# On Utility Optimization in Distributed Multiple Access over a Multi-packet Reception Channel

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***Abstract*—This paper considers distributed medium access control in a wireless multiple access network with an unknown number of users. A multi-packet reception channel is assumed in which all packets should be received successfully if and only if the number of users transmitting in parallel does not exceed a known threshold. We propose a transmission adaptation approach, where, in each time slot, a user estimates the probability of channel availability from the viewpoint of a virtual user and adjusts its transmission probability according to its utility objective and a derived user number estimate. A sufficient condition under which the system should have a unique equilibrium is obtained. Simulation results show that the proposed medium access control algorithm does help users to converge asymptotically to a near optimal transmission probability.**

## I. Introduction

While the problem of distributed medium access control (MAC) in wireless multiple access networks has been extensively investigated in the past several decades, most existing works assumed relatively simple channel models such as the collision channel [1]. Recently, an extended channel coding theory was developed for distributed communication at the physical layer [2]. The new coding theory enabled an enhanced physical-link layer interface [3], which essentially equips each link layer user with multiple transmission options and each link layer receiver with multi-packet reception capability. Consequently, it becomes necessary to develop distributed MAC algorithms to efficiently exploit the advanced communication adaptation options, which, unfortunately, come with sophisticated channel models.

As an early step, in this paper, we consider distributed MAC in a time-slotted wireless multiple access network over a class of multi-packet reception channels with an unknown finite number of homogeneous users. We still assume binary transmission options in the sense that, in each time slot, a user either idles or transmits a packet. Assume that users are backlogged with messages and know the success/failure status of their own packet transmissions. In each time slot, a user measures the "crowdedness" of the channel, which is defined as the conditional success probability of a virtual packet transmission. A transmission probability target is then calculated based on the utility objective and a derived user number estimate. We propose a distributed transmission probability adaptation model and show that it falls into the classical stochastic approximation framework. Under the assumption that each user can obtain the true crowdedness measure of the channel, we obtain a sufficient condition for users' probabilities to asymptotically converge to the unique system equilibrium that maximizes a class of utility functions. In cases when channel crowdedness cannot be measured locally, we propose a two-step approach for each user to interpret crowdedness of the channel from its own transmission probability and conditional packet success probability. We show that the two-step approach can be simplified into one-step in terms of giving the direction of increasing/decreasing the transmission probability. Although we are not able to obtain a convergence proof for cases when users need to interpret channel crowdedness, simulation results show that the proposed MAC algorithm does lead the system to the desired system equilibrium.

## II. Problem Formulation

Consider a time-slotted distributed multiple access network with $K$ homogeneous users. The value of $K$ is assumed to be unknown to all users. In each time slot, a user either idles or transmits a packet. Assume that packet transmissions should be successful if and only if no more than $L \leq K$ users transmit in parallel, where $L > 0$ is a known parameter. Note that the multi-packet reception channel model, which includes the collision channel as a special case, can be derived from a class of physical layer channels [3].

We assume that users are backlogged with messages. In each time slot, user $k$ chooses whether to transmit a packet or not according to a probability parameter $p_k$. We assume that a user gets feedback from the receiver about whether each transmitted packet is successfully received, and this is the only feedback information available. Consequently, user $k$ is able to measure the conditional success probability of its own packet transmissions, and we denote this quantity by $q_k$. Under the assumption that all users should eventually transmit with an identical probability $p$, and the corresponding conditional success probability being denoted by $q$, users intend to maximize the following symmetric network utility

$$U(p,q) = pT(q), \quad \text{with } q = \sum_{i=0}^{L-1} \binom{K-1}{i} p^i (1-p)^{K-1-i}, \tag{1}$$

where $T(q)$ in (1) is a function of $q$.

Since the network utility defined in (1) is a function of the unknown user number, a user has to estimate the user number first in order to calculate the corresponding transmission probability target that maximizes the utility. Assume that, in time slot $n$, user $k$ obtains a transmission probability target $\tilde{p}_k(n)$, and then updates its transmission probability by $p_k(n+1) = (1-\alpha)p_k(n) + \alpha\tilde{p}_k(n)$, where $\alpha > 0$ is a constant step size. Because packet transmissions of the users are random, $\tilde{p}_k(n)$ is a random variable. Let $\hat{p}_k(n) = E_n[\tilde{p}_k(n)]$, where $E_n[.]$ denotes the conditional expectation operation given system state at the beginning of time slot $n$. We say the probability update reaches an equilibrium if and only if $p_k(n) = \hat{p}_k(n)$ for all $k$.

We use $\boldsymbol{p}(n) = [p_1(n), \cdots, p_K(n)]^T$ to denote the transmission probabilities of all users. Similarly, let $\tilde{\boldsymbol{p}}(n)$ and $\hat{\boldsymbol{p}}(n)$ denote the actual and the expected transmission probability targets of all users, respectively. The stochastic probability update can be written as

$$\boldsymbol{p}(n+1) = (1-\alpha)\boldsymbol{p}(n) + \alpha\tilde{\boldsymbol{p}}(n), \quad (2)$$

with the conditional expectation satisfying

$$E_n[\boldsymbol{p}(n+1)] = (1-\alpha)\boldsymbol{p}(n) + \alpha\hat{\boldsymbol{p}}(n). \quad (3)$$

Note that $\hat{\boldsymbol{p}}(\boldsymbol{p}(n))$ is a function of $\boldsymbol{p}(n)$. According to the stochastic approximation theory [4], when step size $\alpha$ is small enough and under certain conditions, dynamics of the probability update can be approximated by the following ordinary differential equation

$$\frac{d\boldsymbol{p}(t)}{dt} = -\left[\boldsymbol{p}(t) - \hat{\boldsymbol{p}}(\boldsymbol{p}(t))\right]. \quad (4)$$

A weak sense convergence property of the system is stated in the following theorem.

**Theorem 1:** Assume that function $\hat{\boldsymbol{p}}(\boldsymbol{p})$ is Lipschitz continuous, i.e., there exists a constant $K_1 > 0$, such that

$$\|\hat{\boldsymbol{p}}(\boldsymbol{p}_1) - \hat{\boldsymbol{p}}(\boldsymbol{p}_2)\| \le K_1\|\boldsymbol{p}_1 - \boldsymbol{p}_2\|. \quad (5)$$

If ordinary differential equation (4) has a unique stable equilibrium at $\boldsymbol{p}^*$, then for any $\epsilon > 0$, there exists a constant $K_2 = K_2(\epsilon)$ such that the following inequality holds under the stochastic probability update given in (2).

$$\limsup_{n\to\infty} P(\|\boldsymbol{p}(n) - \boldsymbol{p}^*\| \ge \epsilon) \le K_2\alpha. \quad (6)$$

Theorem 1 is implied by [4, Theorems 2.3].

In the rest of the paper, we will investigate the condition under which (4) possesses a unique stable equilibrium. For the sake of simple discussion, we assume no measurement noise, i.e., $\tilde{\boldsymbol{p}}(n) = \hat{\boldsymbol{p}}(n)$.

## III. Convergence with True Measurement of Channel Crowdedness

Let us define "channel crowdedness", denoted by $q_v(\boldsymbol{p})$, as the conditional success probability of a virtual packet transmission. Or in other words, $q_v$ is the probability that an additional packet can be sent successfully through the channel in a time slot. For example, under the collision channel model, $q_v(\boldsymbol{p}) = \prod_{i=1}^{K}(1-p_i)$. Under the multi-packet reception channel if $p_1 = \cdots = p_K = p$, for another example, $q_v = \sum_{i=0}^{L-1}\binom{K}{i}p^i(1-p)^{K-i}$. In this section, we assume that $q_v$ can be measured by every user in the system. This assumption holds for the collision channel when $L = 1$, because each user, say user $k$, can first measure its own conditional packet success probability, denoted by $q_k = \prod_{i\ne k}(1-p_i)$, and then calculate $q_v = (1-p_k)q_k$. While the assumption does not hold for multi-packet reception channels with $L > 1$, we will show by computer simulations in Section V that the system can still converge to the same equilibrium with users using an interpreted channel crowdedness variable.

Consider the optimization of symmetric utility given in (1). When all users transmit with an identical probability $p$, conditional success probability $q$ of each user is given by $q = \sum_{i=0}^{L-1}\binom{K-1}{i}p^i(1-p)^{K-1-i}$. We intend to design a distributed MAC algorithm such that transmission probabilities of all users should converge asymptotically to $p^* = \frac{x^*}{K+1}$, where $x^*$ is obtained from

$$x^* = \operatorname*{argmax}_x xT\left(e^{-x}\sum_{i=0}^{L-1}\frac{x^i}{i!}\right). \quad (7)$$

Note that $p^*$ is a near optimal solution to the utility maximization problem for large $K$[1].

Since the value of $K$ is unknown, each user needs to maintain a user number estimate, denoted by $\hat{K}$, which is not necessarily integer-valued. Given $\hat{K}$, transmission probability target of the user is set at $\hat{p} = \frac{x^*}{\hat{K}+1}$. To explain how each user should obtain $\hat{p}$ and $\hat{K}$, we need to define a function $q_v^*(\hat{p})$, which can be viewed as a function of $\hat{p}$ or $\hat{K}$. Let $\lfloor\hat{K}\rfloor \le \hat{K}$ be the integer closest to but no larger than $\hat{K}$. Let $\lambda = \lfloor\hat{K}\rfloor + 1 - \hat{K}$. We define $q_v^*(\hat{p})$ as

$$\begin{aligned} q_v^*(\hat{p}) = \lambda\sum_{i=0}^{L-1}\binom{\lfloor\hat{K}\rfloor}{i}\hat{p}^i(1-\hat{p})^{\lfloor\hat{K}\rfloor-i} \\ +(1-\lambda)\sum_{i=0}^{L-1}\binom{\lfloor\hat{K}\rfloor+1}{i}\hat{p}^i(1-\hat{p})^{\lfloor\hat{K}\rfloor+1-i} \\ \text{with } \hat{K} = \frac{x^*}{\hat{p}} - 1, \lambda = \lfloor\hat{K}\rfloor + 1 - \hat{K}. \end{aligned} \quad (8)$$

In each time slot, we assume a user should first measure $q_v$, which is the conditional success probability of a virtual packet transmission. The transmission probability target $\hat{p}$ should then be obtained from $q_v^*(\hat{p}) = q_v$, with only two exceptions. First, if $q_v \ge q_v^*(\frac{x^*}{L+1})$, we set $\hat{p} = \frac{x^*}{L+1}$. Second, if $q_v \le q_v^*(0)$, we set $\hat{p} = 0$. It is easy to verify that, the probability target function $\hat{p}(\boldsymbol{p})$, as a function of the user transmission probability vector $\boldsymbol{p}$, is Lipschitz continuous.

The following theorem shows the monotonicity property of function $q_v^*(\hat{p})$, and this key property will be used in the convergence proof of the proposed distributed MAC algorithm.

**Theorem 2:** If $x^* \le L$, then $q_v^*(\hat{p})$ defined in (8) is non-decreasing in $\hat{p}$, for $\hat{p} \in \left[0, \frac{x^*}{L+1}\right]$.

[1]We choose to target at the equilibrium of $p^* = \frac{x^*}{K+1}$ instead of $\frac{x^*}{K}$ because the latter choice does not always yield the desired monotonicity property given in Theorem 2.

*Proof:* Let us write $q_v^*(\hat{K})$ as a function of $\hat{K} = \frac{x^*}{\hat{p}} - 1$. To prove the theorem, it is sufficient to show that $q_v^*(\hat{K})$ is non-increasing in $\hat{K}$ between $(\lfloor \hat{K} \rfloor, \lfloor \hat{K} \rfloor + 1]$ for all $\hat{K} \geq L$.

Define $N = \lfloor \hat{K} \rfloor$, $\lambda = \lfloor \hat{K} \rfloor + 1 - \hat{K}$. Assume that $N \geq L$ is fixed. We can write $q_v^*$ as a function of $\lambda$ and $\hat{p}$.

$$q_v^*(\hat{p}, \lambda) = \lambda \sum_{i=0}^{L-1} \binom{N}{i} \hat{p}^i (1-\hat{p})^{N-i} + (1-\lambda) \sum_{i=0}^{L-1} \binom{N+1}{i} \hat{p}^i (1-\hat{p})^{N+1-i}. \quad (9)$$

Because $\hat{p} = \frac{x^*}{N+2-\lambda}$, we can take the derivative of $q_v^*$ with respect to $\lambda$ as follows.

$$\frac{dq_v^*(\lambda)}{d\lambda} = \frac{\partial q_v^*(\hat{p}, \lambda)}{\partial \lambda} + \frac{d\hat{p}}{d\lambda} \frac{\partial q_v^*(\hat{p}, \lambda)}{\partial \hat{p}} = \frac{\partial q_v^*(\hat{p}, \lambda)}{\partial \lambda} + \frac{\hat{p}}{N+2-\lambda} \frac{\partial q_v^*(\hat{p}, \lambda)}{\partial \hat{p}}. \quad (10)$$

Note that

$$\sum_{i=0}^{L-1} \binom{N+1}{i} \hat{p}^i (1-\hat{p})^{N+1-i} = \hat{p} \sum_{i=0}^{L-2} \binom{N}{i} \hat{p}^i (1-\hat{p})^{N-i} + (1-\hat{p}) \sum_{i=0}^{L-1} \binom{N}{i} \hat{p}^i (1-\hat{p})^{N-i}. \quad (11)$$

From (9) and (11), we get

$$\frac{\partial q_v^*(\hat{p}, \lambda)}{\partial \lambda} = \sum_{i=0}^{L-1} \binom{N}{i} \hat{p}^i (1-\hat{p})^{N-i} - \sum_{i=0}^{L-1} \binom{N+1}{i} \hat{p}^i (1-\hat{p})^{N+1-i} = \hat{p} \sum_{i=0}^{L-1} \binom{N}{i} \hat{p}^i (1-\hat{p})^{N-i} - \hat{p} \sum_{i=0}^{L-2} \binom{N}{i} \hat{p}^i (1-\hat{p})^{N-i} = \binom{N}{L-1} \hat{p}^L (1-\hat{p})^{N+1-L}, \quad (12)$$

and

$$\frac{\partial q_v^*(\hat{p}, \lambda)}{\partial \hat{p}} = -\lambda \binom{N}{L-1} \hat{p}^{L-1} (1-\hat{p})^{N-L} (N+1-L) - (1-\lambda) \binom{N+1}{L-1} \hat{p}^{L-1} (1-\hat{p})^{N+1-L} (N+2-L). \quad (13)$$

Because $(N+1)\hat{p} \leq x^* \leq L$, we have

$$\frac{\binom{N+1}{L-1} \hat{p}^{L-1} (1-\hat{p})^{N+1-L} (N+2-L)}{\binom{N}{L-1} \hat{p}^{L-1} (1-\hat{p})^{N-L} (N+1-L)} = \frac{(N+1)(1-\hat{p})}{N+1-L} = \frac{(N+1)(N+2-\lambda-x^*)}{(N+1-L)(N+2-\lambda)} \geq 1, \quad (14)$$

which implies that

$$\frac{\partial q_v^*}{\partial \hat{p}} \geq -\binom{N+1}{L-1} \hat{p}^{L-1} (1-\hat{p})^{N+1-L} (N+2-L). \quad (15)$$

Consequently, combining (10), (12) and (15) yields

$$\frac{dq_v^*}{d\lambda} \geq \binom{N}{L-1} \hat{p}^L (1-\hat{p})^{N+1-L} - \binom{N+1}{L-1} \frac{\hat{p}^L}{N+2-\lambda} (1-\hat{p})^{N+1-L} (N+2-L) = \binom{N}{L-1} \hat{p}^L (1-\hat{p})^{N+1-L} \left(1 - \frac{N+1}{N+2-\lambda}\right) \geq 0. \quad (16)$$

Conclusion of the theorem then follows. ■

Theorem 2 implies that as the network size increases, while maximizing the symmetric network utility, the conditional success probability of a virtual packet transmission should decrease if users' transmission probabilities are chosen appropriately. This is consistent with our original purpose of defining the corresponding quantity as the measure of "channel crowdedness".

The following theorem shows that, if "channel crowdedness" $q_v$ can be measured locally by each user, then for a class of network utility functions, the proposed distributed MAC algorithm converges to the unique equilibrium of $p^* = \frac{x^*}{K+1}$ for all users.

**Theorem 3:** Assume $K \geq L$. Assume that $q_v$, which is the conditional success probability of a virtual packet transmission, can be measured by each user in the system. Let users calculate their transmission probability target $\tilde{p} = \hat{p}$ by equalling $q_v^*(\hat{p}) = q_v$ with $\hat{p} \in [0, \frac{x^*}{L+1}]$. Suppose that $T(q)$ in (1) satisfies the following conditions.

$$T(1) > 0, \quad \frac{dT(q)}{dq} > 0, \quad T(q) \leq q \frac{dT(q)}{dq}. \quad (17)$$

Then distributed MAC algorithm given in (2) converges to $p^* = \frac{x^*}{K+1}$ for all users, and this is the unique stable equilibrium of the ordinary differential equation given in (4).

*Proof:* Define

$$q_\infty(x) = e^{-x} \sum_{i=0}^{L-1} \frac{x^i}{i!}. \quad (18)$$

According to (7), $x^*$ must satisfy the following equality.

$$T(q_\infty) + x \frac{dT(q_\infty)}{dq_\infty} \frac{dq_\infty(x)}{dx} \bigg|_{x=x^*} = 0. \quad (19)$$

Since $T(q)$ satisfies the conditions in (17), if $x > L$, then

$$T(q_\infty) + x \frac{dT(q_\infty)}{dq_\infty} \frac{dq_\infty}{dx} \bigg|_{x=x^*} \leq \frac{dT(q_\infty)}{dq_\infty} \left(q_\infty + x \frac{dq_\infty}{dx}\right) \bigg|_{x=x^*} = \frac{dT(q_\infty)}{dq_\infty} e^{-x} \sum_{i=0}^{L-1} \frac{x^i}{i!} (1+i-x) \bigg|_{x=x^*} < 0. \quad (20)$$

Therefore, $0 < x^* \le L$ must hold. According to Theorem 2, function $q_v^*(\hat{p})$ is non-decreasing in $\hat{p}$.

Because the "channel crowdedness" measure $q_v$ is common to all users, their derived transmission probability targets $\hat{p}$ must be identical. Consequently, all users must have the same transmission probability at any equilibrium of the system. Let $p$ be the user transmission probability at an arbitrary equilibrium. From $p = \hat{p}$, we know that $p$ must satisfy

$$q_v^*(p) = q_v(p) \tag{21}$$

Because the left-hand-side of equation (21) is non-decreasing in $p$ and the right-hand-side is decreasing in $p$, the solution to (21) must be unique. Therefore, (21) has a unique solution at $p^* = \frac{x^*}{K+1}$. ∎

As an example, utility functions satisfying condition (17) include the widely considered utility of symmetric system throughput weighted by a transmission energy cost $E \in [0, 1)$, i.e., $U(p, q) = pq - pE$ with $T(q) = q - E$.

## IV. Convergence with Channel Crowdedness Interpretation

Consider an arbitrary user in the system, say user $k$. Let $q_k$ be the conditional packet transmission success probability of user $k$. For a general multi-packet reception channel, the channel crowdedness variable $q_v$ can be written as

$$q_v = (1 - p_k)q_k + p_k d_k, \tag{22}$$

where $d_k$ is the probability that channel can support an additional virtual packet transmission conditioned on user $k$ transmitting a packet. When $L > 1$, $q_v$ cannot be measured by user $k$ unless additional feedback information is available to enable the measurement of $d_k$. In this section, we show that a simple probability updating algorithm can be developed if user $k$ interprets $q_v$ from $p_k$ and $q_k$. We will show in Section V by computer simulations that such a revised MAC algorithm can still lead the system to the desired unique equilibrium.

Given $\hat{p} = \frac{x^*}{\hat{K}+1}$, $\lambda = \lfloor \hat{K} \rfloor + 1 - \hat{K}$. Let us define the following two functions.

$$\begin{aligned} q^*(\hat{p}) &= \lambda \sum_{i=0}^{L-1} \binom{\lfloor \hat{K} \rfloor - 1}{i} \hat{p}^i (1-\hat{p})^{\lfloor \hat{K} \rfloor - 1 - i} \\ &+ (1-\lambda) \sum_{i=0}^{L-1} \binom{\lfloor \hat{K} \rfloor}{i} \hat{p}^i (1-\hat{p})^{\lfloor \hat{K} \rfloor - i}, \\ d^*(\hat{p}) &= \lambda \sum_{i=0}^{L-2} \binom{\lfloor \hat{K} \rfloor - 1}{i} \hat{p}^i (1-\hat{p})^{\lfloor \hat{K} \rfloor - 1 - i} \\ &+ (1-\lambda) \sum_{i=0}^{L-2} \binom{\lfloor \hat{K} \rfloor}{i} \hat{p}^i (1-\hat{p})^{\lfloor \hat{K} \rfloor - i}. \end{aligned} \tag{23}$$

We propose a two-step approach for user $k$ to calculate its transmission probability target $\hat{p}_k$. Key idea of the approach is to interpret channel crowdedness $q_v$ by writing $d_k$ in (22) as a function of $q_k$.

**Step 1:** In each time slot, user $k$ first measures its conditional packet success probability $q_k$. Then user $k$ should obtain an intermediate probability variable $\breve{p}_k$ from

$$q^*(\breve{p}_k) = q_k, \tag{24}$$

with one exception: if $q_k \le q^*(0)$, user $k$ should set $\breve{p}_k = 0$.

**Step 2:** In the second step, user $k$ should first interpret channel crowdedness $q_v$ as

$$\hat{q}_v = (1 - p_k)q_k + p_k d^*(\breve{p}_k). \tag{25}$$

User $k$ should then obtain the transmission probability target $\hat{p}_k$ from

$$q_v^*(\hat{p}_k) = \hat{q}_v, \tag{26}$$

with two exceptions: if $\hat{q}_v \ge q_v^*(\frac{x^*}{L+1})$, user $k$ should set $\hat{p}_k = \frac{x^*}{L+1}$, while if $\hat{q}_v \le q_v^*(0)$, user $k$ should set $\hat{p}_k = 0$.

While we are not yet able to obtain a convergence proof for the distributed MAC algorithm with the two-step approach, the following theorem shows that the two-step approach can actually be simplified to one step in the sense that user $k$ can directly use $\breve{p}_k$ obtained in the first step as its transmission probability target.

**Theorem 4:** Assume that $K \ge L$ and $x^* \le L$. If users calculate their transmission probability targets using the two-step approach, then for each user, say user $k$, $q_k \ge q^*(p_k)$ implies $\hat{p}_k \ge p_k$ and $q_k \le q^*(p_k)$ implies $\hat{p}_k \le p_k$.

*Proof:* By following a similar proof of Theorem 2, we can see that both functions $q^*(\hat{p})$ and $q_v^*(\hat{p})$ are non-decreasing in $\hat{p}$. Consequently, if $q_k \ge q^*(p_k)$, in the first step of the two-step approach, we must have $\breve{p}_k \ge p_k$. Because $q^*(\breve{p}) \ge d^*(\breve{p})$, we get the following inequality

$$q_v^*(\breve{p}) = (1-\breve{p})q^*(\breve{p}) + \breve{p}d^*(\breve{p}) \le (1-p_k)q^*(\breve{p}) + p_k d^*(\breve{p}) = \hat{q}_v. \tag{27}$$

From monotonicity of $q_v^*(.)$, $\breve{p}_k \ge p_k$ implies $q_v^*(\breve{p}) \ge q_v^*(p_k)$. Consequently, we must have

$$\hat{q}_v \ge q_v^*(\breve{p}) \ge q_v^*(p_k). \tag{28}$$

This implies that, in the second step of the two-step approach, we will get $\hat{p}_k \ge p_k$. Similarly, $q_k \le q^*(p_k)$ implies $\hat{p}_k \le p_k$. ∎

Theorem 4 implies that, if the two-step approach can indeed lead the system to the desired equilibrium, then as an equivalent alternative, in each time slot, user $k$ can simply compare its own conditional transmission success probability $q_k$ with $q^*(p_k)$ to determine whether $p_k$ should be increased or decreased.

## V. Simulation Results

In this section, we use computer simulations to demonstrate convergence of the distributed MAC algorithm proposed in Section IV.

**Example 1:** In the first example, we consider a multiple access system with $K = 12$ users. Assume that the multiple access channel can support no more than $L = 3$ users transmitting in parallel. We choose $T(q) = q$, which means that users intend to maximize the symmetric system throughput. $x^*$ in this case is calculated from (7) to be $x^* = 2.27$. If all users transmit with the same probability $p$, conditional packet success probability experienced by each user should equal $q = \sum_{i=0}^{2} \binom{K-1}{i} p^i (1-p)^{K-1-i}$. Transmission probabilities of the users are initialized randomly. In each time slot, each user, say user $k$, measures its conditional packet success probability $q_k$. We assume that such a measurement is precise. User $k$ then obtains the transmission probability target $\hat{p}_k$ from $q^*(\hat{p}_k) = q_k$. As explained in Section IV, this is equivalent to the two-step approach of calculating the transmission probability target. Step size of the probability update is set at $\alpha = 0.05$. Convergence of the sum throughput of the system is illustrated in Figure 1. The dashed line represents the sum throughput if each user transmits with probability $p^* = \frac{x^*}{K+1} = 0.17$. The dash-dotted line represents the optimal sum throughput obtained by $KU^* = \max_{0\le p\le 1} Kp \sum_{i=0}^{2} \binom{K-1}{i} p^i (1-p)^{K-1-i}$. In this case, since $K = 12$ is much larger than $L = 3$, sum throughput at the system equilibrium is very close to the optimal sum symmetric throughput of the system.

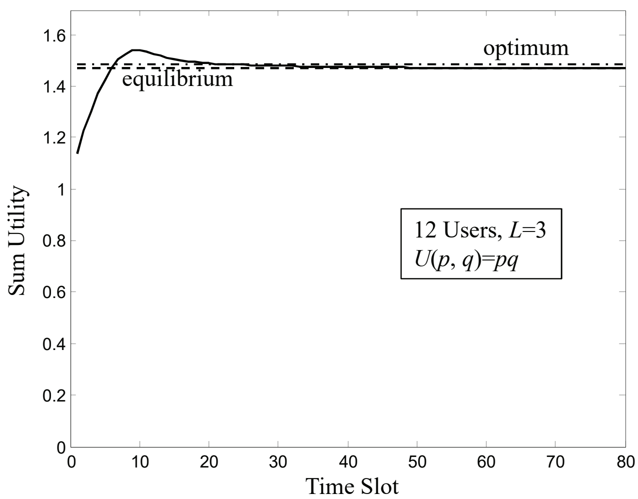


Fig. 1. Sum throughput of system with $K = 12$ users and $L = 3$.

**Example 2:** In the second example, we consider a multiple access system with $K = 10$ users. Assume that the multiple access channel can support no more than $L = 5$ users transmitting in parallel. Utility function of the system is chosen to be $U(p, q) = pq - 0.5p$, which is the symmetric system throughput weighted by a transmission energy cost. $x^*$ in this case equals $x^* = 2.62$. In this example, we initialize transmission probabilities of all users at zero. As in the first example, in each time slot, each user uses the one-step approach to calculate its transmission probability target. Step size of the probability update is set at $\alpha = 0.05$. Convergence of the sum network utility is illustrated in Figure 2. The dashed line represents the sum network utility at the system equilibrium corresponding to $p^* = \frac{x^*}{K+1} = 0.24$. The dash-dotted line represents the optimal sum throughput obtained by $KU^* = \max_{0\le p\le 1} Kp \left( \sum_{i=0}^{5} \binom{K-1}{i} p^i (1-p)^{K-1-i} - 0.5 \right)$. In this case sum utility at the system equilibrium is about 9% below the optimum value.

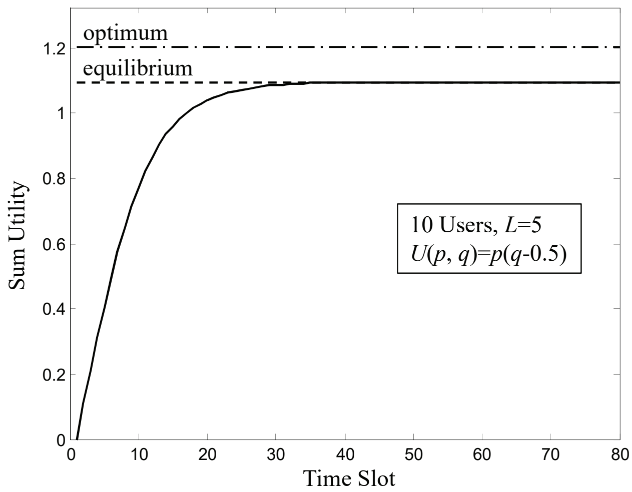


Fig. 2. Sum utility of system with $K = 10$ users and $L = 5$.

## VI. Conclusion

We investigated the problem of distributed MAC in a wireless multiple access network with an unknown finite number of homogeneous users over a class of multi-packet reception channels. We presented a transmission probability adaptation model that falls into the stochastic approximation framework. We defined "channel crowdedness" as the conditional success probability of a virtual packet transmission. Under the assumption that channel crowdedness can be measured locally, we obtained a sufficient condition for transmission probabilities of all users to converge to a unique equilibrium that maximizes a class of network utility functions. In cases when the assumption does not hold, we proposed a simple approach for each user to interpret channel crowdedness and showed by computer simulations that the proposed approach can still lead the system to the desired unique equilibrium.

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