An Improved Complex Sphere Decoder for V-BLAST Systems

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Abstract—**A new complex sphere decoding algorithm is presented for signal detection in V-BLAST systems, which has a computational cost that is significantly lower than that of the original complex sphere decoder (SD) for a wide range of SNRs. Simulation results on a 64-QAM system with 23 transmit and 23 receive antennas at an SNR per bit of 24 dB show that the new sphere decoding algorithm obtains the ML solution with an average cost that is at least 6 times lower than that of the original complex SD. Further, the new algorithm also shows robustness with respect to the initial choice of sphere radius.**

Index Terms—**Branch and bound, complex sphere decoder, ML detection, V-BLAST.**

I. Introduction

THE V-BLAST multiple antenna architecture [1] is a well-known method for achieving high spectral efficiencies over a rich-scattering environment. Since high data rates are achieved by employing many transmit antennas and/or higher order signal constellations, brute-force maximum likelihood (ML) detection is infeasible for practical systems. However, because many communication systems operate at moderate-to-high SNRs, reduced search algorithms can often obtain the ML solution with an average computational cost that is significantly lower than that of a brute-force approach. Most notable among the reduced search algorithms is the sphere decoder (SD) [2], which has been applied to the ML detection problem for multiple antenna systems only recently [3]–[5].

In early applications of the SD to multiple antenna systems, the complex system was decoupled into its real and imaginary parts to form a real-valued system, whose dimension was twice that of the original system. This decoupling is permissible when the signal constellation has a lattice representation, such as in QAM. Hence, for other complex constellations, such as PSK, the real-valued SD is inefficient, since computational resources are wasted anytime infeasible candidates arise in the solution process.

Recently in [5], a complex SD was introduced, which eliminates the need to decouple the system, and thus solves the problem of infeasible candidates. Since the complex SD avoids decoupling, it also possesses a speed advantage over the real-valued SD. However, like the real-valued SD, the complex SD does not utilize statistical information about the system model to speed-up the solution process. The only time statistical information is employed is in the selection of the intial sphere radius, which has a significant impact on the complexity of the algorithm.

Previously, in [6], a fast optimal depth-first branch and bound (BB) algorithm was proposed for the detection of binary symbols in synchronous CDMA channels that provided a substantial speed-up over a real-valued SD. In this letter, we extend the key ideas developed in [6] to the signal detection problem in V-BLAST systems, and present an improved complex sphere decoding algorithm. The new algorithm differs from the original SD in the following ways:

1) The use of the system ordering criterion proposed in [7] to maximize the probability that the first feasible solution is optimal. The application of this idea to reduced search algorithms was originally proposed in [6] and represents the most important step in the new algorithm.
2) The use of a minimum mean square error criterion in the selection of candidates.
3) The application of a "best-first" search strategy, which attempts to find new feasible solutions with as few number of steps as possible.

Simulation results show that the new complex SD offers a substantial reduction in the computational cost over that of [5], and is relatively insensitive to the choice of initial sphere radius.

The rest of the paper is organized as follows. In Section II, the system model for the new algorithm is given. The algorithm is outlined in Section III. In Section IV, the efficiency of the algorithm is demonstrated with simulation results, and the paper concludes in Section V with a brief summary.

II. System Model

Consider a symbol-synchronized multiple antenna system with n_T transmit antennas and n_R receive antennas where we take $n_R \geq n_T$. Assuming narrow band transmission, the received signal vector is given by

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{a} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H}$ is the $n_R \times n_T$ complex-valued channel matrix, $\boldsymbol{a}$ is the vector of transmitted symbols, and $\boldsymbol{n}$ is a zero mean complex white Gaussian noise vector with covariance matrix equal to $2\sigma^2\boldsymbol{I}$. Assuming a rich-scattering environment, the elements of the channel matrix are i.i.d complex Gaussian with zero mean. In addition, we also assume that transmissions are orgnaized into bursts of L symbol durations $(L \gg 1)$, where $\boldsymbol{H}$ is constant during the burst, but changes randomly from one burst to

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the next (quasistatic channel). The channel is assumed to be unknown at the transmitters, but is assumed to be estimated accurately at the recievers through the use of embedded training symbols in each burst.

Multiplying both sides of (1) from the left by $\boldsymbol{H}^*$, where "$*$" denotes the conjugate transpose, we obtain

$$\boldsymbol{y} = \boldsymbol{G}\boldsymbol{a} + \boldsymbol{w} \tag{2}$$

where $\boldsymbol{y} = \boldsymbol{H}^*\boldsymbol{r}, \boldsymbol{G} = \boldsymbol{H}^*\boldsymbol{H}$, and $\boldsymbol{w}$ is a complex Gaussian noise with zero mean and covariance $2\sigma^2\boldsymbol{G}$. Applying the permuation matrix $\boldsymbol{P}$ to (2), which is defined by Theorem 1 of [7], we obtain[1]

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{G}}\boldsymbol{b} + \bar{\boldsymbol{w}} \tag{3}$$

where $\bar{\boldsymbol{y}} = \boldsymbol{P}\boldsymbol{y}, \bar{\boldsymbol{G}} = \boldsymbol{P}\boldsymbol{G}\boldsymbol{P}^T, \boldsymbol{b} = \boldsymbol{P}\boldsymbol{a}$ is the reordered signal vector, and $\bar{\boldsymbol{w}} = \boldsymbol{P}\boldsymbol{w}$. Let $\bar{\boldsymbol{G}} = \boldsymbol{F}^*\boldsymbol{F}$, where $\boldsymbol{F}$ is a lower triangular matrix. Multiplying both sides of (3) from the left by $(\boldsymbol{F}^*)^{-1}$, we obtain

$$\tilde{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{b} + \boldsymbol{v} \tag{4}$$

where $\tilde{\boldsymbol{y}} = (\boldsymbol{F}^*)^{-1}\bar{\boldsymbol{y}}$ and $\boldsymbol{v} = (\boldsymbol{F}^*)^{-1}\bar{\boldsymbol{w}}$ is a complex white Gaussian noise with zero mean and covariance $2\sigma^2\boldsymbol{I}$.

III. The New Complex Sphere Decoder

A. Preliminaries

1) Initial Sphere Radius: Since $\|\boldsymbol{v}\|^2/\sigma^2 = \|\tilde{\boldsymbol{y}} - \boldsymbol{F}\boldsymbol{b}\|^2/\sigma^2 \sim \chi^2$ with $2n_T$ degrees of freedom, a reasonable choice for the initial sphere radius C_0 is to choose it to satisfy $\Pr\{\|\boldsymbol{v}\|^2 \le C_0\} = 0.99$. Using α as a tuning parameter, we set $C_0 = 2\alpha\sigma^2 n_T$ and select α according to

$$\int_0^{\alpha n_T} \frac{x^{(n_T-1)}}{\Gamma(n_T)} e^{-x}\, dx = 0.99 \tag{5}$$

2) Computation of Coordinate Bounds: The bounds on the elements of $\boldsymbol{b}$ are obtained using the equations derived for the original complex SD [5]. The idea of [5] is to represent the signal constellation as concentric rings, each ring intersecting points of the constellation. Let ρ be the radius associated with a particular ring. Hence, any constellation point on that ring can be represented as $\rho e^{i\theta}$ where $i = \sqrt{-1}$ and $0 \le \theta < 2\pi$ is the phase angle associated with some constellation point on that ring. Define

$$\lambda_k = \left(\tilde{y}_k - \sum_{j=1}^{k-1} F_{kj}\hat{b}_j\right) \Big/ F_{kk} \tag{6}$$

where F_{kj} is element (k, j) of $\boldsymbol{F}$ and $\hat{b}_j$ is a tentative decision on b_j. Also, define the quantity

$$\eta = \frac{1}{2\rho|\lambda_k|}\left[\rho^2 + |\lambda_k|^2 - \frac{C}{F_{kk}^2}\right] \tag{7}$$

[1]As a consequence of assuming a quasistatic channel, the compuational cost of optimally ordering the system is negligible since the system only needs to be ordered once at the start of each new burst.

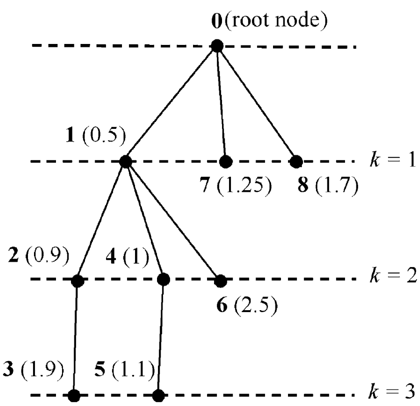

Fig. 1. Example tree formed by new complex sphere decoder.

where C is the current sphere radius. If $\eta < -1$, all the signal points on the ring are candidates for b_k. If $\eta > 1$, none of the points on the ring are candidates for b_k. For $-1 \le \eta \le 1$, only points which are located within the arc of the ring defined by the angle range

$$[\theta_{\lambda_k} - \cos^{-1}(\eta), \theta_{\lambda_k} + \cos^{-1}(\eta)] \tag{8}$$

are candidates for b_k where θ_{λ_k} is the phase angle of λ_k. In (8), it is assumed that $0 \le \cos^{-1}(\cdot) \le \pi$. To obtain the complete set of candidates for b_k, the aforementioned process is applied to all rings in the constellation.

B. Search Strategy

In the original SD method, once a new feasible solution is obtained, the search process starts over from the root node in the tree using the new smaller radius. In the new algorithm, an alternate search strategy is applied. Rather than restarting the search from the root node, the search resumes with the path that is closest to completion. The motivation behind this is to form new feasible solutions as quickly as possible. We illustrate this with a short example for $n_T = 3$. In the example in Fig. 1, the number in boldface by a node is the order of visitation of that node and the number in parenthesis is the total cost accrued to that point. After obtaining its first feasible solution via path $0 \to 1 \to 2 \to 3$, the algorithm selects node 4 from level 2 (the order of node selection for a level is decided by a "best-first" rule, that is, select the node that will raise the cost the least). This selection leads to a new feasible solution via path $0 \to 1 \to 4 \to 5$ with a cost of 1.1. After obtaining the new feasible solution and updating the cost, the algorithm selects the remaining node 6 in level 2 and finds that it has exceeded the new cost and terminates the search in level 2. Having exhausted all the nodes in level 2, the search resumes in level 1 with nodes 7 and then 8, but terminates there when the accrued cost for both nodes is found to exceed the new best cost. In our simulations, we have observed that this search strategy, when applied to an optimally ordered system, offers an improved complexity over a traditional SD search.

C. The Algorithm

The algorithm for the new complex SD is given as follows:

1) Compute $\tilde{\boldsymbol{y}}$ as given by (2)–(4) and $C = 2\alpha\sigma^2 n_T$ where α is given by (5). Set flag $= 0$.

2) Initialize $k=0, \boldsymbol{z}=\tilde{\boldsymbol{y}}, \xi=0$. Also, initialize the candidate sets $S_j=\emptyset$ for $j=1,2,\ldots n_T-1$, and set $\hat{\boldsymbol{b}}=\mathbf{0}$. If flag $=1$, set $C=1.2C$.
3) Set $k=k+1$ and $\lambda_k=z_k/F_{kk}$. Obtain the candidates (if any) for b_k using (7) and (8).
4) If candidates are found, goto step 7. Otherwise, goto step 5.
5) If the candidate sets are nonempty, goto step 13. Otherwise, goto step 6.
6) If no solution has been found yet, set flag $=1$ and goto step 2. Otherwise, STOP and report the "current-best" solution.
7) For each candidate x found in step 3, compute $\delta=|\lambda_k-x|^2$. Denote the smallest δ as δ_k. Let the set S_k comprise the candidates with δ's not equal to δ_k and arrange them in increasing order of their δ's so that the first candidate in S_k is associated with the smallest δ and the last with the largest. For the set S_k, store the quantities $\boldsymbol{z}, \{\hat{b}_i\}_{i=1}^{k-1}$, and ξ. For each candidate in S_k, store their associated δ's.
8) Set $\hat{b}_k$ equal to the candidate associated with δ_k.
9) Set $\xi=\xi+F_{kk}^2\delta_k$. If $\xi<C$, goto step 11. Otherwise, goto step 10.
10) If the candidate sets are nonempty, goto step 13. Otherwise, goto step 6.
11) If $k<n_T$, set $z_{k+1}=z_{k+1}-\sum_{j=1}^{k}F_{k+1,j}\hat{b}_j$ and goto step 3.
12) If $k=n_T$, update the "current-best" solution with $\hat{\boldsymbol{b}}$ and let $C=\xi$. If the candidate sets are nonempty, goto step 13. Otherwise, STOP and report the "current-best" solution.
13) Select the nonempty candidate set closest to the last tree level. From this candidate set, remove the first element and set the quantities $\boldsymbol{z}, \xi, \delta$, and $\{\hat{b}_i\}_{i=1}^{k}$ equal the values associated with the set and the chosen candidate. Set k equal to the level of the candidate set and $\delta_k=\delta$ and goto step 9.

IV. Simulation Results

In our simulations, the burst length was set equal to 100 symbol durations and the channel matrix $\boldsymbol{H}$ is generated randomly from one burst to the next with i.i.d elements $H_{ij}\sim\mathcal{CN}(0,1)$, where the real and imaginary parts are independent with zero mean and equal variance. Finally, we adjust $\sigma^2=(n_T\bar{E}_s)/(2\log_2(q))10^{-\mathrm{SNR}/10}$ where $\bar{E}_s$ is the average signal energy of the constellation and q is the size of the constellation.

Example 1: In the first example, we consider a 64-QAM system with SNR per bit equal to 24 dB. The average complexity of the new algorithm for decoding 100 transmit vectors of length n_T is compared against that of the original complex SD and the zero-forcing V-BLAST optimal order detector [1] as n_T is varied from 2 to 23 with $n_R=n_T$. The initial sphere radius of the original complex SD was adjusted so that the probability of finding a point within the sphere was 0.99. In the eventuality that no point was found within the sphere, the initial sphere radius was increased by a factor of 1.2 and the search process was restarted. From Fig. 2, we observe that the complexity of the new complex SD is actually lower than that of the VBLAST detector. At $n_T=23$, the original complex SD has a computational cost that is at least 6 times greater than that of the new algorithm.

Example 2: In the second example, we consider a 64-QAM system with $n_T=n_R=23$ and SNR per bit equal to 26 dB. The average complexity of the new algorithm for decoding 100 transmit vectors is compared against that of the original complex SD as the initial squared sphere radius is varied. From Fig. 3, we note that the computational cost of the complex SD increases very quickly as the initial sphere radius is increased while the cost of the new algorithm remains reasonably flat. The robustness to the initial radius by the new algorithm is mainly due to the system ordering, which lowers the probability of making an

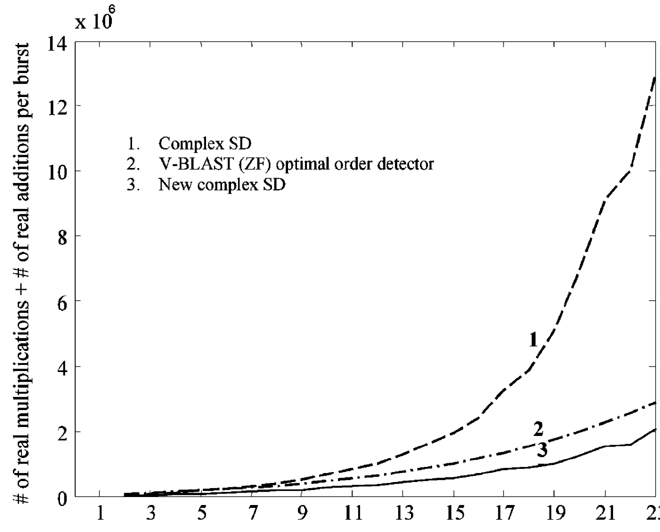

Fig. 2. Example 1: Average computational cost versus n_T. ($n_T=n_R$, 64-QAM, SNR per bit $=24$ dB, 500 Monte Carlo runs.)

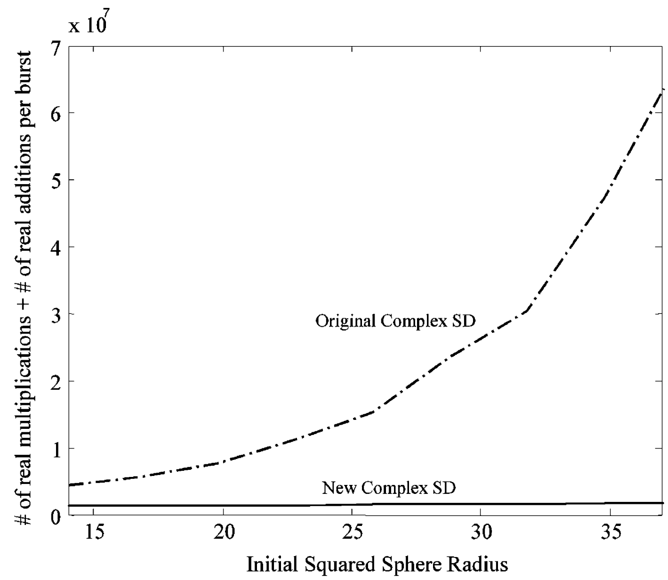

Fig. 3. Example 2: Average computational cost versus initial squared sphere radius. ($n_T=n_R=23$, 64-QAM, SNR per bit $=26$ dB, 500 Monte Carlo runs.)

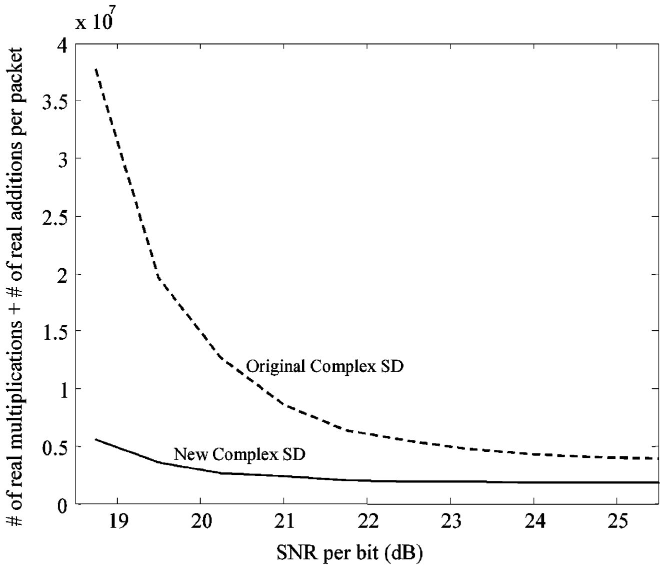

Fig. 4. Example 3: Average computational cost versus SNR per bit. ($n_T = n_R = 30$, 16-QAM, 500 Monte Carlo runs.)

early error in the tree; the upshot of this is a significantly smaller search space.

Example 3: In the third example, we consider a 16-QAM system with $n_T = n_R = 30$. The average complexity of the new algorithm for decoding 100 transmit vectors is compared against that of the original complex SD as the SNR per bit is varied from 18.5 dB to 25.5 dB. From Fig. 4, we note that the computational cost of the new complex SD remains low for the range of SNRs unlike the original algorithm.

V. Summary

A new complex SD is proposed, which utilizes the key idea of [6] along with other modifications to greatly speed-up the ML search process. Simulation results show that the new algorithm solves the ML detection problem with an average computational cost that is significantly lower than the original complex SD and is relatively robust to the initial sphere radius.

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