Iterative Multiuser Decoding Based on Probabilistic Data Association

Peng Hui Tan, Lars K. Rasmussen and Jie Luo

Abstract

We propose a low complexity soft-input/soft-output multiuser detector based on probabilistic data association. This new scheme has a total computational complexity of $\mathcal{O}(K^2) + \mathcal{O}(2^\nu)$ per user, which is the same as the complexity of linear soft-input/soft-output multiuser detectors. Numerical results demonstrate that the proposed detector, together with rate 1/2 error control coding, outperforms linear alternatives and achieves single-user performance at loads beyond $2N$ users, where N is the processing gain.

I. Introduction

The optimal decoding scheme for convolutional coded code-division multiple-access (CDMA) systems described in [1] requires a computational complexity of $\mathcal{O}(2^{K\nu})$. Here K is the number of users and ν is the code constraint length. This is prohibitive for practical use, thus motivating the study of low-complexity, suboptimal decoders. By viewing a synchronous (respectively, asynchronous) CDMA channel as a block code (respectively, convolution code), an iterative decoding scheme that exchanges soft information between the multiuser detector and the forward error control (FEC) decoders can be employed. For coded CDMA systems, the multiuser detector determines the *a posteriori* probabilities (APPs) for the code bits, which are passed on to the single-user decoders [2]–[4]. Each user decodes the corresponding FEC code independently, using a full complexity APP algorithm [5]. With iterative decoding, the performance of the coded multiuser system approaches single user performance at moderate to high signal-to-noise ratios. The complexity of these methods, $\mathcal{O}(2^K) + \mathcal{O}(2^\nu)$, is, however, still prohibitive for channels with a medium to large number of users.

Many iterative decoding schemes for coded CDMA have been proposed. In [6], a soft interference cancellation structure is proposed as an approximation to the APP detector. The extrinsic information for user k is computed based on the probability density function of the matched filter output given soft symbol estimates of other users. Here, the multiple access interference (MAI) is approximated by a Gaussian random variable. To further suppress the residual interference, linear minimum mean-square error (LMMSE) filtering can be applied after cancellation [4, 7, 8].

The contributions of our paper are as follows. A low complexity suboptimal soft-input/soft-output (SISO) multiuser detector based on probabilistic data association (PDA) is proposed for iterative decoding of convolutional coded multiuser data. This new scheme has a total computational complexity of $\mathcal{O}(K^2)+\mathcal{O}(2^\nu)$ per user, which is comparable to the complexity of iterative decoding based on linear SISO multiuser detectors. The PDA method, originally suggested for target tracking [9], was first introduced to multiuser detection for synchronous CDMA in [10]. The decision variables of the users are modelled as binary random variables, and the MAI is approximated as Gaussian noise with an appropriate covariance matrix. The APP associated with each code bit of a user is updated sequentially given the associated APPs of the other users.

P. H. Tan {phtan@ce.chalmers.se} and L. K. Rasmussen are with the Dept. of Computer Engineering, Chalmers University of Technology, Göteborg, Sweden. L. K. Rasmussen is also with the Institute for Telecommunications Research (ITR), University of South Australia, Adelaide, Australia. J. Luo is with the Institute for Systems Research, University of Maryland, College Park, Maryland, USA. P. H. Tan is supported by the Swedish Research Council under Grant 27-1999-390 and by Personal Computing and Communication (PCC++) under Grant PCC-0201-09. L. K. Rasmussen is supported by the Australian Government under ARC Grant DP0344856. Part of this work was performed while P. H. Tan was visiting ITR.

II. System Description

For conceptual ease, we consider a symbol-synchronous coded CDMA system with K active users. The PDA detector can also be modified to suit an asynchronous system [11]. Each user is assigned a normalized modulation waveform, consisting of N chips. The binary information data sequence for user k, $\{b_k\}$ (for $k = 1, \ldots, K$) is encoded by an FEC encoder with rate R_k. Each user transmits a sequence of $R_k L$ information symbols, corresponding to code symbol frames of length L symbols. The encoded sequence $\{d_k\}$ for each user is independently interleaved and modulated onto binary phase-shift keying code symbols of duration T. Each code symbol is further modulated by a spreading waveform $s_k(t)$ and transmitted over the channel. It is assumed that $s_k(t)$ is a unit energy waveform with support only in the interval $[0, T]$.

The received signal is described by the superposition of the signals from the K active users and additive white Gaussian noise,

$$r(t) = \sum_{k=1}^{K} A_k \sum_{i=0}^{L-1} d_k(i) s_k(t - iT) + n(t),$$

where A_k is the amplitude of user k, and $n(t)$ is an additive white Gaussian noise process with power spectral density $N_0/2$. To obtain a sufficient statistic for all the code bits of all the users, $r(t)$ is passed through a bank of filters matched to the signal waveforms $s_k(t)$ for $k = 1, \ldots, K$ and sampled at bit rate. The sampled signal at bit interval i can be expressed by the K-tuple vector

$$\mathbf{y}(i) = \mathbf{R}(i)\mathbf{A}(i)\mathbf{d}(i) + \mathbf{n}(i) \tag{1}$$

where $\mathbf{R}$ is the correlation matrix, $\mathbf{A} = \text{diag}[A_1, \ldots, A_K]$ is a diagonal matrix with received amplitudes, $\mathbf{d}(i) = [d_1(i), \ldots, d_K(i)]^{\mathrm{T}}$ is the data bit vector, and $\mathbf{n}(i)$ is a zero mean Gaussian noise vector with $E[\mathbf{n}(i)\mathbf{n}(i)^{\mathrm{T}}] = \sigma^2 \mathbf{R}$ [4].

The structure of the iterative multiuser receiver is shown in Fig. 1 [4]. It consists of a SISO multiuser

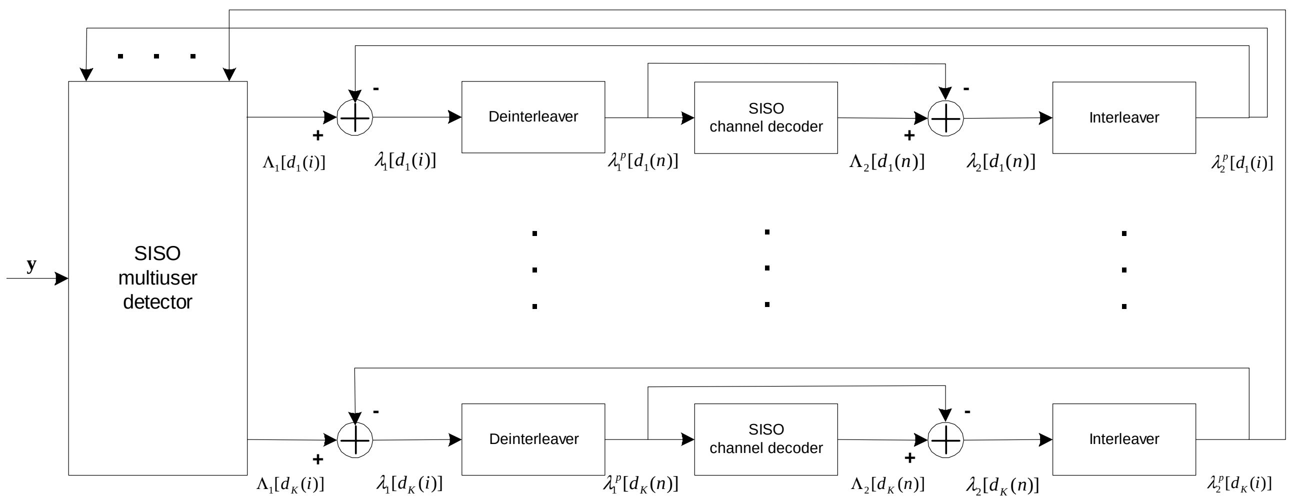

Fig. 1. A coded CDMA system with iterative multiuser receiver.

detector followed by K parallel single-user SISO FEC decoders. The SISO multiuser detector delivers *a posteriori* log-likelihood ratios (LLRs) for every code bit of every user,

$$\Lambda_1[d_k(i)] = \log \frac{P[d_k(i) = +1 | \mathbf{y}(i), \{\lambda_2^p[d_k(i)]\}_{i=0}^{L-1}]}{P[d_k(i) = -1 | \mathbf{y}(i), \{\lambda_2^p[d_k(i)]\}_{i=0}^{L-1}]} = \underbrace{\log \frac{p[\mathbf{y}(i) | d_k(i) = +1]}{p[\mathbf{y}(i) | d_k(i) = -1]}}_{\lambda_1[d_k(i)]} + \underbrace{\log \frac{P[d_k(i) = +1]}{P[d_k(i) = -1]}}_{\lambda_2^p[d_k(i)]}. \tag{2}$$

The first term in (2), denoted by $\lambda_1[d_k(i)]$, represents the extrinsic information that will be determined by the SISO multiuser detector. The second term in (2), denoted by $\lambda_2^p[d_k(i)]$, represents the *a priori* LLR of $d_k(i)$. For iterative decoding, $\lambda_2^p[d_k(i)]$ is computed by the SISO FEC decoder of user k in the previous iteration, interleaved, and then fed back to the SISO multiuser detector. For the first iteration, no prior information is available; we have $\lambda_2^p[d_k(i)] = 0$ for all k and i. The extrinsic information $\lambda_1[d_k(i)]$ is deinterleaved and then fed into the FEC decoder of user k as the *a priori* information of the code bit.

Based on the *a priori* LLR $\lambda_1[d_k(i)]$ obtained from the SISO multiuser detector and the trellis structure of the channel code, the SISO FEC decoder of user k computes the *a posteriori* LLR of each code bit

$$\Lambda_2[d_k(n)] = \log \frac{P[d_k(n) = +1 | \{\lambda_1^p[d_k(n)]\}_{n=0}^{L-1}]}{P[d_k(n) = -1 | \{\lambda_1^p[d_k(n)]\}_{n=0}^{L-1}]} = \lambda_2[d_k(n)] + \lambda_1^p[d_k(n)].$$

The extrinsic information $\lambda_2[d_k(n)]$ is then interleaved and fed back to the SISO multiuser detector as the *a priori* information of the code bits in the next iteration. The *a posteriori* LLR $\Lambda_2[d_k(n)]$ is used to make a decision on the decoded bit at the last iteration.

III. Iterative Multiuser Detector

A. Full-Complexity SISO Multiuser Detector

For notational ease, we now omit the symbol index i. From (1) and (2), the extrinsic information $\lambda_1[d_k]$ delivered by the SISO multiuser detector is given by [4]

$$\lambda_1[d_k] = \log \frac{p(\mathbf{y}|d_k = +1)}{p(\mathbf{y}|d_k = -1)} = \frac{2A_k y_k}{\sigma^2} + \log \frac{\sum_{\mathbf{d}:d_k=+1} f(\mathbf{d})}{\sum_{\mathbf{d}:d_k=-1} f(\mathbf{d})}, \tag{3}$$

where

$$f(\mathbf{d}) = \exp\left[-\mathbf{d}^{\mathrm{T}}\mathbf{ARAd}/(2\sigma^2)\right] \prod_{j \neq k} \left[1 + d_j \tanh\left(A_j y_j/\sigma^2\right)\right] \left[1 + d_j \tanh\left(\lambda_2^p[d_j]/2\right)\right]. \tag{4}$$

It is seen from (4) that the extrinsic information $\lambda_1[d_k]$ at the output of the SISO multiuser detector consists of two terms. The first term is the received observation value of the desired user $A_k y_k/\sigma^2$, and the second term is the information extracted from the observations of the user users $\{y_j\}_{j \neq k}$ as well as the corresponding *a priori* information $\{\lambda_2^p[d_j]\}_{j \neq k}$.

B. Probabilistic Data Association SISO

The summations in the numerator and denominator of (3) are over all possible data vectors with $d_k = +1$ and $d_k = -1$, respectively. The computational complexity of the full-complexity SISO multiuser therefore grows exponentially with the number of users. As a low-complexity alternative, we propose a SISO multiuser detector based on PDA.

The PDA SISO detector is based on approximating the MAI as Gaussian noise. It can therefore be derived based on any linearly transformation of the sufficient statistic since it has been shown in [12] that the pdf of the output of any linear filter is well approximated by a Gaussian distribution. The derivation is however simplified if the input signal is decorrelated. The system model is thus reformulated as

$$\mathbf{z} = \mathbf{A}^{-1}\mathbf{R}^{-1}\mathbf{y} = d_k \mathbf{e}_k + \sum_{j \neq k} d_j \mathbf{e}_j + \mathbf{v}. \tag{5}$$

where $\mathbf{e}_k = [0, 0, ...0, 1, 0, ..., 0]^{\mathrm{T}}$ with a one in position k. For any user k, we associate a probability P_k as the current estimate of the probability that $d_k = +1$, while $1 - P_k$ is the corresponding estimate for $d_k = -1$. These probabilities $\{P_k\}$ are initialized to 0.5. Based on the current values of $\{P_j\}_{j \neq k}$, we update P_k by

$$P_k = P[d_k = +1 | \mathbf{z}, \{P_j\}_{j \neq k}]. \tag{6}$$

The computational complexity of (6) is still growing exponentially with the number of users. To avoid that, we define the effective noise as

$$\mathbf{u}_k = \sum_{j \neq k} d_j \mathbf{e}_j + \mathbf{v}$$

and approximate it by a Gaussian random vector with mean and covariance given by

$$\mathbf{m}_k = E[\mathbf{u}_k] = \sum_{j \neq k} 2(P_j - 1)\mathbf{e}_j$$

and

$$\mathbf{\Omega}_k = Cov[\mathbf{u}_k] = \sum_{j \neq k} 4P_j(1 - P_j)\mathbf{e}_j\mathbf{e}_j^{\mathrm{T}} + \sigma^2 \mathbf{A}^{-1}\mathbf{R}^{-1}\mathbf{A}^{-1}, \tag{7}$$

respectively. The updated LLR for user k is given by

$$\log \frac{P_k}{1 - P_k} = -2(\mathbf{m}_k - \mathbf{z})^{\mathrm{T}}\mathbf{\Omega}_k^{-1}\mathbf{e}_k + \lambda_2^p[d_k], \tag{8}$$

where $\lambda_2^p[d_k]$ is the *a priori* LLR provided by the SISO FEC decoder. This procedure is then repeated for all users sequentially. The algorithm continues until all the probabilities $\{P_k\}$ have converged or the maximum number of stages[1] has been reached. For the numerical examples presented in Section IV, the PDA SISO multiuser detector usually converges in less than 5 stages. Efficient numerical schemes with computational complexity of the order of $\mathcal{O}(K^2)$ for updating $\mathbf{\Omega}_k^{-1}$ is presented in [10]. The overall complexity of each stage in the PDA SISO is then $\mathcal{O}(K^3)$ for all K users.

After the completion of the PDA SISO multiuser detector, the extrinsic information delivered is (from (8))

$$\tilde{\lambda}_1[d_k] = \log \frac{p(\mathbf{z}|d_k = +1, \{P_j\}_{j \neq k})}{p(\mathbf{z}|d_k = -1, \{P_j\}_{j \neq k})} = -2(\mathbf{m}_k - \mathbf{z})^{\mathrm{T}}\mathbf{\Omega}_k^{-1}\mathbf{e}_k,$$

which is delivered to the SISO FEC decoders in parallel for processing and completion of one iteration of the iterative decoder.

Using iterative decoding techniques, it is possible to approach single-user performance even when the system is overloaded, i.e., $K > N$. To overcome the singularities that will inevitably occur in $\mathbf{R}$, we let $\mathbf{R} = \mathbf{R} + \delta\mathbf{I}$. As demonstrated by the numerical examples in Section IV, the performance of the PDA SISO multiuser detector does not deteriorate as long as δ is reasonably small. A more elegant approach to avoiding this problem is to apply the PDA algorithm directly on the chip matched filtered output, demonstrating that it is not a flaw inherent to the PDA SISO detector.

IV. NUMERICAL RESULTS

In the examples presented, all users employ the same rate $1/2$ constraint length $\nu = 2$ convolutional code with generators (5,7) in octal notation, and have the same block size of 500 information bits.

To evaluate the performance of the PDA detector, we consider a 14-user system with random signature sequences of length $N = 8$. In Fig. 2, we observe that the BER performance approaches the single user performance after 7 iterations. In Fig. 3, the BER performance against the number of iterations is compared for the PDA and the LMMSE SISO multiuser detectors [4, 7] at an $E_b/N_0 = 5$ dB. The LMMSE SISO multiuser detector is among the best linear detectors suggested in connection with iterative decoding of coded CDMA [8]. We observe that the PDA SISO multiuser detector requires less iterations and thus lower computational complexity, to obtain the same BER performance as the LMMSE SISO. In addition, when $K = 17$, the PDA SISO is able to approach single–user performance, whereas the LMMSE SISO fails to converge for $K = 16$. The results in Fig. 3 even indicate that the PDA SISO may approach single–user performance for $K = 18$, given a large number of iterations.

[1] One stage is K iterations of the PDA SISO multiuser detector where all the users $\{P_k\}$ have been updated once.

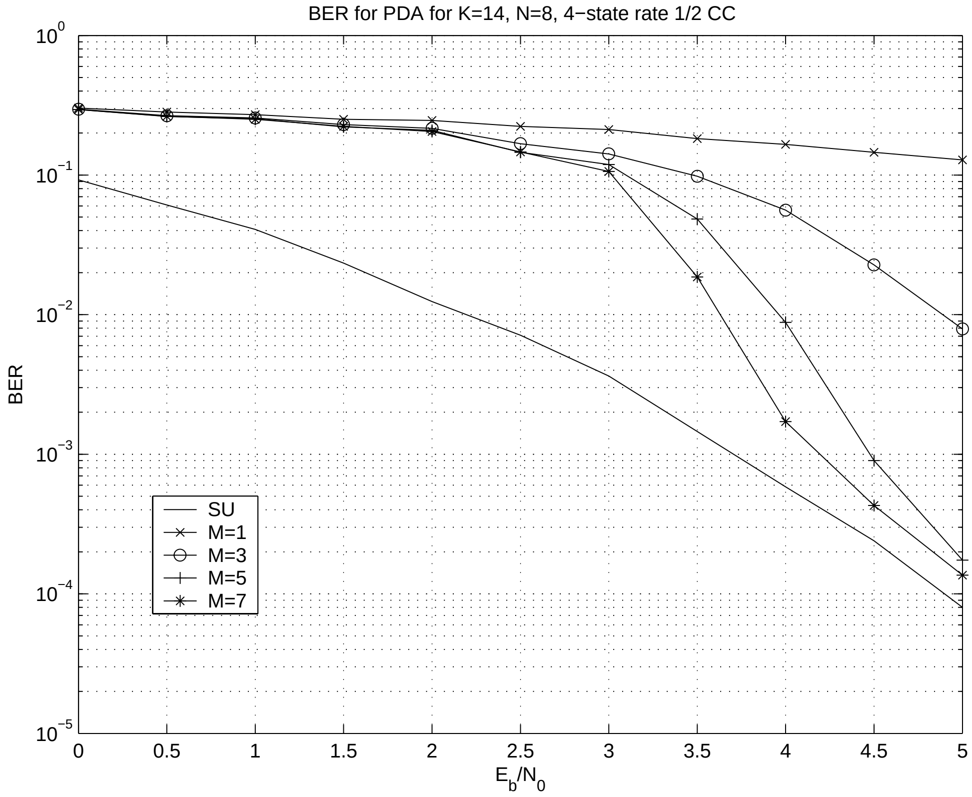

Fig. 2. BER performance of the PDA SISO multiuser detector after a different number of iterations M. $K = 14$, $N = 8$.

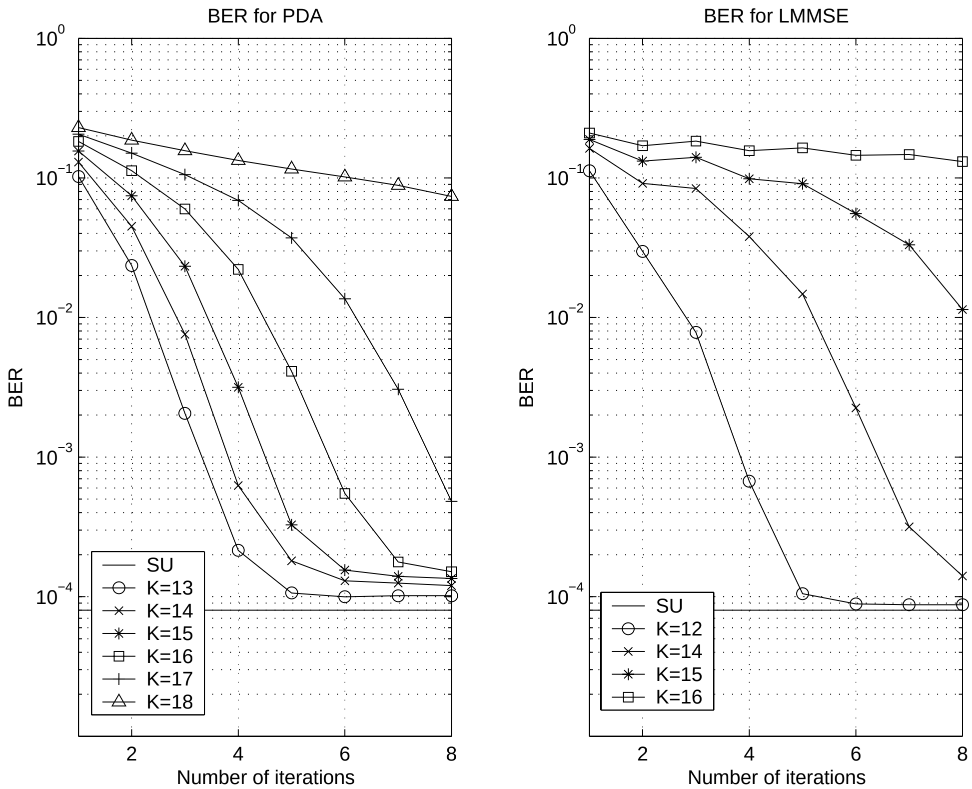

Fig. 3. Comparison of BER performance of the PDA and the LMMSE SISO multiuser detectors for systems with high load. $N = 8$, $E_b/N_0 = 5$ dB.

V. Conclusion

Full complexity APP multiuser detection is recognized as being prohibitively complex for a moderate number of active users. Here, we have presented a low complexity alternative based on probabilistic data association. This PDA SISO multiuser detector has a total computational complexity of the same order as linear SISO multiuser detectors, but has been shown to significantly outperform these linear alternatives. Using rate 1/2 error control coding, single-user performance is achieved for loads beyond $2N$ users, where N is the processing gain.

References

[1] T. R. Giallorenzi and S. G. Wilson, "Multiuser ML sequence estimator for convolutional coded asynchronous DS-CDMA systems," *IEEE Trans. Commun.*, vol. 44, pp. 997–1008, Aug. 1996.

[2] M. Moher, "Multi-user detection for DS-CDMA communications," *IEEE Trans. Commun.*, vol. 46, pp. 870–880, July 1998.

[3] M. C. Reed, C. B. Schlegel, P. D. Alexander, and J. A. Asenstorfer, "Iterative multiuser detection for CDMA with FEC: Near-single-user performance," *IEEE Trans. Commun.*, vol. 46, pp. 1693–1699, Dec. 1998.

[4] X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, vol. 47, pp. 1046–1061, July 1999.

[5] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimum decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. 20, pp. 284–287, 1974.

[6] P. D. Alexander, A. J. Grant, and M. C. Reed, "Iterative detection in code-division multiple-access with error control coding," *Eur. Trans. Telecommun.*, vol. 9, pp. 1046–1061, Oct. 1998.

[7] H. El-Gamal and E. Geraniotis, "Iterative multiuser detection for coded CDMA signals in AWGN and fading channels," *IEEE J. Selected Areas Commun.*, vol. 18, pp. 30–41, 2000.

[8] L. K. Rasmussen, A. J. Grant, and P. D. Alexander, "Recursive filters for iterative multiuser decoding," *in Proc. Int. Symposium Inf. Theory '01 (Lausanne, Switzerland)*, p. 219, June 2002.

[9] Y. Bar-Shalom and E. Tse, "Tracking in A Clusttered Environment with Probabilistic Data Association," *Automatic*, vol. 11, pp. 451-460, Sept. 1975.

[10] J. Luo, K. Pattipati, P. Whillett, and F. Hasegawa, "Near optimal multiuser detection in synchronous CDMA," *IEEE Comm. Lett.*, vol. 5, pp. 361–363, Sept. 2001.

[11] J. Luo, K. Pattipati, and P. Whillett, "A sliding window PDA for asynchronous CDMA and a proposal for deliberate asynchronicity," *submitted to IEEE Trans. Commun.*, 2002.

[12] D. Guo, S. Verdú, and L. K. Rasmussen, "Asymptotic normality of linear multiuser receiver outputs," *IEEE Trans. Inform. Theory*, to appear Dec. 2002.