A Generalized Probabilistic Data Association Detector for Multiple Antenna Systems

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***Abstract*—The Probabilistic Data Association (PDA) method for multiuser detection (MUD) over synchronous CDMA channels is extended to the signal detection problem in V-BLAST systems. Computer simulations show that the algorithm has an error probability that is significantly lower than that of the V-BLAST optimal order detector and has a computational complexity that is cubic in the number of transmit antennas.**

***Index Terms*—Code-division multiple access (CDMA), Probabilistic Data Association (PDA), QAM, V-BLAST.**

I. INTRODUCTION

IN THIS LETTER, we extend the Probabilistic Data Association (PDA) detector, originally for MUD in synchronous code division multiple access (CDMA) [2], to V-BLAST systems [1], and propose a generalized PDA (GPDA) algorithm for complex modulation. The GPDA detector is presented for the special case of square/rectangular (sqr/rect) QAM, but extension to other constellations is straightforward and follows the basic PDA implementation of [3] which requires the computation of q probabilities for each transmit symbol, where q is the size of the constellation.

In the case of sqr/rect q-QAM, the new algorithm differs from the direct PDA approach of [3] by reducing the number of probabilities associated with each transmit symbol. As an apparent consequence of reducing the number of probabilities (hypotheses) for sqr/rect QAM, the new detector shows an improved error probability over the direct PDA approach used in [3]. A further advantage of this method is that it offers a reduced computational cost over that of [3] for the case when the number of receive antennas is greater than or equal to the number of transmit antennas; this advantage, which is true for any complex modulation scheme, is achieved by returning to the formulation of the original PDA algorithm, which works from the "decorrelated signal model" [2]. Computer simulations show that the proposed algorithm significantly outperforms the V-BLAST optimal order detector [1] and has a computational complexity that is cubic in the number of transmit antennas.

II. SYSTEM MODEL

Consider a symbol-synchronized multiple antenna system with n_T transmit antennas and n_R receive antennas where we take $n_T \leq n_R$ [1]. In the V-BLAST architecture, the input is a single bit stream that is demultiplexed into n_T substreams, and each substream is mapped to a sequence of complex modulation symbols and transmitted by its respective antenna. It is assumed that the same constellation is used for each substream, and that transmissions are organized into bursts of L symbol durations over a quasistatic Rayleigh fading channel which remains constant for the duration of the burst but changes randomly from one burst to the next. The channel is assumed to be unknown at the transmitters, but is assumed to be estimated accurately at the receivers through the use of embedded training symbols in each burst. The received baseband signal at each instant of time is given by

$$\boldsymbol{r} = \boldsymbol{H}a + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{H}$ is the $n_R \times n_T$ channel matrix whose (i,j)th element, h_{ij}, is the fading between transmitter j and receiver i; $\boldsymbol{a} = [a_1 \ a_2 \ \cdots \ a_{n_T}]^T$ is the transmit vector of sqr/rect QAM symbols, where each symbol in the contellation is transmitted with equal probability; and $\boldsymbol{v}$ is an $n_R \times 1$ complex-valued white Gaussian noise vector with zero mean and covariance matrix equal to $2\sigma^2\boldsymbol{I}$. Assuming a rich scattering model, the elements of the channel matrix $\boldsymbol{H}$ are i.i.d. complex Gaussian with zero mean.

To obtain the system model for the sqr/rect QAM version of the GPDA detector, we begin by transforming (1) into the real-valued vector equation

$$\tilde{\boldsymbol{r}} = \tilde{\boldsymbol{H}}\tilde{\boldsymbol{a}} + \tilde{\boldsymbol{v}} \tag{2}$$

where

$$\tilde{\boldsymbol{r}} = [\Re\{\boldsymbol{r}^T\} \quad \Im\{\boldsymbol{r}^T\}]^T \tag{3}$$

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \Re\{\boldsymbol{H}\} & -\Im\{\boldsymbol{H}\} \\ \Im\{\boldsymbol{H}\} & \Re\{\boldsymbol{H}\} \end{bmatrix} \tag{4}$$

$$\tilde{\boldsymbol{a}} = [\Re\{\boldsymbol{a}^T\} \quad \Im\{\boldsymbol{a}^T\}]^T \tag{5}$$

$$\tilde{\boldsymbol{v}} = [\Re\{\boldsymbol{v}^T\} \quad \Im\{\boldsymbol{v}^T\}]^T \tag{6}$$

Next we multiply (2) from the left by $\tilde{\boldsymbol{H}}^T$ to obtain

$$\boldsymbol{y} = \boldsymbol{G}\tilde{\boldsymbol{a}} + \boldsymbol{n} \tag{7}$$

where $\boldsymbol{y} = \tilde{\boldsymbol{H}}^T\tilde{\boldsymbol{r}}, \boldsymbol{G} = \tilde{\boldsymbol{H}}^T\tilde{\boldsymbol{H}}$, and $\boldsymbol{n} = \tilde{\boldsymbol{H}}^T\tilde{\boldsymbol{v}}$. Note that because the elements of $\boldsymbol{H}$ are modeled as i.i.d. complex Gaussian, $\tilde{\boldsymbol{H}}$ will have full column rank almost surely, and, consequently the symmetric matrix $\boldsymbol{G}$ will be positive definite almost surely. For this reason, (7) is analogous to the synchronous CDMA system in [5].

[1]Extending the proposed algorithm for $n_T > n_R$ is a straightforward matter of adapting the overloaded version of the PDA method (see [4]) to the current problem.

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The model for the sqr/rect QAM version of the GPDA detector is obtained by multiplying (7) from the left by $\boldsymbol{G}^{-1}$ to yield[2]

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{a}} + \tilde{\boldsymbol{n}} = \boldsymbol{e}_i \tilde{a}_i + \sum_{j \neq i} \boldsymbol{e}_j \tilde{a}_j + \tilde{\boldsymbol{n}} \tag{8}$$

In (8), $\tilde{\boldsymbol{y}} = \boldsymbol{G}^{-1}\boldsymbol{y}, \tilde{\boldsymbol{n}} = \boldsymbol{G}^{-1}\boldsymbol{n}$, and $\boldsymbol{e}_i$ is a column vector whose ith element is 1 and whose other elements are 0.

III. (Square/Rectangular QAM) GPDA Detector

A. Basic Algorithm

In the reformulated multi-antenna model (8), we treat the elements of $\tilde{\boldsymbol{a}}$ as independent multivariate random variables where the ith element, $\tilde{a}_i$, is a member of one of two possible sets:

$$\tilde{a}_i \in \begin{cases} \mathcal{S}_{\Re} = X_i = \{x_m(i)\}, & i \in [1, n_T] \\ \mathcal{S}_{\Im} = X_i = \{x_m(i)\}, & i \in [n_T + 1, 2n_T] \end{cases} \tag{9}$$

In (9), $\mathcal{S}_{\Re}$ and $\mathcal{S}_{\Im}$ are the sets of distinct values that can be assumed by the real and imaginary parts of the QAM symbols, respectively. For any element $\tilde{a}_i$, we associate a vector $\boldsymbol{p}(i)$ whose mth element, $p_m(i)$, is the current estimate of the posterior probability that $\tilde{a}_i = x_m(i)$. Since direct evaluation of $\text{Prob}\{\tilde{a}_i = x_m(i) \,|\, \tilde{\boldsymbol{y}}\}$ is computationally prohibitive, the new algorithm attempts to estimate it by using the Gaussian "forcing" idea [2] to approximate $\text{Prob}\{\tilde{a}_i = x_m(i) \,|\, \tilde{\boldsymbol{y}}, \{\boldsymbol{p}(j)\}_{\forall j \neq i}\}$, which will serve as the updated value for $p_m(i)$.

Now to estimate the associated probabilities for an arbitrary element $\tilde{a}_i$, we treat all other elements $\tilde{a}_j$ $(j \neq i)$ as multivariate random variables, and, from (7), we define

$$\boldsymbol{N}_i = \sum_{j \neq i} \boldsymbol{e}_j \tilde{a}_j + \tilde{\boldsymbol{n}} \tag{10}$$

as the effective noise on $\tilde{a}_i$, and approximate it as a Gaussian noise with matched mean and covariance

$$\bar{\boldsymbol{N}}_i = \sum_{j \neq i} \boldsymbol{e}_j E[\tilde{a}_j]$$
$$\boldsymbol{\Omega}_i = \sum_{j \neq i} \boldsymbol{e}_j \boldsymbol{e}_j^T \text{Var}[\tilde{a}_j] + \sigma^2 \boldsymbol{G}^{-1} \tag{11}$$

where $\bar{\boldsymbol{N}}_i = E[\boldsymbol{N}_i]$ and $\boldsymbol{\Omega}_i = \text{Cov}[\boldsymbol{N}_i]$. In (11), $E[\tilde{a}_j]$ and $\text{Var}[\tilde{a}_j]$ are given by

$$E[\tilde{a}_j] = \sum_m x_m(j) p_m(j)$$
$$\text{Var}[\tilde{a}_j] = \sum_m x_m^2(j) p_m(j) - (E[\tilde{a}_j])^2 \tag{12}$$

Let $\boldsymbol{\theta}_i = \tilde{\boldsymbol{y}} - \bar{\boldsymbol{N}}_i$ and

$$\begin{aligned} \alpha_m(i) &= (\boldsymbol{\theta}_i - 0.5\boldsymbol{e}_i x_m(i))^T \boldsymbol{\Omega}_i^{-1} \boldsymbol{e}_i x_m(i) \\ &= \left(\sum_j [\theta_i]_j \left[\Omega_i^{-1}\right]_{ji} - 0.5 x_m(i) \left[\Omega_i^{-1}\right]_{ii} \right) x_m(i) \\ &= \left(\delta_i - 0.5 x_m(i) \left[\Omega_i^{-1}\right]_{ii}\right) x_m(i) \end{aligned} \tag{13}$$

where $[\theta_i]_j$ is the jth element of $\boldsymbol{\theta}_i$; $[\Omega_i^{-1}]_{ji}$ is element (j, i) of $\boldsymbol{\Omega}_i^{-1}$; and $\delta_i = \sum_j [\theta_i]_j [\Omega_i^{-1}]_{ji}$. The posterior probability $p_m(i)$ is then given as

$$p_m(i) = \frac{\exp[\alpha_m(i)]}{\sum_l \exp[\alpha_l(i)]} = \frac{\exp[\alpha_m(i) - \alpha(i)]}{\sum_l \exp[\alpha_l(i) - \alpha(i)]} \tag{14}$$

where $\alpha(i) = \max(\{\alpha_m(i)\}_{\forall m})$. The basic procedure for the proposed GPDA detector is as follows.

1) Based on the matrix $\boldsymbol{G}$ in (7), obtain the optimal detection sequence proposed for the decision feedback detector in [5] (specifically, [5, Theorem 1]) and denote the sequence as $\{k_i\}_{i=1}^{2n_T}$.
2) Initialize the probabilities as
$$p_m(i) = 1/|X_i| \quad \forall m \, \forall i$$
and set the iteration counter $z = 1$.
3) Initialize $i = 1$.
4) Based on the current values of $\{\boldsymbol{p}(k_j)\}_{k_j \neq k_i}$, use the Gaussian "forcing" idea to compute
$$\text{Prob}\{\tilde{a}_{k_i} = x_m(k_i) \,|\, \tilde{\boldsymbol{y}}, \{\boldsymbol{p}(k_j)\}_{k_j \neq k_i}\} \; \forall m$$
and set the results equal to the corresponding elements of $\boldsymbol{p}(k_i)$.
5) If $i < 2n_T$, let $i = i + 1$ and goto step 4. Otherwise, goto step 6.
6) If $\forall i, \boldsymbol{p}(i)$ has converged, goto step 7. Otherwise, let $z = z + 1$ and return to step 3.
7) For $j = 1, \ldots, n_T$, make a decision $\hat{a}_j$ for a_j via

$$\Re\{\hat{a}_j\} = x_l(j), \quad l = \arg\max_d\{p_d(j)\}$$
$$\Im\{\hat{a}_j\} = x_m(j + n_T), \quad m = \arg\max_d\{p_d(j + n_T)\} \tag{15}$$

Under the decoupled system in (8), the GPDA detector computes $2\sqrt{q}$ and $\sqrt{2q} + \sqrt{q/2}$ probabilities per transmit symbol, respectively, for square and rectangular q-QAM. The overall complexity of the new algorithm can be substantially reduced by applying the computational "speed-up" tactics presented in [2].

B. Complexity

The computational efficiency for the proposed method for the case when $n_R \geq n_T$ is achieved by decorrelating the elements of the signal vector. If we take square q-QAM as an example, the complexity of the proposed algorithm using the matrix speed-up tactic of [2] is approximately $\mathcal{O}(8n_T^3)$ real operations for updating the inverse of the covariance matrix per iteration, and $\mathcal{O}(4n_T^2 + 2\sqrt{q}n_T)$ real operations for evaluating (13) per iteration. This is actually a worst case complexity, since the size of the problem may be lowered by the use of successive cancellation [2]. When the full number of probabilities[3] are evaluated by the GPDA detector, the complexities for the aforementioned parts per iteration is $\mathcal{O}(n_T^3)$ and $\mathcal{O}(n_T^2 + qn_T)$ complex operations, respectively. By comparison, the PDA implementation of [3] has a worst case complexity of $\mathcal{O}(n_T n_R^2)$ complex operations for updating the inverse of the covariance matrix per iteration, and $\mathcal{O}(qn_T n_R^2)$ complex operations for evaluating the symbol probabilities per iteration. Hence for $n_R \geq n_T$, the PDA

[2]Note that the choice of (8) as the system model for the GPDA algorithm is only for computational efficiency. At the cost of raising the complexity, (7) may be used as an alternate model for GPDA.

[3]The "full number of probabilities" is the number of probabilities that are evaluated per symbol if the system is not decoupled into its real and imaginary parts. Similarly, when we refer to the "reduced number of probabilities", this is the number of probabilities that are evaluated per symbol when the system is decoupled.

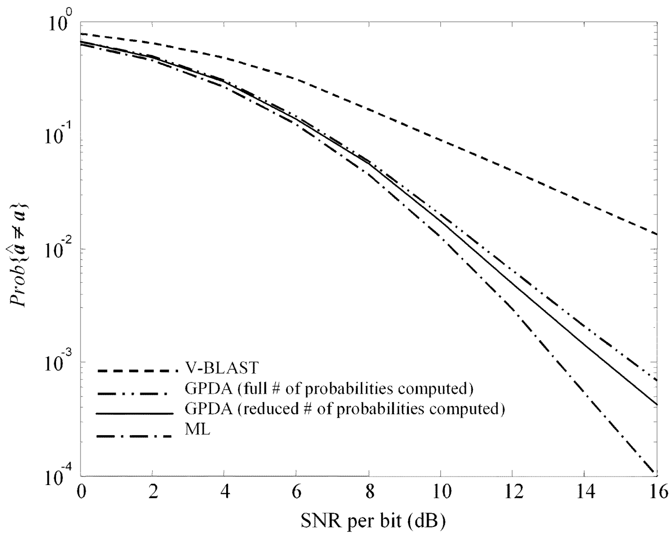

Fig. 1. Comparison of error probabilities for 4-QAM with $n_T = n_R = 4 \times 10^6$ Monte Carlo Runs.

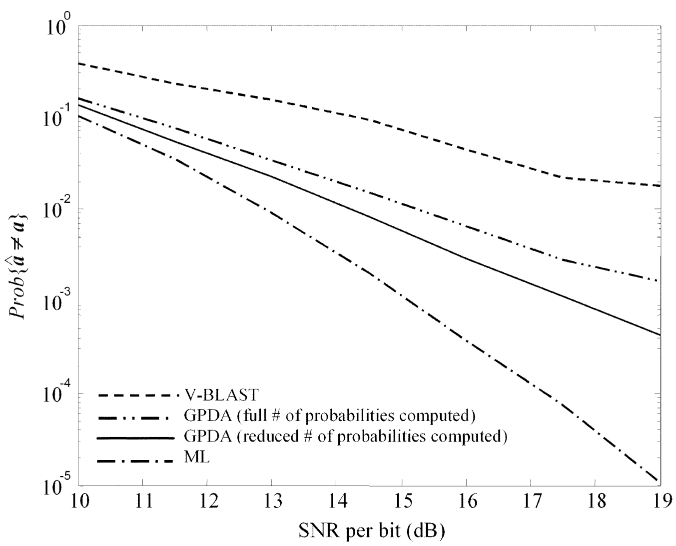

Fig. 2. Comparison of of error probabilities for 8-QAM with $n_T = n_R = 6$. 7.5×10^5 Monte Carlo Runs.

implementation used in this paper is more efficient than that of [3].

The number of iterations required for convergence is largely influenced by the SNR. From our simulations, we have observed that for SNRs of 12 dB and above, the algorithm typically converges within 5 iterations (as in [2] and [3]).

IV. Simulation Results

In our simulations, the burst length was set equal to 100 symbol durations. The elements of the channel matrix $\boldsymbol{H}$ are modeled as i.i.d. $\mathcal{N}(0, 0.5) + \sqrt{-1}\mathcal{N}(0, 0.5)$ and we generate them randomly from one burst to the next (quasistatic channel). We set $\sigma^2 = (n_T \bar{E}_s)/(2 \log_2(q))10^{(-\mathrm{SNR}/10)}$, where $\bar{E}_s$ is the average signal energy of the sqr/rect q-QAM constellation. For each example, we set $n_T = n_R$.

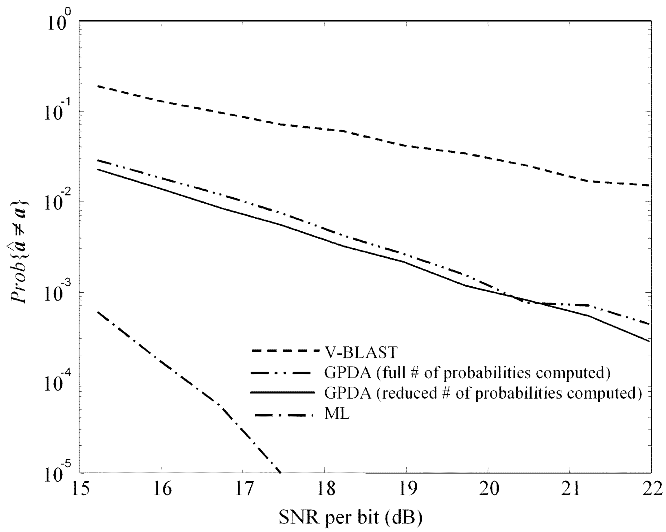

Fig. 3. Comparison of of error probabilities for 16-QAM with $n_T = n_R = 12$. 2×10^5 Monte Carlo Runs.

In Figs. 1–3, we compare the error probabilities of the GPDA detector for the case when the reduced number of probabilities are computed for each transmit symbol as well as for the case when the full number of probabilities are computed (direct PDA) for $q = 4, 8$, and 16 respectively; the results for the (zero-forcing) V-BLAST optimal order detector [1] and the ML detector are also included.

V. Conclusion

A low complexity algorithm based on the PDA method of [2] has been proposed for signal detection in V-BLAST systems, which has an error probability that is significantly lower than that of the V-BLAST detector. For sqr/rect QAM systems the GPDA detector exploits the constellation's product-form structure, and thereby offers an improved error probability as compared to a direct PDA approach. For other constellations, such as PSK, where the real and imaginary parts can only be decoupled at the risk of producing invalid symbols, the full number of probabilities should be evaluated.

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