, 7 and 9. Computational complexity of the decoding algorithm is reduced to the order of $O\left(\max_{i\in\{1,\cdots,L\}}\sum_{\boldsymbol{g}\in C_i\cap\mathcal{R}}\exp\left(N\left(\sum_{k=1}^{K} r_k(g_k)\right)\right)\right)$. Note that, the complexity reduction due to channel code estimation may not appear to be significant in the above expression. However, such a picture can change easily if the complexity scaling law in the codeword length can be reduced from exponential to polynomial. Similar to the collision detection problem, the channel code estimation problem can also become ill-posed [3]. The solution to such an issue is to follow the idea of "operation margin" definition and, for every code index region, to mark some other regions as its detection margin. In stead of distinguishing code index vectors between different regions, one can relax the detection problem and only require the receiver to distinguish code index vectors inside a region from those outside the region and the detection margin.

VI. Conclusion

We presented a generalized channel coding model for random multiple access communication systems and derived its performance limitations and tradeoff bounds. The key idea behind these results is the extension of communication error definition beyond the classical meaning of decoding failure. Our results demonstrated that, by matching the communication error definition with that of the unexpected system outcome, classical channel coding theorems can potentially be extended to a wide range of communication modules, especially those in distributed wireless network. It is our hope that the results and the analytical framework presented in this paper can serve as a bridge toward rigorous understandings on the impact of channel coding to the organization and operation of distributed wireless systems.

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