A Generalized Probabilistic Data Association Detector for Multiple Antenna Systems

D. Pham, K.R. Pattipati, P. K. Willett

Abstract

The Probabilistic Data Association (PDA) method for multiuser detection (MUD) over synchronous CDMA channels is extended to the signal detection problem in V-BLAST systems. Computer simulations show that the algorithm has an error probability that is significantly lower than that of the V-BLAST optimal order detector and has a computational complexity of $\mathcal{O}(8n_T^3)$, where n_T is the number of transmit antennas.

I. Introduction

MULTIPLE antenna systems have attracted considerable attention in the communication community due to their extraordinary capacity gains, which were revealed in the information theoretic studies of [2] [3]. A well known approach to realizing the high spectral efficiencies of multi-antenna systems was proposed by Bell Labs with the introduction of their V-BLAST (Vertical-Bell Laboratories Layered Space-Time) wireless communications architecture. The optimal solution to the V-BLAST communications architecture is given by a Maximum Likelihood (ML) detector, whose complexity is exponential in the number of transmit antennas (with a base equal to the size of the complex signal constellation). Due to the extreme complexity of the ML detector, a suboptimal solution based on the idea of nulling and interference cancelation was originally proposed for the V-BLAST architecture [5] [6]. Although the original V-BLAST detector (V-BLAST optimal order detector) can be said to offer a good trade-off between complexity and performance, there is nonetheless a large gap between the proability of error of the aforementioned detector and that of the ML detector.

In this paper, we extend the PDA detector, originally for MUD in synchronous CDMA [4], to V-BLAST systems [5], and propose a Generalized PDA (GPDA) algorithm for complex modulation. The GPDA detector is presented for the special case of square/rectangular (sqr/rect) QAM, but extension to other constellations is straightforward and follows the basic PDA implementation of [7] which requires the computation of q probabilities for each transmit symbol, where q is the size of the constellation.

In the case of sqr/rect q-QAM, the new algorithm differs from the direct PDA approach of [7] by

The authors are with the Electrical and Computer Engineering Department, University of Connecticut, Storrs, CT 06269, USA. This work was supported by the ONR under contract #N00014-00-1-0101 and NASA under contract #NAG2-1635 (e-mail:krishna@engr.uconn.edu).

reducing the number of probabilities associated with each transmit symbol to a bare minimum; for sqr/rect QAM, the GPDA detector respectively computes $2\sqrt{q}$ and $\sqrt{2q} + \sqrt{q/2}$ probabilities per transmit symbol. As a consequence of reducing the number of probabilities (hypotheses) for sqr/rect QAM, the new detector attains an improved error probability over a direct PDA approach which evaluates q probabilities per symbol. However, the primary difference between the new PDA algorithm and that of [7] is in the implementation, specifically for the case when the number of receive antennas is greater than or equal to the number of transmit antennas[1]. For the aforementioned case, the complexity of the PDA method is reduced significantly from that of [7] by returning to the formulation of the original PDA algorithm [4].

Computer simulations show that the proposed algorithm significantly outperforms the V-BLAST optimal order detector [5] and has a complexity of $\mathcal{O}(8n_T^3)$ for the case of sqr/rect QAM.

II. System Model

Consider a symbol-synchronized multiple antenna system with n_T transmit antennas and n_R receive antennas where we take $n_T \leq n_R$ [2]. The total power launched by each transmitter is proportional to $1/n_T$, so that the total radiated power is constant and independent of n_T. In the V-BLAST architecture, the input is a single bit stream that is demultiplexed into n_T substreams, and each substream is mapped to a sequence of complex modulation symbols and transmitted by its respective antenna. It is assumed that the same constellation is used for each substream, and that transmissions are organized into bursts of L symbol durations over a quasi-static Rayleigh fading channel which remains constant for the duration of the burst but changes randomly from one burst to the next. The channel is assumed to be unknown at the transmitters, but is assumed to be estimated accurately at the receivers through the use of embedded training symbols in each burst. The received baseband signal at each instant of time is given by

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{a} + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{H}$ is the $n_R \times n_T$ channel matrix whose (i,j)th element, h_{ij}, is the fading between transmitter j and receiver i; $\boldsymbol{a} = [a_1 \quad a_2 \quad \cdots \quad a_{n_T}]^T$ is the transmit vector of sqr/rect QAM symbols, where each symbol in the contellation is transmitted with equal probability; and $\boldsymbol{v}$ is an $n_R \times 1$ complex-valued white Gaussian noise vector with zero mean and covariance matrix equal to $2\sigma^2\boldsymbol{I}$. Assuming a rich scattering model, the elements of the channel matrix $\boldsymbol{H}$ are i.i.d. complex Gaussian with zero mean.

[1]Results from information theory (e.g., [2]) show that this condition is necessary if the capacity of a multi-antenna channel is to grow at least linearly with the number of transmit antennas.

[2]Extending the proposed algorithm for $n_T > n_R$ is a straightforward matter of adapting the overloaded version of the PDA method (see [8]) to the current problem.

To obtain the system model for the sqr/rect QAM version of the GPDA detector, we begin by transforming (1) into the real-valued vector equation

$$\tilde{\mathbf{r}} = \tilde{\boldsymbol{H}}\tilde{\mathbf{a}} + \tilde{\mathbf{v}} \tag{2}$$

where

$$\tilde{\mathbf{r}} = \begin{bmatrix} \Re\{\boldsymbol{r}^T\} & \Im\{\boldsymbol{r}^T\} \end{bmatrix}^T \tag{3}$$

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \Re\{\boldsymbol{H}\} & -\Im\{\boldsymbol{H}\} \\ \Im\{\boldsymbol{H}\} & \Re\{\boldsymbol{H}\} \end{bmatrix} \tag{4}$$

$$\tilde{\mathbf{a}} = \begin{bmatrix} \Re\{\boldsymbol{a}^T\} & \Im\{\boldsymbol{a}^T\} \end{bmatrix}^T \tag{5}$$

$$\tilde{\mathbf{v}} = \begin{bmatrix} \Re\{\mathbf{v}^T\} & \Im\{\mathbf{v}^T\} \end{bmatrix}^T \tag{6}$$

Next we multiply (2) from the left by $\tilde{\boldsymbol{H}}^T$ to obtain

$$\boldsymbol{y} = \boldsymbol{G}\tilde{\mathbf{a}} + \boldsymbol{n} \tag{7}$$

where $\boldsymbol{y} = \tilde{\boldsymbol{H}}^T\tilde{\mathbf{r}}$, $\boldsymbol{G} = \tilde{\boldsymbol{H}}^T\tilde{\boldsymbol{H}}$, and $\boldsymbol{n} = \tilde{\boldsymbol{H}}^T\tilde{\mathbf{v}}$. Note that because the elements of $\boldsymbol{H}$ are modeled as i.i.d. complex Gaussian, $\tilde{\boldsymbol{H}}$ will almost always have full column rank, and, consequently the symmetric matrix $\boldsymbol{G}$ will be positive definite with probability nearly one. For this reason, (7) is analogous to the synchronous CDMA system in [9].

The model for the sqr/rect QAM version of the GPDA detector is obtained by multiplying (7) from the left by $\boldsymbol{G}^{-1}$ to yield[3]

$$\tilde{\boldsymbol{y}} = \tilde{\mathbf{a}} + \tilde{\boldsymbol{n}} = \boldsymbol{e}_i\tilde{a}_i + \sum_{j\neq i}\boldsymbol{e}_j\tilde{a}_j + \tilde{\boldsymbol{n}} \tag{8}$$

In (8), $\tilde{\boldsymbol{y}} = \boldsymbol{G}^{-1}\boldsymbol{y}$, $\tilde{\boldsymbol{n}} = \boldsymbol{G}^{-1}\boldsymbol{n}$, and $\boldsymbol{e}_i$ is a column vector whose ith element is 1 and whose other elements are 0.

III. The (Square/Rectangular QAM) GPDA Detector

A. Basic Algorithm

In the reformulated multi-antenna model (8), we treat the elements of $\tilde{\mathbf{a}}$ as independent multivariate random variables where the ith element, $\tilde{a}_i$, is a member of one of two possible sets:

$$\tilde{a}_i \in \begin{cases} \mathcal{S}_{\Re} = X_i = \{x_m(i)\}, & i \in [1,\ n_T] \\ \mathcal{S}_{\Im} = X_i = \{x_m(i)\}, & i \in [n_T + 1,\ 2n_T] \end{cases} \tag{9}$$

[3]Note that the choice of (8) as the system model for the GPDA algorithm is only for computational efficiency. At the cost of raising the complexity, (7) may be used as an alternate model for GPDA.

In (9), $\mathcal{S}_\Re$ and $\mathcal{S}_\Im$ are the sets of distinct values that can be assumed by the real and imaginary parts of the QAM symbols respectively. For any element $\tilde{a}_i$, we associate a vector $\boldsymbol{p}(i)$ whose mth element, $p_m(i)$, is the current estimate of the posterior probability that $\tilde{a}_i = x_m(i)$. Since direct evaluation of $Prob\{\tilde{a}_i = x_m(i)|\tilde{\boldsymbol{y}}\}$ is computationally prohibitive, the new algorithm attempts to estimate it by using the Gaussian "forcing" idea [4] to approximate $Prob\{\tilde{a}_i = x_m(i)|\tilde{\boldsymbol{y}},\ \{\boldsymbol{p}(j)\}_{\forall j\neq i}\}$, which will serve as the updated value for $p_m(i)$.

An important factor in the performance of the proposed algorithm is the order in which the probability vectors $\{\boldsymbol{p}(i)\}_{\forall i}$ are updated. As in [4], we use the optimal decision feedback ordering [9] for the update sequence determined from the matrix $\boldsymbol{G}$ in (7).

Now to estimate the associated probabilities for an arbitrary element $\tilde{a}_i$, we treat all other elements $\tilde{a}_j$ $(j \neq i)$ as multivariate random variables, and, from (7), we define

$$\boldsymbol{N}_i = \sum_{j\neq i} \mathbf{e}_j \tilde{a}_j + \tilde{\boldsymbol{n}} \tag{10}$$

as the effective noise on $\tilde{a}_i$, and approximate it as a Gaussian noise with matched mean and covariance:

$$\begin{aligned} \bar{\boldsymbol{N}}_i &= \sum_{j\neq i} \mathbf{e}_j E[\tilde{a}_j] \\ \boldsymbol{\Omega}_i &= \sum_{j\neq i} \mathbf{e}_j \mathbf{e}_j^T Var[\tilde{a}_j] + \sigma^2 \boldsymbol{G}^{-1} \end{aligned} \tag{11}$$

where $\bar{\boldsymbol{N}}_i = E[\boldsymbol{N}_i]$ and $\boldsymbol{\Omega}_i = Cov[\boldsymbol{N}_i]$. In (11), $E[\tilde{a}_j]$ and $Var[\tilde{a}_j]$ are given by

$$\begin{aligned} E[\tilde{a}_j] &= \sum_m x_m(j) p_m(j) \\ Var[\tilde{a}_j] &= \sum_m x_m^2(j) p_m(j) - (E[\tilde{a}_j])^2 \end{aligned} \tag{12}$$

Now, defining

$$\boldsymbol{\theta}_i = \tilde{\boldsymbol{y}} - \bar{\boldsymbol{N}}_i \tag{13}$$

and

$$\alpha_m(i) = (\boldsymbol{\theta}_i - 0.5\mathbf{e}_i x_m(i))^T \boldsymbol{\Omega}_i^{-1} \mathbf{e}_i x_m(i) \tag{14}$$

we obtain

$$\begin{aligned} p_m(i) &= \frac{exp[\alpha_m(i)]}{\sum_l exp[\alpha_l(i)]} \\ &= \frac{exp[\alpha_m(i) - \alpha(i)]}{\sum_l exp[\alpha_l(i) - \alpha(i)]} \end{aligned} \tag{15}$$

where $\alpha(i) = \max(\{\alpha_m(i)\}_{\forall m})$. The basic procedure for the proposed GPDA detector is as follows.

1. Based on the matrix $\boldsymbol{G}$ in (7), obtain the optimal detection sequence proposed for the decision feedback detector in [9] (specifically theorem 1 of [9]) and denote the sequence as $\{k_i\}_{i=1}^{2n_T}$.

2. Initialize the probabilities as

$$p_m(i) = 1/|X_i| \quad \forall m, \ \forall i$$

and set the iteration counter $z = 1$.

3. Initialize $i = 1$.

4. Based on the current values of $\{\boldsymbol{p}(k_j)\}_{k_j \neq k_i}$, use the Gaussian "forcing" idea to approximate

$$Prob\{\tilde{a}_{k_i} = x_m(k_i)|\tilde{\boldsymbol{y}}, \{\boldsymbol{p}(k_j)\}_{k_j \neq k_i}\} \ \forall m$$

and set the results equal to the corresponding elements of $\boldsymbol{p}(k_i)$.

5. If $i < 2n_T$, let $i = i + 1$ and goto step 4. Otherwise, goto step 6.

6. If $\forall i$, $\boldsymbol{p}(i)$ has converged, goto step 7. Otherwise, let $z = z + 1$ and return to step 3.

7. For $j = 1, \ldots, n_T$, make a decision $\hat{a}_j$ for a_j via

$$\begin{aligned} \Re\{\hat{a}_j\} &= x_l(j), \quad l = \arg\max_d \{p_d(j)\} \\ \Im\{\hat{a}_j\} &= x_m(j + n_T), \quad m = \arg\max_d \{p_d(j + n_T)\} \end{aligned} \tag{16}$$

B. Computational Refinements

The complexity of the new algorithm can be substantially reduced by utilizing the computational "speed-up" tactics of [4].

B.1 Speed-Up–Matrix Arithmetic

As noted in [4], the inverse of $\boldsymbol{\Omega}_j$ can be evaluated by applying the Sherman-Morrison-Woodbury formula [10] twice consecutively. A similar idea can also be applied to avoid direct calculation of $\bar{\boldsymbol{N}}_j$. Given $\boldsymbol{\Omega}_i^{-1}$ and $\bar{\boldsymbol{N}}_i$, $\boldsymbol{\Omega}_j^{-1}$ and $\bar{\boldsymbol{N}}_j$ are calculated as follows.

1. Define auxillary variables $\boldsymbol{\Omega}$ and $\bar{\boldsymbol{N}}$ as

$$\begin{aligned} \boldsymbol{\Omega} &= \boldsymbol{\Omega}_i + \mathbf{e}_i \mathbf{e}_i^T Var[\tilde{a}_i] \\ \bar{\boldsymbol{N}} &= \bar{\boldsymbol{N}}_i + \mathbf{e}_i E[\tilde{a}_i] \end{aligned} \tag{17}$$

2. Compute $\boldsymbol{\Omega}^{-1}$ via

$$\boldsymbol{\Omega}^{-1} = \boldsymbol{\Omega}_i^{-1} - \frac{\boldsymbol{\Omega}_i^{-1}\mathbf{e}_i\mathbf{e}_i^T\boldsymbol{\Omega}_i^{-1}Var[\tilde{a}_i]}{1 + \mathbf{e}_i^T\boldsymbol{\Omega}_i^{-1}\mathbf{e}_iVar[\tilde{a}_i]} \tag{18}$$

3. Compute $\mathbf{\Omega}_j^{-1}$ and $\bar{\mathbf{N}}_j$ via

$$\begin{aligned} \mathbf{\Omega}_j^{-1} &= \mathbf{\Omega}^{-1} + \frac{\mathbf{\Omega}^{-1}\boldsymbol{e}_j\boldsymbol{e}_j^T\mathbf{\Omega}^{-1}Var[\tilde{a}_j]}{1-\mathbf{e}_j^T\mathbf{\Omega}^{-1}\mathbf{e}_jVar[\tilde{a}_j]} \\ \bar{\mathbf{N}}_j &= \bar{\boldsymbol{N}} - \mathbf{e}_jE[\tilde{a}_j] \end{aligned} \tag{19}$$

B.2 Speed-Up–Successive Cancellation

In our simulations, we have observed that the algorithm generally converges within 3 to 7 iterations for $SNR < 14dB$, and within 1 to 3 iterations for $SNR > 14dB$. However, the overall complexity can be high if one or two elements of $\tilde{\mathbf{a}}$ exhibit slow convergence. To reduce the complexity in these instances, successive cancellation [4] is applied after each iteration.

After the zth iteration, define D to be the set of elements that satisfy

$$\max\left(\{p_m(i)\}_{\forall m}\right) \geq 1-\epsilon \quad \forall i \in D \tag{20}$$

where ϵ is a small positive number. For each $\tilde{a}_i \in D$, fix their values as

$$\tilde{a}_i = x_l(i), \quad l = \arg\max_d\{p_d(i)\} \quad \forall i \in D \tag{21}$$

Define $\bar{D}$ as the complement of D. In the $(z+1)$st iteration, we only update the probabilities of the members of $\bar{D}$ using

$$\boldsymbol{G}_{\bar{D}\bar{D}}^{-1}(\boldsymbol{y}_{\bar{D}} - \boldsymbol{G}_{\bar{D}D}\tilde{\mathbf{a}}_D) = \tilde{\mathbf{a}}_{\bar{D}} + \tilde{\mathbf{n}}_{\bar{D}} \tag{22}$$

as the new system model where $\boldsymbol{G}_{\bar{D}D}$ is the sub-block matrix of $\boldsymbol{G}$ whose rows correspond to the members of $\bar{D}$ and whose columns correspond to the members of D. In (22), $\tilde{\mathbf{n}}_{\bar{D}}$ is the Gaussian noise for the sub-system model with zero mean and covariance matrix equal to $\sigma^2\boldsymbol{G}_{\bar{D}\bar{D}}^{-1}$.

C. Complexity

If we take square q-QAM as an example, the complexity of the proposed algorithm using the afforementioned matrix speed-up tactic [4] for one iteration is $\mathcal{O}(8n_T^3 + 4\sqrt{q}n_T^2)$ real operations. This is actually a worst case complexity since the size of the problem can be lowered by employing successive cancellation [4]. By comparison, the PDA implementation of [7] involves $\mathcal{O}(n_Tn_R^2 + qn_Tn_R^2)$ complex operations. For $n_R \geq n_T$, the proposed PDA algorithm is considerably more efficient than that of [7].

IV. Simulation Results

In our simulations, the burst length was set equal to 100 symbol durations. The elements of the channel matrix $\boldsymbol{H}$ are modeled as i.i.d. complex Gaussian with zero mean and variance 0.5 per dimension and we generate them randomly from one burst to the next. We adjust $\sigma^2 = \frac{n_T\bar{E}_s}{2log_2(q)}10^{(-SNR/10)}$, where

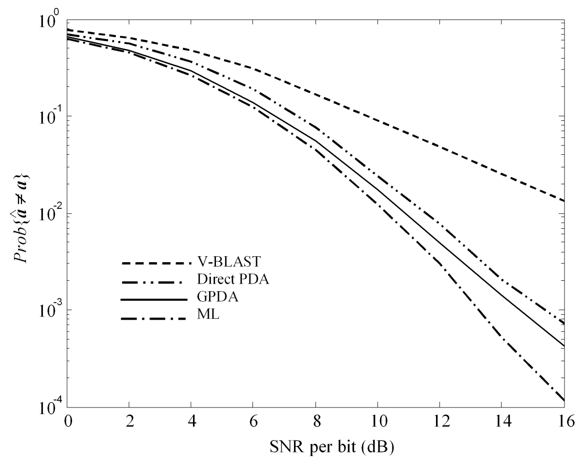

Fig. 1. Comparison of error probabilities for 4-QAM with $n_T = n_R = 4$. 10^6 Monte Carlo Runs

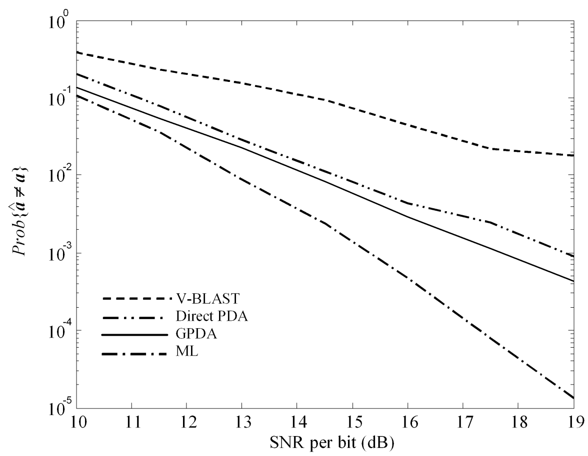

Fig. 2. Comparison of of error probabilities for 8-QAM with $n_T = n_R = 6$. 750000 Monte Carlo Runs

$\bar{E}_s$ is the average signal energy of the sqr/rect q-QAM constellation. For the successive cancellation part of the GPDA detector, we used $\epsilon = \frac{0.01\sigma^2}{2\bar{E}_s}$. For each example, we set $n_T = n_R$.

In Figure 1, we compare the error probability of the GPDA detector with $q = 4$ and $n_T = 4$ for the case when the reduced number of probabilities is computed for each transmit symbol as well as the case when the full number of probabilities are computed (direct PDA). The results of the (zero-forcing) optimal order V-BLAST [5] and ML detectors are also included. Note that when the full number of probabilities are evaluated by the GPDA detector, the complexity per iteration is $\mathcal{O}(n_T^3 + qn_T^2)$ complex operations, which is still significantly lower than the PDA implementation of [7]. In Figure 2, we show the results of the afforementioned detectors for the case when $q = 8$ and $n_T = 6$; figure 3 shows the results (minus the ML curve) when $q = 32$ and $n_T = 14$.

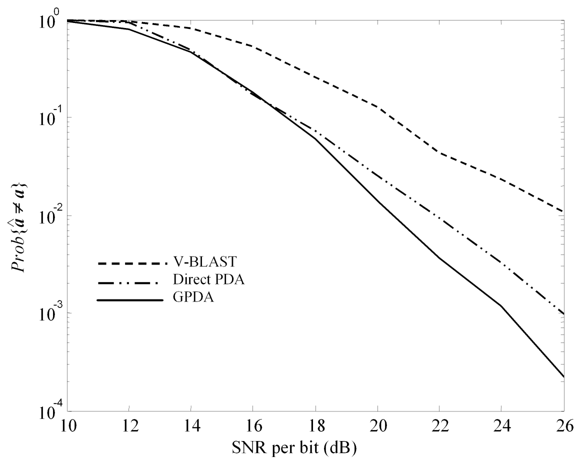

Fig. 3. Comparison of error probabilities for 32-QAM with $n_T = n_R = 14$. 50000 Monte Carlo Runs

V. Summary

A low complexity algorithm based on the PDA method of [4] has been proposed for signal detection in V-BLAST systems which has an error probability that is significantly lower than the V-BLAST detector. For sqr/rect QAM systems, the GPDA detector shows improved performance over a direct PDA approach by reducing the number of probabilities associated with each transmit symbol. For other constellations, such as PSK, where the real and imaginary parts can only be decoupled at the risk of producing invalid symbols, the full number of probabilities should be evaluated.

References

[1] G.J. Foschini, "Layered Space-Time Architecture for Wireless Communication in a Fading Environment when Using Multi-element Antennas", Bell Labs Tech. J., pp. 41-59, Autumn 1996.

[2] G.J. Foschini, M.J. Gans, "On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas", *Wireless Pers. Commun.*, vol. 6, pp. 311-334, March 1998.

[3] E. Telatar, "Capacity of Multiantenna Gaussian Channels", AT& T Bell Laboratories, Tech. Memo., June 1995.

[4] J. Luo, K. Pattipati, P. Willett and F. Hasegawa, "Near Optimal Multiuser Detection in Synchronous CDMA using Probabilistic Data Association", *IEEE Comm. Lett.* , vol. 5, pp. 361-363, Sept. 2001.

[5] G.D. Golden, C.J. Foschini, R.A. Valenzuela and P.W. Wolniansky, "Detection Algorithm and Initial Laboratory Results Using V-BLAST Space-Time Communication Architecture", *Electronic Letters*, vol. 35, pp. 14-15, Jan. 1999.

[6] P.W. Wolniansky, G.J. Foshini, G.D. Golden, R.A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel", *Proc. 1998 Int. Symp. on Advanced Radio Technologies*, Boulder, Colorado, 9-11, Sept. 1998.

[7] S. Liu and Z. Tian, "Near-Optimum Soft Decision Equalization for Frequency Selective MIMO Channels", to appear in *IEEE Transactions on Signal Processing*.

[8] J. Luo "Improved Methods for Multiuser Detection in Code-Division Multiple Access Systems" PhD disseration, University of Connecticut, May 2002.

[9] M.K. Varanasi, "Decision Feedback Multiuser Detection: a Systematic Approach", *IEEE Trans. Inform. Theory*, vol. 45, pp. 219–240, Jan. 1999.

[10] W. W. Hager, "Updating the Inverse of a Matrix", *SIAM Review*, vol. 31, No. 2, pp. 221-239, 1989.