# Large raindrops against melting hail: calculation of specific differential attenuation, phase and reflectivity

M. Thurai✉, E. Chobanyan, V.N. Bringi and B.M. Notaroš

The specific differential attenuation caused by large raindrops and a special form of melting hail is computed and studied. Raindrops of smoothed conical shapes and hailstones modelled by an ice sphere surrounded by a water torus are considered. For the former, scattering computation is carried out by the T-matrix method, while the higher-order method of moments in the surface integral equation formulation is used for the latter. Results show a much higher specific differential attenuation factor for hail particles, with the smallest hail showing values similar to the largest raindrops. This can explain the higher than expected differential attenuation observed by C-band radars in intense storms as reported in some previous studies. Results also show that the melting hail particles exhibit higher specific differential phase factor than large raindrops, but have similar differential reflectivity.

*Introduction:* Raindrops and melting hail differ not only in their shapes and sizes but also in terms of their dielectric properties. Previous polarimetric radar observations at C-band (frequency near 5 GHz) have identified regions of 'hot spots' where a much larger differential attenuation between the horizontal and vertical polarisations have been noted [1] compared with those expected for rain.

In this Letter, we use the T-matrix method as well as a numerical technique based on the method of moments (MoM) in the surface integral equation (SIE) formulation [2], which enables accurate and efficient scattering matrix calculations of asymmetric hydrometeors of electrically large sizes. We compare the specific differential attenuation ($A_{dp}$) 'factor' between large raindrops and a specific form of melting hail where the initial stage of melting ice is assumed to occur around the 'equator' as has been observed in wind-tunnel experiments [3]. Here, the $A_{dp}$ factor is defined as $\mathrm{Im}\{f_{hh} - f_{vv}\}$, where $f$ is the forward-scattering amplitude at $\boldsymbol{h}$ (horizontal) and $\boldsymbol{v}$ (vertical) polarisations ($\boldsymbol{v}$ is aligned along the zenith and $\boldsymbol{v} \times \boldsymbol{h}$ gives the direction of propagation, i.e. the eigenpolarisations of the medium composed of a random distribution of identical and aligned particles). Also considered are the corresponding specific differential phase, $K_{dp}$, factor, given by $\mathrm{Re}\{f_{hh} - f_{vv}\}$, and the differential reflectivity, $Z_{dr}$, for a single scatterer. Some preliminary results for $A_{dp}$ only (a subset of data in Fig. 4*a*) are presented in a summary form in [4].

*Raindrop and melting hail shapes and dimensions:* According to thousands of two-dimensional (2D)-video disdrometer measurements of large raindrops, the shapes have been shown to take on a 'smoothed' conical shape, given in [5] as

$$x,\ y = c_1\sqrt{1-\left(\frac{z}{c_2}\right)^2}\left[\cos^{-1}\left(\frac{z}{c_3 c_2}\right)\right]\left[c_4\left(\frac{z}{c_2}\right)^2+1\right] \qquad (1)$$

where $x$, $y$ and $z$ are the Cartesian coordinates and the parameters $c_1$, $c_2$, $c_3$ and $c_4$ are constants calculated as

$$\begin{aligned} c_1 &= \frac{1}{\pi}\left(0.02914D_{eq}^2 + 0.9263D_{eq} + 0.07791\right),\\ c_2 &= -0.01938D_{eq}^2 + 0.4698D_{eq} + 0.09538,\\ c_3 &= -0.06123D_{eq}^3 + 1.3880D_{eq}^2 + 10.41D_{eq} + 28.34,\\ c_4 &= -0.01352D_{eq}^3 + 0.2014D_{eq}^2 - 0.8964D_{eq} + 1.226,\ 4 < D_{eq} \le 7\mathrm{mm}\\ c_4 &= 0 \text{ if } 1.5\mathrm{mm} \le D_{eq} \le 4\mathrm{mm} \end{aligned} \qquad (2)$$

with $D_{eq}$ being a diameter of the equivolumetric sphere. A 3D shape of a 7 mm drop, obtained by the above formulas, is shown in Fig. 1, together with the projection onto the $x$–$z$ and $y$–$z$ planes. They represent the most probable shape (taking into account drop oscillations).

By contrast, in Fig. 2, we show a special form of melting hail, represented by an ice-sphere in the centre, surrounded by a water torus, with also a thin layer of water over the top and bottom of the ice sphere.

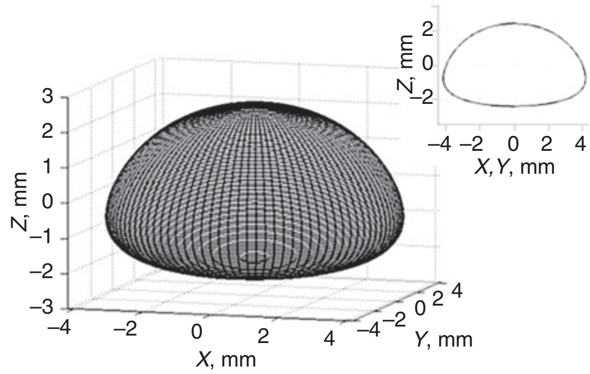


**Fig. 1** *3D shape of 7 mm $D_{eq}$ raindrop obtained using (1) and (2)*

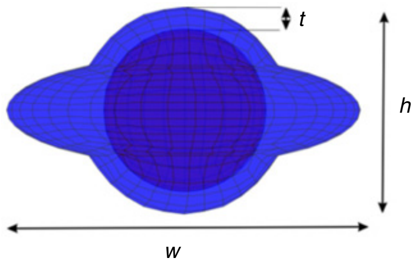


**Fig. 2** *MoM-SIE scattering model, using 1047 curved quadrilateral patches in Fig. 3, of melting hailstone represented by ice sphere (darker shaded area) surrounded by water torus (lighter shaded area), with thin layer of water also over top and bottom of ice sphere. Simulation requires 4028 unknowns and 30 s of computational time*

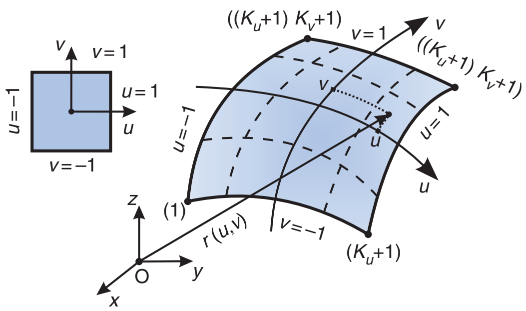


**Fig. 3** *Generalised curved parametric quadrilateral MoM-SIE patch defined by (3), with square parent domain also shown*

For the water drop, $D_{eq}$ is chosen to range from 2 to 10 mm. For the melting hail model, we adopt the dimensions given in Fig. 7.46 in [6] representing the aircraft probe measurement of the melting hail silhouette. For such a case, $w$ in Fig. 2 is set to 16.79 mm, $h$ to 9.75 mm and $t$ to 0.4 mm. The complex dielectric constants of water and ice are adopted to be $\varepsilon_r^{water} = 72.5 - j22.3$ and $\varepsilon_r^{ice} = 3.14 - j0.004$, respectively.

*Electromagnetic scattering modelling of raindrops and hail:* For raindrops, we use the well-established T-matrix method [7]. For melting hail, we use the higher-order MoM-SIE method [2]. According to this method, the external (between dielectric and air) and internal (between dielectric layers or regions) dielectric boundary surfaces of a raindrop or a melting hailstone are modelled by the generalised curved quadrilateral patches shown in Fig. 3, using the Lagrange interpolation polynomials ($L$) of arbitrary geometrical orders $K_u$ and $K_v$ ($K_u$, $K_v \ge 1$) [2]

$$\boldsymbol{r}(u, v) = \sum_{k=0}^{K_u}\sum_{l=0}^{K_v}\boldsymbol{r}_{kl}L_k^{K_u}(u)L_l^{K_v}(v),\ L_k^K(u) = \prod_{\substack{i=0\\ i\ne k}}^{K}\frac{u-u_i}{u_k-u_i}, \quad -1 \le u, v \le 1. \qquad (3)$$

Electric and magnetic equivalent surface current density vectors, $\boldsymbol{J}_s$ and $\boldsymbol{M}_s$, over the patches are approximated using divergence-conforming hierarchical polynomial vector basis functions $\boldsymbol{f}$ [2], as follows:

$$\boldsymbol{J}_s(u,v)=\sum_{i=0}^{N_u}\sum_{j=0}^{N_v-1}\alpha_{uij}\boldsymbol{f}_{uij}+\sum_{i=0}^{N_u-1}\sum_{j=0}^{N_v}\alpha_{vij}\boldsymbol{f}_{vij}, \quad (4)$$

$$\boldsymbol{M}_s(u,v)=\sum_{i=0}^{N_u}\sum_{j=0}^{N_v-1}\beta_{uij}\boldsymbol{f}_{uij}+\sum_{i=0}^{N_u-1}\sum_{j=0}^{N_v}\beta_{vij}\boldsymbol{f}_{vij}, \quad (5)$$

where $N_u$ and $N_v$ ($N_u, N_v \geq 1$) are the adopted arbitrarily high degrees of the current approximation, and $\{\alpha\}$ and $\{\beta\}$ are unknown current-distribution expansion coefficients. These coefficients are determined by a Galerkin-type direct solution to the SIEs based on the boundary conditions for the tangential components of the total (incident plus scattered) electric and magnetic fields [2] on all dielectric surfaces in the raindrop or melting hailstone models.

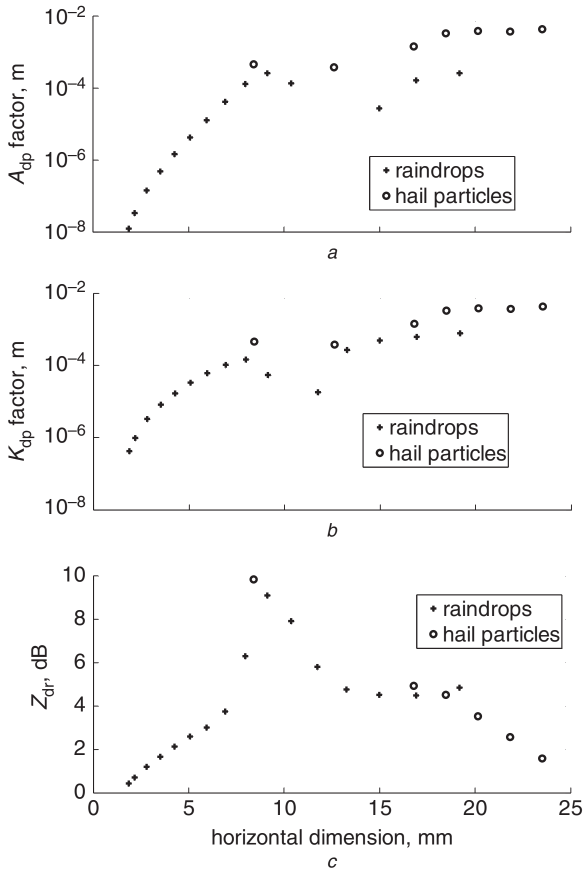


**Fig. 4** *Calculations at C-band for raindrops compared with melting hail particles, with hydrometeor shapes given in Figs. 1 and 2. Scattering from raindrops, in Fig. 1, is calculated using T-matrix method [7], whereas higher-order MoM-SIE modelling based on (3)–(5) is used for melting hail, in Fig. 2*

*a* $A_{dp}$ factor
*b* $K_{dp}$ factor
*c* $Z_{dr}$

*Results:* Fig. 4*a* shows the C-band (5.625 GHz) $A_{dp}$ factor calculated for raindrops and melting hail. For both types of hydrometeors, the *x*-axis represents the maximum horizontal dimension. For rain, the range of the $D_{eq}$ considered is 1.5–9 mm. For the hail particles, the dimensions of *h*, *w* and *t* in Fig. 2 are varied from 50 to 150% of the original value. The *x*-axis for these hydrometeors represents *w* in Fig. 2.

The resonance region for rain can be seen at around $D_{eq}$ of 6–8 mm (i.e. horizontal dimensions of ~10–15 mm), which is expected at the C-band. Also noticeable are the higher values of the $A_{dp}$ factor for hail particles, with the smallest hail showing values similar to the largest raindrops, except near the resonance region. Hence, the presence of hail with shapes as in Fig. 2 can explain the higher than expected differential attenuation reported at C-band, as in [1], in some intense storms.

Fig. 4*b* compares the corresponding $K_{dp}$ factor, and once again we note that it tends to be generally higher for hail than rain. This too agrees with the $K_{dp}$ observations reported in [1]. Fig. 4*c* compares the differential reflectivity, $Z_{dr}$ (see [6] for definition), and here we see hail particles giving values more comparable to large raindrops which can reach 9–10 dB in the resonance region.

Our results also highlight the advantages of using the higher-order MoM-SIE method for calculating scattering amplitudes of hydrometeors other than raindrops. For validation, calculations were made using commercial codes high frequency structure simulator (HFSS) and HFSS-IE as well for the melting hail particles. Very similar results were obtained, and hence, for clarity, these results are not included in Fig. 4.

*Conclusion:* In this Letter, scattering characteristics of large raindrops and melting hail particles have been computed using the T-matrix and MoM-SIE numerical techniques and have been compared against each other and discussed. Since the raindrop shapes can be assumed smooth and rotationally symmetric, which can be described by an empirical formula, their polarimetric radar parameters have been calculated using the fast T-matrix code. However, for the special form of melting hailstones considered in our case, a more advanced and general numerical technique, the MoM-SIE approach, has been used, because of their relatively complex shapes and material composition. Our results show that the melting hailstones, represented by an ice sphere surrounded by a water torus, exhibit specific differential attenuation factors that are noticeably higher than those for the large raindrops. This finding can be used as an explanation for the very high differential attenuation factors reported by C-band radars in intense storms [1]. The hail particles also exhibit a higher specific differential phase factor than large raindrops, but have similar differential reflectivity.

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One or more of the Figures in this Letter are available in colour online.

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