Entire-domain polynomial approximation of volume currents in the analysis of dielectric scatterers

B.D. Popović
B.M. Notaroš

Indexing terms: Dielectric scatterers, Wave propagation

Abstract: A method is proposed for the analysis of electrically medium-sized scatterers made of inhomogeneous, imperfect dielectric. The scatterer is modelled by parallelepipeds of arbitrary shapes and sizes, that can be positioned and interconnected arbitrarily, which enables efficient approximation of very diverse scatterer shapes. The current-density vector is approximated by entire-domain three-dimensional polynomials of arbitrary degree with complex coefficients within individual parallelepipeds, even if the dielectric inside them is continuously inhomogeneous. The coefficients are determined by the point-matching solution of the integral equation for the total current-density vector inside the scatterer. Agreement between the results obtained by the proposed method and those from other sources is found to be excellent, both in the far-field and current distribution. When compared with other available methods, however, the proposed method requires much less unknowns per $\lambda^3_{in\,diel}$, although it is neither conceptually nor computationally more complicated than any of them.

1 Introduction

In addition to its theoretical importance, analysis of dielectric scatterers is of great practical interest. It is required in numerous areas of application, such as wave propagation in the presence of dielectric inhomogeneities, analysis of radar targets, electromagnetic-wave interactions with biological systems, medical applications of electromagnetic waves (e.g. hyperthermia and microwave imaging), and electromagnetic coupling between communication antennas and neighbouring dielectric bodies.

Methods based on solving volume integral equations using subdomain approximation for induced volume currents are widely used to analyse electrically medium-sized dielectric scatterers of arbitrary shape and inhomogeneity [1–3]. To solve any practical problem, however, subdomain approximations result in a very large number of unknown parameters to be determined. Consequently, serious problems are frequently encountered with computer memory requirements, the necessary computing time and stability of the solution.

Entire-domain approximation of current has been used successfully in the analysis of metallic antennas and scatterers for some time [4, 5]. It was found that such a kind of approximation demands the least number of unknowns possible for a given problem. As far as the authors are informed, a general method for the analysis of dielectric scatterers based on entire or almost-entire domain approximation of either actual volume currents or equivalent surface currents, that would ensure such a minimal number of unknowns, is not available.

The principal contributions of this paper, that appears to offer the first entire-domain method for the analysis of dielectric scatterers (possibly lossy and inhomogeneous), can be summarised as follows: First, the basic elements for the approximation of geometry are simple macro-parallelepipeds (although micro-parallelepipeds can also be used as a special case) of arbitrary shapes and sizes, that can be positioned and interconnected arbitrarily. This simple model enables surprisingly efficient modelling of quite diverse scatterer shapes. Secondly, three-dimensional polynomial current approximation of arbitrary degree (including a constant function as a special case) with unknown complex coefficients is used inside individual parallelepipeds, and the unknown coefficients are determined by point-matching. Consequently, when compared with other available methods (which all use subdomain approximations), the proposed method requires considerably less unknowns for a given problem. For example, for larger bodies of simpler forms a sufficient degree of approximation in one dimension is at the most three per wavelength in the dielectric, while all subdomain approximations require an equivalent degree of approximation greater than ten, i.e., approximately as many as 30 times more unknowns. Finally, the proposed method enables the analysis of a scatterer (or a large part of the scatterer) with continually inhomogeneous dielectric as a single unit (parallelepiped), without partitioning it into subdomains with approximately constant permittivity. In cases where the complex permittivity is a well-behaved function of co-ordinates the method yields surprisingly convergent and accurate results.

2 Entire-domain method for solution of integral equation for volume current distribution

Consider a body made of imperfect inhomogeneous dielectric situated in a vacuum in incident time-harmonic electromagnetic field of angular frequency ω. We assume

Paper 1825H (E11), first received 6th June and in revised form 23rd November 1994

The authors are with the Department of Electrical Engineering, University of Belgrade, PO Box 816, 11001 Belgrade, Yugoslavia

that the incident electric-field vector $\boldsymbol{E}_i$, relative permittivity ε_r and conductivity σ of the body are known functions of co-ordinates, and that permeability at all points is μ_0.

The total (conduction plus polarisation) current-density vector, $\boldsymbol{J}$, inside the body is obtained as

$$\boldsymbol{J} = j\omega\varepsilon_0(\varepsilon_{er} - 1)(\boldsymbol{E}_i + \boldsymbol{E}) \tag{1}$$

In this equation, $\varepsilon_{er} = \varepsilon_r - j\sigma/(\omega\varepsilon_0)$ is the equivalent complex relative permittivity and $\boldsymbol{E}$ represents the electric-field vector due to volume currents (of density $\boldsymbol{J}$) and volume charges [of density $\rho = (j/\omega)$ div $\boldsymbol{J}$] throughout the volume v of the body, and to surface charges [of density $\rho_s = -(j/\omega)\boldsymbol{J}\cdot\boldsymbol{n}$] over all surfaces S of abrupt changes in ε and/or σ, where $\boldsymbol{n}$ is the reference unit vector normal to S. (Note that div $\boldsymbol{J} = 0$ only in homogeneous media.) The electric-field vector, $\boldsymbol{E}$, can be expressed in terms of induced currents and charges, assumed to exist in a vacuum. Therefore eqn. 1 becomes an integral equation in a single unknown, the total current-density vector $\boldsymbol{J}$:

$$\frac{j}{\omega\varepsilon_0}\left\{\frac{\boldsymbol{J}}{(\varepsilon_{er}-1)} - \int_v [\beta^2\boldsymbol{J}g(R) + \operatorname{div}\boldsymbol{J}\operatorname{grad}g(R)]\,dv + \int_S \boldsymbol{J}\cdot\boldsymbol{n}\operatorname{grad}g(R)\,ds\right\} = -\boldsymbol{E}_i \tag{2}$$

Here, $g(R) = e^{-j\beta R}/4\pi R$ is the free-space Green function, R is distance between the source and field points and $\beta = \omega\sqrt{(\varepsilon_0\mu_0)} = 2\pi/\lambda$ is the free-space phase coefficient.

Eqn. 2 can be solved numerically by any of the procedures belonging to the method of moments [6]. Before proceeding to that step, we first approximate the body by m parallelepipeds of convenient sizes and shapes, as shown in Fig. 1 for $m = 9$. Any surface of abrupt change in ε_{er} must coincide with a side of a parallelepiped.

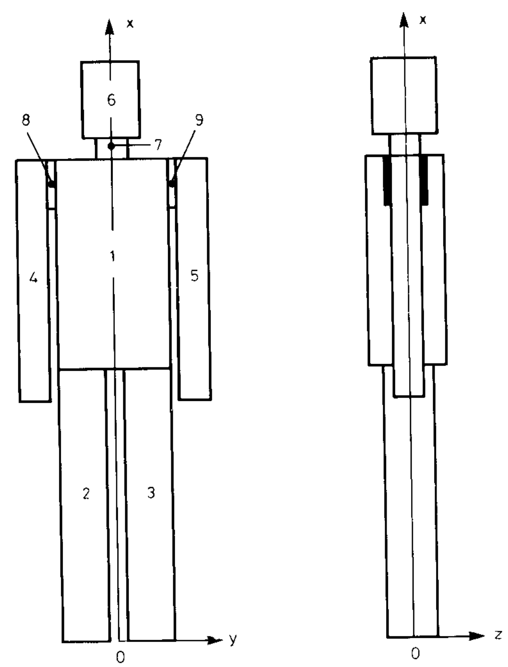

Fig. 1 *Parallelepiped model of a man exposed to incident electromagnetic field*

Next, approximate every component of the vector $\boldsymbol{J}$ in the lth parallelepiped by a polynomial in the local cartesian co-ordinates x_l, y_l and z_l,

$$J_{lx} = \sum_{i=0}^{n_{lx}}\sum_{j=0}^{n_{ly}}\sum_{k=0}^{n_{lz}} a_{lxijk}\,x_l^i y_l^j z_l^k$$

$$\begin{Bmatrix} -d_{lx} \leqslant x_l \leqslant d_{lx} \\ -d_{ly} \leqslant y_l \leqslant d_{ly} \\ -d_{lz} \leqslant z_l \leqslant d_{lz} \end{Bmatrix} \quad l = 1, 2, \ldots, m \tag{3}$$

with analogous expressions for J_{ly} and J_{lz}, where $2d_{lx}$, $2d_{ly}$ and $2d_{lz}$ are the edge lengths of the lth parallelepiped. The constants a_{lxijk}, a_{lyijk}, a_{lzijk} are unknown complex coefficients to be determined, and n_{lx}, n_{ly} and n_{lz} are the adopted degrees of the polynomials.

Finally, we substitute the approximate current distribution in eqn. 3 into the integral eqn. 2, and stipulate that the equation be satisfied at $N/3$ points throughout the dielectric body, where N is the total number of unknowns. The matching points in the lth parallelepiped are equidistant along each local co-ordinate. This results in a system of linear algebraic equations in N unknown current-distribution coefficients, which can be solved by any of the available direct or iterative methods. In the proposed method, the Gaussian elimination procedure is used for that purpose.

Polynomial basis functions enable extremely efficient recursive evaluation of the system matrix elements resulting in the fill time of the matrix that increases with N much more slowly than N^2. (Note that fill time of matrix in subdomain solutions is always proportional to N^2.) Numerical integration was performed by the Gauss–Legendre integration formula. For field points close to or coinciding with a source point, the principal (static) parts of the integrals were extracted and computed analytically.

3 Numerical results

In all the examples presented below no use of symmetry was made. All the results were obtained on a PC386 computer, 25 MHz, with a coprocessor 80387 and 4 Mbyte RAM.

In the first group of examples, consider a dielectric scatterer in the form of a parallelepiped in the field of a linearly polarised plane wave of electric field intensity $\boldsymbol{E}_i$ (Fig. 2). As the first example, consider a cube ($a = b = c$)

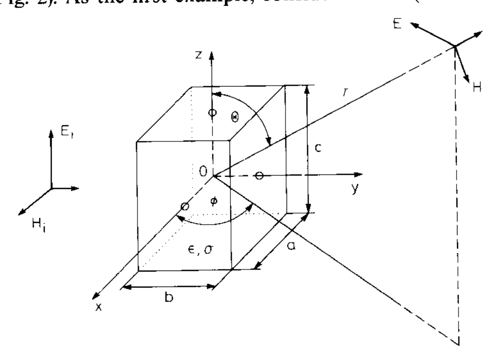

Fig. 2 *Dielectric scatterer in the form of a parallelepiped in the field of a linearly polarised plane wave*

The centre of the cartesian co-ordinate system coincides with the parallelepiped centre

made of a homogeneous lossless dielectric ($\varepsilon_r = 9$, $\sigma = 0$). Let the cube edge length be one-fifth of the free-space wavelength, λ, i.e. three-fifths of the wavelength in the dielectric, $\lambda_{in\,diel}$. Fig. 3 shows the scattered far field in planes $\phi = 0°$ and $\phi = 90°$. The degrees of polynomials were adopted to be $n_x = n_y = n_z = 3$ ($m = 1$) (a total of

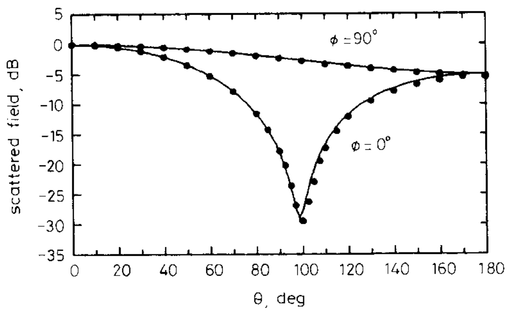

Fig. 3 *Scattered field, 20 log E, for a homogeneous perfect-dielectric cube of side λ/5, against θ, in two characteristic planes*

$\varepsilon_r = 9$, $\sigma = 0$, $a = b = c$, $E_i = 377e^{-j\beta x}i_z$ (V/m), $f = 300$ MHz

—— This method, 192 unknowns

●●● Reference 2, 1536 unknowns

$N = 192$ scalar complex unknowns). The results are compared with those obtained by the subdomain approximation, with the scatterer approximated by 512 small cubes with uniform current distribution, resulting in 1536 unknowns [2]. It is seen that agreement of the two sets of results is very good, in spite of the number of unknowns in the proposed method being for almost an order of magnitude less than in the other.

To illustrate accuracy of the resulting current distribution, i.e. of the near field, consider a rodlike scatterer (Fig. 2, $a = 0.05\lambda$, $b = 0.1\lambda$, $c = 1.25\lambda$) made of a homogeneous perfect dielectric ($\varepsilon_r = 4$, $\sigma = 0$). Fig. 4 shows the distribu-

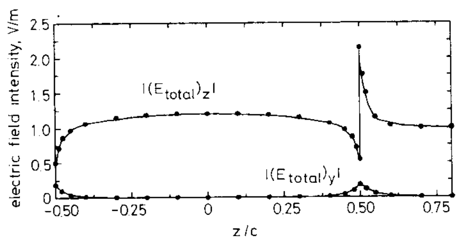

Fig. 4 *Distribution of the total electric field inside and in the vicinity of a homogeneous perfect-dielectric rodlike scatterer, along the line $x_0 = 0$, $y_0 = b/4$*

$a = 0.05\lambda$, $b = 0.1\lambda$, $c = 1.25\lambda$, $\varepsilon_r = 4$, $\sigma = 0$, $E_i = 1e^{-j\beta x}i_z$ (V/m), $f = 300$ MHz

—— This method, 21 unknowns

●●● Reference 1

tion of the total field, $\boldsymbol{E}_{total}$ (the incident field plus that of the induced currents and charges), inside the scatterer and in its vicinity. The polynomial degrees adopted were $n_x = n_y = 0$, $n_z = 6$ ($m = 1$) (a total of only $N = 21$ unknowns). The results are compared with those from Reference 1, where every field component in the scatterer was approximated by locally constant functions in equal orthorhombic cells, and the point-matching method was used. Very good agreement of the two sets of results is observed, the degree of the polynomial along the z-axis, $n_z = 6$, being much smaller than the equivalent degree of approximation (about 50) from Reference 1.

The next example is aimed at demonstrating rapid and stable convergence of the results with increasing degree of the polynomial approximation. Fig. 5 shows the current distribution along a scatterer in the form of a parallelepiped of square cross-section (Fig. 2, $a = b = 5$ cm) with the degree n_z in the polynomial approximation as parameter. The material adopted was a dielectric of high

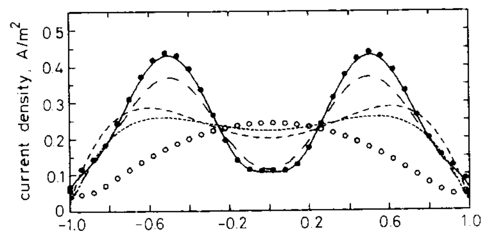

Fig. 5 *Modulus of the total current density, $|J_z|$, along the axis of a scatterer in the form of a parallelepiped of square crosssection [$a = b = 5$ cm, $c = 1$ m, $E_i = 1e^{-j\beta x}i_z$ (V/m), $f = 600$ MHz] made of a lossy dielectric ($\varepsilon_r = 71$, $\sigma = 4.4$ S/m, i.e. $\varepsilon_{re} = 71 - j131.82$), with the degree n_z of polynomial approximation as a parameter*

○○○ $n_z = 2$

------- $n_z = 4$

- - - - $n_z = 5$

— — $n_z = 8$

●●● $n_z = 10$

—— $n_z = 16$

relative permittivity ($\varepsilon_r = 71$) with significant losses ($\sigma = 4.4$ S/m) which can be used to replicate a biological tissue, and the scatterer length was adopted to be two wavelengths in a vacuum, $c = 2\lambda$. Very accurate results are obtained with a single polynomial of degree $n_z = 10$.

As an example of the analysis of piecewise homogeneous dielectric bodies, consider a parallelepiped scatterer (Fig. 2, $a = b = 0.05\lambda$, $c = \lambda$) consisting of two parts of equal length $c/2$, made of different imperfect dielectrics ($\varepsilon_{er1} = 3 - j4$, $\varepsilon_{er2} = 8 - j6$). Fig. 6 shows the distribution

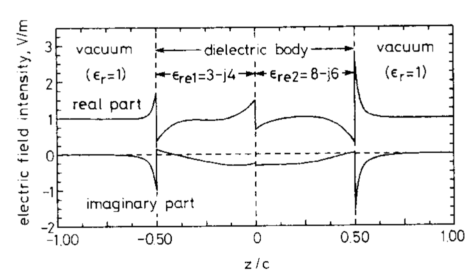

Fig. 6 *Distribution of the total field E_{total} along the axis of a cylindrical scatterer, consisting of two homogeneous parts of length c/2 and complex permittivities $\varepsilon_{er1} = 3 - j4$, $\varepsilon_{er2} = 8 - j6$*

$a = b = 0.05\lambda$, $c = \lambda$, $E_i = 1e^{-j\beta x}i_z$ (V/m), $f = 300$ MHz

The field on the axis has only the z-component

of the total electric field along the scatterer axis. The parameters of the approximaton adopted were $n_{1x} = n_{1y} = n_{2x} = n_{2y} = 0$, $n_{1z} = n_{2z} = 4$ ($m = 2$) (a total of $N = 30$ unknowns).

To check the accuracy of the results shown in Fig. 6, the boundary conditions for the vector $\varepsilon_e \boldsymbol{E}_{total}$ were considered. The ratios obtained numerically of the two relevant normal field components at the boundary surfaces $z = -c/2$, $z = 0$ and $z = c/2$ are 2.99963-j3.99992, 1.91987 + j0.559844 and 7.99758 − j5.99763, respectively, while the corresponding exact ratios are $3 - j4$, $1.92 + j0.56$ and $8 - j6$. It can be seen that agreement between the two sets of results is excellent, in spite of quite small number of unknowns used.

Let us consider an inhomogeneous scatterer, the permittivity of which is a continuously varying function of space co-ordinates. This problem can be solved by approximating the body by many small homogeneous dielectric cells. However, the integral eqn. 2 enables a continuously inhomogeneous body to be treated as a whole, at least theoretically. The next example is aimed at demonstrating that with the entire-domain polynomial approximation this conclusion is also numerically correct.

Consider a square cylindrical scatterer (Fig. 2, $a = b = 0.16$ cm) of finite length $c = 3.2$ cm $= 1.008\lambda$. The relative permittivity of the perfect dielectric ($\sigma = 0$) is the following function of the z-co-ordinate:

$$\varepsilon_r(z) = \varepsilon_{r1} - (\varepsilon_{r1} - 1)\frac{z^2}{h^2} \quad -h \leqslant z \leqslant h \quad h = c/2 \tag{4}$$

where ε_{r1} is a constant. Fig. 7 shows the distribution of the vector $\boldsymbol{J}$ along the scatterer axis. Shown in the Figure

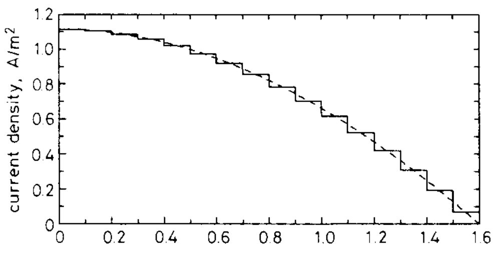

Fig. 7 *Modulus of the current-density vector along the axis of a continuously inhomogeneous scatterer in the form of a parallelepiped, with relative permittivity given in eqn. 4 with $\varepsilon_{r1} = 3$ and $\sigma = 0$*

$a = b = 0.16$ cm, $c = 3.2$ cm $= 1.008\lambda$, $E_i = 1e^{-j\beta x}i_z$ (V/m), $f = 9.45$ GHz
——— piecewise-constant current approximation with $m = 32$ (96 unknowns)
– – – – entire-domain polynomial current approximation with $m = 1$ (21 unknowns)

are the results obtained with two different approaches. In the first approach the body was divided into $m = 32$ equal parallelepipeds of volume $a \times b \times c/m$, and the permittivity in all of them was approximated by a constant, equal to the value of the function 4 at the parallelepiped centre. A constant current density in all the small domains was adopted, i.e. $n_{lx} = n_{ly} = n_{lz} = 0$, $l = 1, 2, \ldots, m$, resulting in a total of $N = 96$ unknowns. In the second case, the body was treated as a single piece ($m = 1$) and the polynomial approximation with $n_x = n_y = 0$, $n_z = 6$ (a total of $N = 21$ unknowns) was adopted. Excellent agreement between the two sets of results is observed.

For the next example, consider an inhomogeneous right cylinder, of elliptical cross-section shown in Fig. 8. The relative permittivity of the cylinder with respect to the co-ordinate system in Fig. 8 is given by

$$\varepsilon_r(x, y, z) = \varepsilon_{r1} - (\varepsilon_{r1} - 1)\left[\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2\right] \tag{5}$$

and $\sigma = 0$. The cylinder with $a = 0.64\lambda$, $b = 0.32\lambda$ and $d = 0.08\lambda$ was modelled by $m = 7$ parallelepipeds, as indicated in Fig. 8, and the adopted degrees of the polynomial approximation were $n_{1x} = 4$, $n_{2x} = n_{3x} = n_{4x} = n_{5x} = n_{6x} = n_{7x} = 0$, $n_{1y} = n_{2y} = n_{3y} = n_{5y} = n_{6y} = 4$, $n_{4y} = n_{7y} = 2$, $n_{iz} = 0$, $i = 1, 2, \ldots, 7$ (a total of $N = 153$ unknowns). (In modelling of bodies with curved surfaces it is convenient and advisable to use as large parallelepipeds as possible, with adequate degrees of the polynomial approximation of current.) Note that all the parallelepipeds in the model are made of inhomogeneous dielectric.

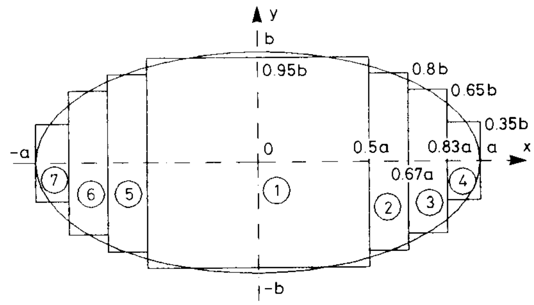

Fig. 8 *Right cylinder of elliptical crosssection, of semimajor and semiminor axes*

a and b, and height d (normal to the plane of the drawing), and its approximation by $m = 7$ parallelepipeds. The cylinder bases are parallel to the plane of the figure

Fig. 9 shows the normalised bistatic scattering cross-section, $S_{bist}(\theta, \phi)\lambda^2$, of the cylinder, in planes $\phi = 0°$ and $\phi = 90°$. The results are compared with those from Reference 1, where a model with equal orthorhombic cells was used in conjunction with piecewise constant field approximation and point-matching method. (Unfortunately, the number of unknowns in [1] was not given but obviously it was much larger than 153.) It can be observed from the Figure that the results obtained by using the polynomial approximation are in very good agreement with the results from Reference 1 previously multiplied by a factor 1/2. Presented in [1] are also the results for the normalised bistatic scattering cross-section of a homogeneous cube, that also need to be multiplied by 1/2 in order to be in agreement with the results obtained by the present method. It is therefore very likely that the results in Reference 1 represent $20\log(S_{bist}/\lambda^2)$, instead of usual $10\log(S_{bist}/\lambda^2)$.

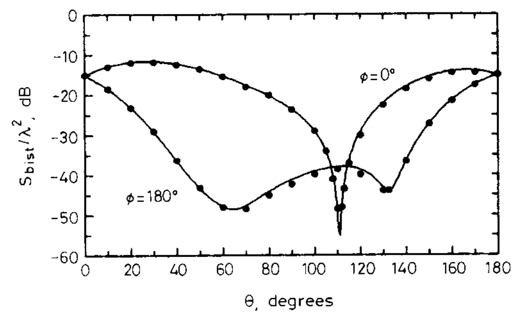

Fig. 9 *Normalised bistatic scattering cross-section of inhomogeneous right elliptical cylinder, of dielectric permittivity given by eqn. 5 with $\varepsilon_{r1} = 2$, in two characteristic planes*

$a = 0.64\lambda$, $b = 0.32\lambda$, $d = 0.08\lambda$, $E_i = (E_{0x}i_x + E_{0y}i_y + E_{0z}i_z)\exp[-j\beta(k_x x + k_y y + k_z z)]$, $f = 300$ MHz, $k_x = 0.5$, $k_y = 0$, $k_z = \sqrt{(3)}2$, $E_{0x} = -\sqrt{(3)}$V/m, $E_{0y} = 0$, $E_{0z} = 1$ V/m]
——— this method
●●● results from Reference 1 multiplied by 1/2

For a final example, consider a model of man in the field of a plane wave of frequency $f = 90$ MHz. The model is 180 cm high, and is made of a homogeneous lossy dielectric of parameters $\varepsilon_r = 76$ and $\sigma = 0.85$ S/m, i.e. $\varepsilon_{re} = 76 - j169.76$. It is constructed with $m = 9$ parallelepipeds, as sketched in Fig. 1. The dimensions of the parallelepipeds are as follows (dimensions are given in centimeters, as $a \times b \times c = \Delta x \times \Delta y \times \Delta z$): parallelepiped 1 – 65 × 34 × 23; 2 and 3 – 85 × 14 × 15; 4 and 5 – 75 × 9 × 9; 6 – 23 × 17 × 19; 7–7 × 10 × 10; 8 and 9 – 15 × 3 × 12. The adopted degrees of the polynomial approximation $n_x \times n_y \times n_z$ are the following: 1–3 × 2 × 2; 2 and 3 – 4 × 2 × 2; 4 and 5 – 4 × 0 × 0; 6–2 × 2 × 2; 7, 8 and 9 – 0 × 0 × 0 (a total of $N = 498$ unknowns). Finally, let the incident electric-field vector be $E_i = 1e^{-j\beta z} i_x$ V/m. (Note that by use of symmetry the number of unknowns for this particular incident field could be halved, i.e. $N_{with\ symmetry} = 249$ unknowns.) Fig. 10 shows the distribution of volume density of the Joule

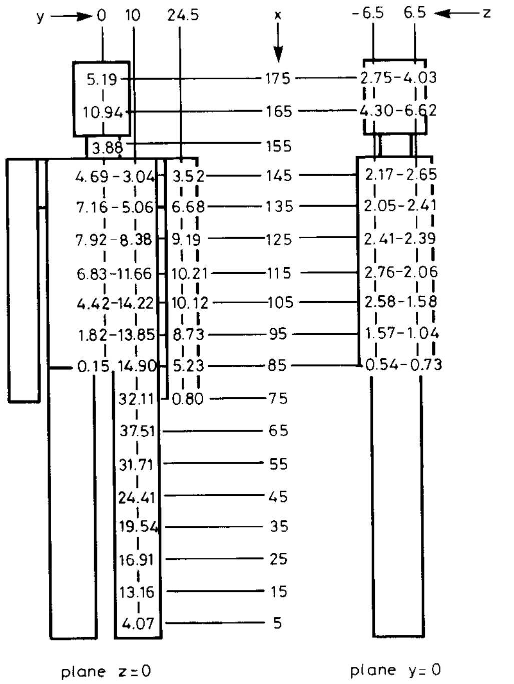

Fig. 10 *Distribution of volume density of the Joule losses, p_j in $(W/m^3) \times 10^{-2}$, in two characteristic planes inside a parallelepiped model of human body exposed to the field of a plane wave*

For the dimensions of the parallelepipeds and other parameters, please see text

losses, $p_j = \sigma \mid E_{total} \mid^2$, in two characteristic planes inside the human body. It can be seen from the Figure that p_j varies greatly throughout the body. Consequently, there is a great variation in the absorption of the electromagnetic energy by different regions inside the body.

To get some feeling of CPU time, t_{CPU}, needed for the proposed entire-domain solution and to compare it to that required by other (subdomain) methods, consider a homogeneous dielectric scatterer in the form of a cube of edge length equal to the wavelength in the dielectric, $a = \lambda_{in\ diel}$. The proposed entire-domain approach enabled very accurate results to be obtained with as few as 192 unknowns. The CPU time was $t_{CPU} = 208.7$ s (155.1 s for the evaluation of the system matrix elements plus 53.6 s for solving the system equations). The reference subdomain solutions were obtained also using the proposed method, but with the lowest-degree polynomial approximation. To have exactly the same number of unknowns (192), the cube was first partitioned into only 4 × 4 × 4 smaller cubes, although obviously this partitioning is not at all sufficient for accurate subdomain analysis. This solution required $t_{CPU} = 262.8$ s. To have, however, a solution of approximately the same accuracy as that of the above entire-domain solution, at least 10 × 10 × 10 partitions of the cube, i.e. 3000 complex unknowns, are necessary. This problem, unfortunately, was far too large for the PC used, but the estimated CPU time in that case exceeded 70 hours.

Of course, the above comparison is valid only for the problem considered and for the particular reference subdomain solution, but it certainly is at least indicative of very modest CPU time requirements of the proposed method.

4 Conclusions

A specific moment method is proposed for the analysis of scatterers made of inhomogeneous imperfect dielectrics, having the following properties:

(i) As far as the authors are informed, it represents the first application of an entire-domain approximation to the analysis of dielectric scatterers of arbitrary shape and inhomogeneity.

(ii) It is based on solving the volume integral equation in the total current density vector, $\boldsymbol{J}$, which is approximated by three-dimensional polynomials with complex coefficients inside individual macro-parallelepipeds used to approximate the scatterer. The solution is obtained via the point-matching method.

(iii) It yields results of the same accuracy as any available subdomain method, but requires considerably less unknowns for a given problem. For larger bodies of simpler forms a sufficient degree of approximation (i.e. polynomial degree plus one) in one dimension is about six per free-space wavelength, and usually much less (at the most three) per wavelength in the dielectric. Consequently, the method has relatively low memory requirements and is comparatively very rapid.

(iv) The macro-parallelepipeds used for modelling a dielectric body can be of arbitrary sizes and shapes and interconnected in an arbitrary manner. Consequently, this simple model is very flexible and a good approximation can be achieved of scatterers having quite diverse shapes.

(v) The degree of the polynomial approximation in any co-ordinate can be quite high (e.g. up to 16) for the solution to remain stable, so that electrically large scatterers can be analysed without partitioning (except for modelling of their geometry).

(vi) It yields accurate results for continually or piecewise inhomogeneous scatterers of arbitrary complex permittivity. Large scatterers (or large parts of scatterers) with continually inhomogeneous dielectric can be treated as a single unit (parallelepiped).

5 References

1 SU, C.C.: 'Electromagnetic scattering by a dielectric body with arbitrary inhomogeneity and anisotropy', *IEEE Trans.*, 1989, **AP-37**, (3), pp. 384–389

2 SARKAR, T.A., ARVAS, E., and PONNAPALLI, S.: 'Electromagnetic scattering from dielectric bodies', *IEEE Trans.*, 1989, **AP-37**, (5), pp. 673–676

3 SCHAUBERT, D.H., WILTON, D.R., and GLISSON, A.W.: 'A tetrahedral modelling for electromagnetic scattering by arbitrarily

shaped inhomogeneous dielectric bodies', *IEEE Trans.*, 1984, **AP-32**, (1), pp. 77–85
4 POPOVIĆ, B.D.: 'CAD of wire antennas and related radiating structures' (Research Studies Press, John Wiley & Sons, Chichester, New York, 1991)
5 POPOVIĆ, B.D., and KOLUNDŽIJA, B.M.: 'Analysis of metallic antennas and scatterers' (IEE Electromagnetic Wave Series, No. 38, London, 1994)
6 HARRINGTON, R.F.: 'Field computation by moment methods' (Macmillan, New York, 1968)