Multiuser Detection in Asynchronous CDMA using PDA

J. Luo, K. Pattipati, P. Willett, F. Hasegawa[1]
ECE Department
University of Connecticut
Storrs, CT 06269
e-mail: krishna@engr.uconn.edu

Abstract — **The Probabilistic Data Association (PDA) method is extended to multiuser detection over asynchronous Code-Division Multiple Access (CDMA) communication channels. PDA models the undecided user signals as binary random variables. By approximating the Multiple Access Interference (MAI) as Gaussian noise with an appropriately evaluated covariance matrix, PDA updates the probabilities associated with user signals iteratively. A sliding window processing is used during the updates in order to avoid considering the entire data. Computer simulations show that the probability of group detection error of the proposed PDA method is very close to the performance lower bound. The computational complexity of the PDA detector is $O((2h+1)K^3)$ per time frame where K is the number of users and $2h+1$ is the width of the processing window. Relations between the proposed PDA detector and the PDA detector for synchronous CDMA is also shown.**

I. Introduction

Due to the NP-hard nature of the general multiuser detection problems in Code-Division Multiple Access (CDMA) communications [1], suboptimal algorithms that provide reliable decisions and ensure polynomial computational costs have been widely studied for over fifteen years. Linear detectors and decision-driven multiuser detectors are the two most popular categories [2]. The multiuser detectors among these two categories include the decorrelator [1], the Minimum Mean Square Error (MMSE) detector [3], the Decision Feedback (DF) detector [4] [7], the multistage detector [5] and the group detector [6] [9]. Compared with the conventional matched-filter detector, these sub-optimal detectors provide significantly better accuracies and the overall computational complexities are $O(K^2)$ where K is the number of users. However, in many cases, there is still a large gap between the performances of these detectors and that of the optimal Maximum Likelihood (ML) detector.

Recently, the Probabilistic Data Association (PDA) detector [8] and the semidefinite relaxation detector [10] have been proposed recently for the multiuser detections of symbol-synchronous CDMA. The probabilities of group detection error of these detectors are very close to that of the optimal ML detector. Although the complexities of both detectors are in the order of $O(K^3)$, the computational cost of the PDA method is much less than the semidefinite relaxation detector. Furthermore, since PDA works with probabilities and gives "soft" outputs, it is naturally flexible and relatively easy to extend to other more complicated CDMA systems.

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In this paper, we extend the PDA method to the multiuser detection in asynchronous CDMA communications. We first treat each bit in asynchronous CDMA as transmitted by different fictitious users and view a K-user M-frame asynchronous CDMA system as a KM-user synchronous system [2]. By exploiting the special structure of the correlation matrix, the PDA multiuser detector for synchronous CDMA is simplified and applied to the equivalent synchronous system. A truncated processing window is introduced to avoid considering the entire transmission data. This is further extended to a sliding window version and directly follows the proposal of the PDA multiuser detector for asynchronous CDMA. Simulation results for both regular and overloaded systems are presented to show the near optimal performance of the proposed detector.

The rest of the paper is organized as follows. The system model and the PDA detector for synchronous CDMA are briefly reviewed in section II. The system model of asynchronous CDMA is given in section III. The PDA method is extended to the asynchronous case by viewing the K-user M-frame asynchronous system as a KM-user synchronous system and exploiting the special structure of the correlation matrix. By further introducing a sliding window processing, the PDA method for asynchronous CDMA is formalized in section IV. Simulation results are given in section V and the paper concludes in section VI.

II. Review of PDA Multiuser Detector for Synchronous CDMA

A discrete-time equivalent model for the matched-filter outputs at the receiver of a K-user synchronous CDMA channel is given by the K-length vector [2]

$$\boldsymbol{y} = \boldsymbol{R}\boldsymbol{W}\boldsymbol{b} + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{R}$ is the symmetric matrix with unity diagonal components; $\boldsymbol{W}$ is a diagonal matrix whose k-th diagonal element, w_k, is the square root of the received signal energy per bit of the k-th user; $\boldsymbol{b} \in \{-1,+1\}^K$ denotes the K-length vector of bits transmitted by the K active users; and $\boldsymbol{v}$ is a real-valued zero-mean Gaussian random vector with a covariance matrix $\sigma^2\boldsymbol{R}$.

Suppose $\boldsymbol{R} = \boldsymbol{L}^T\boldsymbol{L}$ is the Cholesky decomposition of the correlation matrix and $\boldsymbol{L}$ is a lower triangular matrix. The white-noise model of the system is obtained by multiplying $(\boldsymbol{L}^T)^{-1}$ on both sides of (1).

$$\tilde{\boldsymbol{y}} = \boldsymbol{L}\boldsymbol{W}\boldsymbol{b} + \tilde{\boldsymbol{v}} \tag{2}$$

where $\tilde{\boldsymbol{y}} = (\boldsymbol{L}^T)^{-1}\boldsymbol{y}$, and $\tilde{\boldsymbol{v}} = (\boldsymbol{L}^T)^{-1}\boldsymbol{v}$ is a zero mean white Gaussian noise with covariance matrix $\sigma^2\boldsymbol{I}$.

When all the user signals are equally probable, the optimal solution of (2) is the output of a ML detector

$$\Phi_{ML}: \hat{\boldsymbol{b}} = arg \min_{\boldsymbol{b} \in \{-1,+1\}^K} \|\boldsymbol{L}\boldsymbol{W}\boldsymbol{b} - \tilde{\boldsymbol{y}}\|_2^2 \tag{3}$$

Generally, obtaining the ML solution is NP-hard [1] unless the signature correlation matrix has a special structure.

The PDA method suggests that we treat the decision variables $\boldsymbol{b}$ as binary random variables. For any user i, associate a probability P_{bi} with user signal b_i to express the current estimation of the probability that $b_i = 1$. Now, for an arbitrary user signal b_i, treat the other user signals b_j, $(j \neq i)$ as binary random variables and $\sum_{j \neq i} \boldsymbol{l}_j w_j b_j + \tilde{\mathbf{v}}$ as the effective noise, where $\boldsymbol{l}_j$ denotes the j^{th} column of $\boldsymbol{L}$. Consequently, $P(b_i = 1 | \tilde{\boldsymbol{y}}, \{P_{bj}\}_{j \neq i})$ can be obtained from (2); this serves as updated information for user signal b_i. The multistage PDA detector for synchronous CDMA is presented as follows.

(1) Sort users according to the user ordering criterion proposed for decision feedback detector in [7].

(2) $\forall i$, initialize the probabilities as $P_{bi} = 0.5$. Initialize the stage counter $k = 1$.

(3) Initialize the user counter $i = 1$.

(4) Based on the current value of P_{bj} $(j \neq i)$ for user i, update P_{bj} by

$$P_{bi} = P\{b_i = 1 | \tilde{\boldsymbol{y}}, \{P_{bj}\}_{j \neq i}\} \tag{4}$$

(5) If $i < K$, let $i = i + 1$ and goto step (4).

(6) If $\forall i$, P_{bi} has converged, goto step (7). Otherwise, let $k = k + 1$ and return to step (3).

(7) $\forall i$, make a decision on user signal i via

$$b_i = \begin{cases} 1 & P_{bi} \geq 0.5 \\ -1 & P_{bi} < 0.5 \end{cases} \tag{5}$$

In step (4) of the above procedure, we define the effective noise as

$$\boldsymbol{V}_i = \sum_{j \neq i} \boldsymbol{l}_j w_j b_j + \tilde{\mathbf{v}} \tag{6}$$

and approximate it by a Gaussian noise with matched mean and covariance, which are

$$\begin{aligned} E[\boldsymbol{V}_i] &= \sum_{j \neq i} \boldsymbol{l}_j w_j (2P_{bj} - 1) \\ Cov[\boldsymbol{V}_i] &= \sum_{j \neq i} 4P_{bj}(1 - P_{bj}) w_j^2 \boldsymbol{l}_j \boldsymbol{l}_j^T + \sigma^2 \boldsymbol{I} \end{aligned} \tag{7}$$

Computationally efficient numerical schemes for updating (7) are presented in [8].

III. PDA Multiuser Detector for Asynchronous CDMA

Similar to the system model of (1), the asynchronous CDMA system can be described in the z domain by [2]

$$\boldsymbol{y}(z) = \boldsymbol{R}(z)\boldsymbol{W}\boldsymbol{b}(z) + \mathbf{v}(z) \tag{8}$$

where $\boldsymbol{v}$ is a colored Gaussian noise with zero mean and covariance $\sigma^2 \boldsymbol{R}(z)$. The signature correlation matrix $\boldsymbol{R}(z)$ is now composed of three parts [2]

$$\boldsymbol{R}(z) = \boldsymbol{R}[1]^T z + \boldsymbol{R}[0] + \boldsymbol{R}[1] z^{-1} \tag{9}$$

Here $\boldsymbol{R}[0]$ is a symmetric matrix with unity diagonal components and whose off-diagonal components represent the correlation between user signatures at the same time index; and $\boldsymbol{R}[1]$ is a singular matrix whose components represent the signature correlations relating to successive time frames. Denote the component on the i^{th} row and j^{th} column of $\boldsymbol{R}[1]$ by $R[1]_{ij}$: since user signal i in time frame n cannot simultaneously be correlated with that of j in time frame $n-1$ and in time frame $n+1$, we have $R[1]_{ij} R[1]_{ji} = 0$. It is shown in [4] that the correlation matrix $\boldsymbol{R}(z)$ can be factored as

$$\boldsymbol{R}(z) = (\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)(\boldsymbol{F}[0] + \boldsymbol{F}[1] z^{-1}) \tag{10}$$

where $\boldsymbol{F}[0]$ is a lower triangular matrix, and $\boldsymbol{F}[1]$ is singular. Applying the anticausal feed-forward filter $(\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)^{-1}$ to both sides of (8), we obtain the white noise model [4]

$$\tilde{\boldsymbol{y}}(z) = (\boldsymbol{F}[0] + \boldsymbol{F}[1] z^{-1})\boldsymbol{W}\boldsymbol{b}(z) + \tilde{\mathbf{v}}(z) \tag{11}$$

where $\tilde{\boldsymbol{y}}(z) = (\boldsymbol{F}[0]^T + \boldsymbol{F}[1]^T z)^{-1} \boldsymbol{y}(z)$ and $\tilde{\mathbf{v}}$ is a white Gaussian noise vector with zero mean and covariance matrix $\sigma^2 \mathbf{I}$. The corresponding time-domain representation of the white noise model is

$$\tilde{\boldsymbol{y}}(n) = \boldsymbol{F}[0]\boldsymbol{W}\boldsymbol{b}(n) + \boldsymbol{F}[1]\boldsymbol{W}\boldsymbol{b}(n-1) + \tilde{\mathbf{v}}(n) \tag{12}$$

Suppose there are overall M time frames in the transmission. We can view the asynchronous system as an MK-user synchronous system and rewrite (12) as

$$\tilde{\boldsymbol{Y}} = \tilde{\boldsymbol{L}}\tilde{\boldsymbol{W}}\tilde{\boldsymbol{b}} + \tilde{\boldsymbol{V}} \tag{13}$$

Here

$$\begin{aligned} \tilde{\boldsymbol{Y}} &= [\tilde{\boldsymbol{y}}(0)^T, \tilde{\boldsymbol{y}}(1)^T, \ldots, \tilde{\boldsymbol{y}}(M)^T]^T \\ \tilde{\boldsymbol{b}} &= [\boldsymbol{b}(0)^T, \boldsymbol{b}(1)^T, \ldots, \boldsymbol{b}(M)^T]^T \\ \tilde{\boldsymbol{V}} &= [\tilde{\mathbf{v}}(0)^T, \tilde{\mathbf{v}}(1)^T, \ldots, \tilde{\mathbf{v}}(M)^T]^T \\ \tilde{\boldsymbol{W}} &= \begin{bmatrix} \boldsymbol{W} & \mathbf{0} & \ldots \\ \mathbf{0} & \ldots & \mathbf{0} \\ \ldots & \mathbf{0} & \boldsymbol{W} \end{bmatrix} \end{aligned} \tag{14}$$

and

$$\tilde{\boldsymbol{L}} = \begin{bmatrix} \boldsymbol{F}[0] & \mathbf{0} & \ldots & \ldots \\ \boldsymbol{F}[1] & \boldsymbol{F}[0] & \mathbf{0} & \ldots \\ \mathbf{0} & \ldots & \ldots & \ldots \\ \ldots & \mathbf{0} & \boldsymbol{F}[1] & \boldsymbol{F}[0] \end{bmatrix} \tag{15}$$

is the Cholesky decomposition matrix of the equivalent synchronous system.

A. Direct Extension

Apparently, the computational cost of directly applying the PDA method to the equivalent MK-user system is $O\{(MK)^3\}$, which can be very high if M is not small. Fortunately, due to the special structure of the Cholesky decomposition matrix $\tilde{\boldsymbol{L}}$, the probability update in the PDA method can be simplified.

Consider updating the probability associated with user i in time frame n. From (12), we have

$$\begin{aligned} &P_{bi}(n) \\ &= P\left\{ b_i(n) = 1 \,\middle|\, \tilde{\boldsymbol{Y}}, \{P_{bj}(n)\}_{j \neq i}, \{P_{bl}(k)\}_{k \neq n} \right\} \\ &= P\left\{ b_i(n) = 1 \,\middle|\, \begin{matrix} \tilde{\boldsymbol{y}}(n), & \tilde{\boldsymbol{y}}(n+1) \\ \{P_{bj}(n)\}_{j \neq i}, & \{P_{bl}(k)\}_{k = n-1, n+1} \end{matrix} \right\} \end{aligned} \tag{16}$$

Therefore, to update the probability $P_{bi}(n)$, only two observation vectors, $\tilde{\boldsymbol{y}}(n), \tilde{\boldsymbol{y}}(n+1)$, are required. The corresponding observation model from (12) is

$$\begin{bmatrix} \tilde{\boldsymbol{y}}(n) \\ \tilde{\boldsymbol{y}}(n+1) \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}[0] \\ \boldsymbol{F}[1] \end{bmatrix} \boldsymbol{W}\boldsymbol{b}(n) + \begin{bmatrix} \boldsymbol{F}[1]\,\boldsymbol{W}\boldsymbol{b}(n-1) \\ \boldsymbol{F}[0]\,\boldsymbol{W}\boldsymbol{b}(n+1) \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{v}}(n) \\ \tilde{\boldsymbol{v}}(n+1) \end{bmatrix} \tag{17}$$

For user signal $b_i(n)$, define the effective noise as

$$\boldsymbol{V}_i(n) = \sum_{j\neq i} \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix} w_j b_j(n) + \begin{bmatrix} \boldsymbol{F}[1]\,\boldsymbol{W}\boldsymbol{b}(n-1) \\ \boldsymbol{F}[0]\,\boldsymbol{W}\boldsymbol{b}(n+1) \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{v}}(n) \\ \tilde{\boldsymbol{v}}(n+1) \end{bmatrix} \tag{18}$$

where $\boldsymbol{f}_j[0]$ and $\boldsymbol{f}_j[1]$ denote the jth columns of $\boldsymbol{F}[0]$ and $\boldsymbol{F}[1]$, respectively. Consequently, we have

$$\begin{aligned} E[\boldsymbol{V}_i(n)] &= \sum_{j\neq i} \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix} w_j(2P_{bj}(n)-1) \\ &+ \sum_{k=1}^{K} \begin{bmatrix} \boldsymbol{f}_k[1]w_k(2P_{bk}(n-1)-1) \\ \boldsymbol{f}_k[0]w_k(2P_{bk}(n+1)-1) \end{bmatrix} \\ Cov[\boldsymbol{V}_i(n)] &= \sigma^2\boldsymbol{I} \\ &+ \sum_{j\neq i} 4P_{bj}(n)(1-P_{bj}(n))w_j^2 \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix} \begin{bmatrix} \boldsymbol{f}_j[0] \\ \boldsymbol{f}_j[1] \end{bmatrix}^T \\ &+ \sum_{k=1}^{K} 4P_{bk}(n-1)(1-P_{bk}(n-1))w_k^2 \begin{bmatrix} \boldsymbol{f}_k[1]\boldsymbol{f}_k[1]^T & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \\ &+ \sum_{k=1}^{K} 4P_{bk}(n+1)(1-P_{bk}(n+1))w_k^2 \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{f}_k[0]\boldsymbol{f}_k[0]^T \end{bmatrix} \end{aligned} \tag{19}$$

Approximating $\boldsymbol{V}_i(n)$ by a Gaussian noise with matched mean and covariance, it is easy to see that the computational load for updating $P_{bi}(n)$ is $O(K^2)$. Therefore, the overall computational cost of the PDA detector is $O(MK^3)$, i.e., $O(K^3)$ per time frame, the same as in the case of synchronous CDMA.

B. PDA with Sliding Processing Window

As described in section II, PDA updates the associated probabilities iteratively. Therefore, in the above batch method, PDA can do iterations and make decisions on user signals only when the entire transmitted data has been received. This can consequently cause significant delays at the receiver.

Suppose we are only interested in decisions on user signal vector $\boldsymbol{b}(n)$. Consider a truncated processing window that contains user signal vectors $\boldsymbol{b}(m)$, $(n-h \leq m \leq n+h)$, i.e., the width of the processing window is $2h+1$. Due to the limited error propagation in practical systems, it is reasonable to assume that, if h is large enough, the effects of values of user signals outside the processing window on the decisions of $\boldsymbol{b}(n)$ are negligible. Therefore, when making decisions on $\boldsymbol{b}(n)$, one can apply the PDA method and perform iterations only within the truncated processing window.

Notice that the processing windows of user signals in successive time frames differ only slightly. Hence, we can use the probabilities from a processing window as the initial conditions of the PDA method for the next processing window to further simplify the iterative updates. This modifies the truncated-window PDA to a sliding-window PDA. The detailed procedure is described below:

(1) Sort users according to the user ordering and time labeling criterion proposed for decision feedback detector in [11].

(2) $\forall i$ and $\forall n$, initialize the probabilities as $P_{bi}(n) = 0.5$. Initialize the window counter $k = 1$.

(3) Initialize the time frame counter $n = \max\{1, k-h\}$.

(4) Initialize the user counter $i = 1$.

(5) Based on the current values of the associated probabilities, update $P_{bi}(n)$ according to (16).

(5) If $i < K$, let $i = i+1$ and goto step (5).

(6) If $n < \min\{M, k+h\}$, let $n = n+1$ and goto step (4).

(7) If $k > h$, $\forall i$, make a decision on user signal $b_i(k-h)$ via

$$b_i(k-h) = \begin{cases} 1 & P_{bi}(k-h) \geq 0.5 \\ -1 & P_{bi}(k-h) < 0.5 \end{cases} \tag{20}$$

(8) If $k < M+h$, let $k = k+1$ and goto step (3). Otherwise, stop.

The relations between the indices i, n and k in the above procedure are further illustrated in Figure 1.

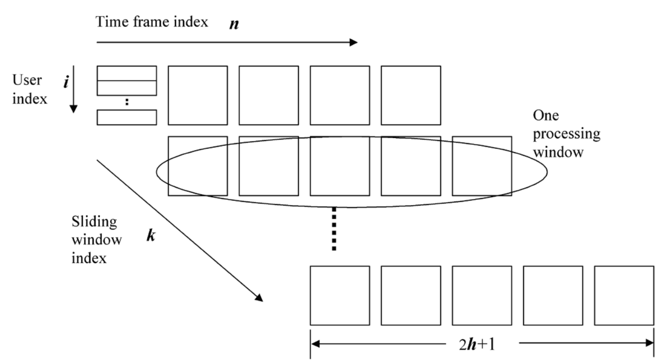

Figure 1: Illustration of the sliding-window PDA

Apparently, the computational complexity of the above PDA detector is $O((2h+1)K^3)$ per time frame.

IV. Simulation Results

In this section, we compare the performances of the Decorrelator, the DF detector and the PDA detector in various situations. The optimal user ordering and time labeling rule proposed in [11] is applied to both the DF and the PDA detectors. By clairvoyantly plugging the true values of $\boldsymbol{b}(n-1)$ and $\boldsymbol{b}(n+1)$ into (17) and applying the ML detection for synchronous CDMA, a performance lower bound is also provided.

Example 1: In the first 3-user example, the correlation matrices $R[0]$, $R[1]$ and the square roots of user signal powers W are randomly chosen as

$$\boldsymbol{R}[0] = \begin{bmatrix} 1.0 & -0.27 & -0.49 \\ -0.27 & 1.0 & 0.55 \\ -0.49 & 0.55 & 1.0 \end{bmatrix}$$

$$\boldsymbol{R}[1] = \begin{bmatrix} 0 & 0 & 0 \\ -0.06 & 0 & 0 \\ 0.16 & -0.01 & 0 \end{bmatrix}$$

$$\boldsymbol{W} = diag(4.48, 4.36, 4.1) \tag{21}$$

The width of the processing window for the PDA detector is chosen to be 3. Figure 2 shows the performance comparison of different algorithms obtained from a simulation of 1000000 Monte-Carlo runs. Similar to the synchronous case [8], the probability of error of the PDA detector is very close to the performance lower bound.

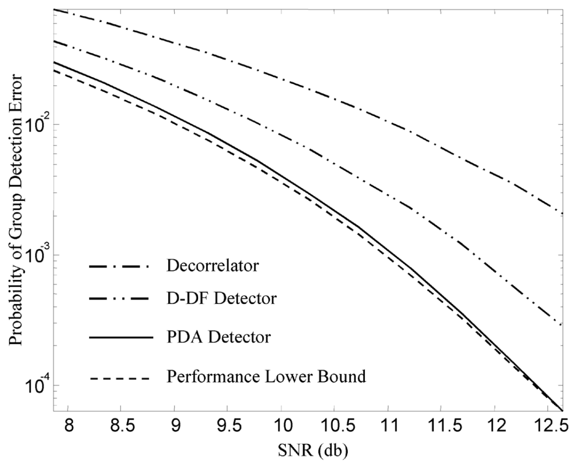

Figure 2: Performance comparison, 3-users, 1000000 Monte-Carlo runs.

Example 2: The second example is an overloaded system with 30 users and 15-length Gold codes as signature sequences. Although the number of users is increased to 30, the width of the processing window for the PDA detector remains at 3. The time delays of the user signals are random and uniformly-distributed within a symbol duration and we use the system model introduced in [12] to generate the signature correlation matrix. The square roots of user signal powers are generated randomly by $w_i \sim N(4.5, 4)$ ($N(.)$ represents the Gaussian distribution) and are limited within the range of $[3, 6]$. Figure 3 shows the performance comparison of different detectors. The performance of the PDA detector is significantly better than the decorrelator and the DF detector. It is also close to the performance lower bound (Notice that the performance lower bound is not necessarily reachable even by the optimal ML detector).

V. Conclusion

The PDA method proposed in [8] has been extended to the multiuser detection over asynchronous CDMA communication channels. With a sliding window of width $2h+1$, the computational complexity of the proposed PDA detector is shown to be $O((2h+1)K^3)$ per time frame where K is the number of users. Simulation results show that the performance of the PDA detector, in terms of the probability of group detection error, is significantly better than the decorrelator and the DF detector; and is also close to the performance lower bound in both regular and overloaded systems.

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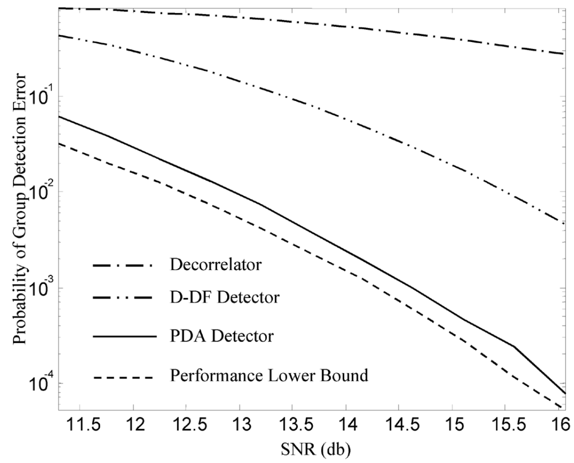

Figure 3: Performance comparison, 30-users, 15-length Gold codes as signature sequences, 1000000 Monte-Carlo runs.

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