Adjoint Methods for Uncertainty Quantification in Applied Computational Electromagnetics: FEM Scattering Examples

Cameron L. Key [1], Aaron P. Smull [1], Donald J. Estep [2], Troy D. Butler [3], and Branislav M. Notaroš [1]

[1] Electrical & Computer Engineering Department, Colorado State University, Fort Collins, CO
camkey@rams.colostate.edu, aarons24@hotmail.com, notaros@colostate.edu

[2] Department of Statistics, Colorado State University, Fort Collins, CO
Donald.Estep@colostate.edu

[3] Department of Mathematical and Statistical Sciences, University of Colorado Denver, Denver, CO
troy.butler@ucdenver.edu

***Abstract* —** We present methods for quantifying uncertainty and discretization error of numerical electromagnetics solvers based on adjoint operators and duality. We briefly introduce the concept of the adjoint operator and describe applications of adjoint solutions for predicting and analyzing numerical error and approximating sensitivity of a given quantity of interest to a given parameter. Forward solutions are based on the higher order finite element method (FEM).

***Index Terms* —** Adjoint methods, computational electromagnetics, finite element method, scattering; radar, sensitivity analysis, uncertainty quantification.

I. INTRODUCTION

Computational error estimation, model sensitivity prediction, and optimization are growing areas of interest in the field of computational electromagnetics. While full wave numerical methods using the finite element method (FEM) are ubiquitous in the field, analyzing solutions from these methods is challenging due to geometric discretization error, field order expansion error, and lengthy computation times. Problems requiring numerical methods typically lack analytical solutions, making computational error difficult to discern from measurement error when comparing model results to real-world data where error analysis is desired. Meanwhile, prohibitive computation times make techniques like Monte-Carlo simulation computationally untenable where model sensitivity analysis and optimization are desired.

This paper focuses on an application of adjoint methods toward sensitivity analysis for computational electromagnetics problems. Specifically, it presents an example of an application of an adjoint operator to quantify sensitivity of a quantity of interest (QoI) to perturbations in an input parameter. We use the sensitivity information to predict the QoI over the parameter domain. The computational modeling, both for forward and reverse solutions, in the given example was performed using a double-higher-order FEM solver [1].

II. THE ADJOINT SOLUTION AND ITS APPLICATIONS

To define the adjoint operator for a given problem, we first put the problem in variational form. If the problem is nonlinear, we use the integral Mean Value Theorem to write the problem in linear form. For the illustrative dielectric scatterer problem, we consider the solution of the Dirichlet boundary value problem:

Find $\mathbf{E}^{sc} \in \mathrm{H}^1$ (the space of L^2 functions with L^2 first derivatives) such that the variational formulation of the following holds:

$$\nabla\times\bar{\bar{\mu}}_r^{-1}\nabla\times\mathrm{E}^{sc}-k_0^2\bar{\bar{\varepsilon}}_r\,\mathrm{E}^{sc}=-\nabla\times\bar{\bar{\mu}}_r^{-1}\nabla\times\mathrm{E}^{inc}+k_0^2\bar{\bar{\varepsilon}}_r\,\mathrm{E}^{inc}, \quad (1a)$$

$$n\times\mathrm{E}^{sc}=0, \quad (1b)$$

where the former equation holds through the volume of the domain and the latter holds on the boundary of the domain. This may be stated in linear operator form as:

$$L\mathbf{v}=\mathbf{f}. \quad (2)$$

The adjoint operator of L is then the operator L^* which satisfies:

$$\langle\mathbf{u}, L\mathbf{v}\rangle=\langle L^*\mathbf{u},\mathbf{v}\rangle, \quad (3)$$

with angle brackets denoting the L^2 inner product between two vectors. The adjoint problem may then be formulated as:

$$L^*\mathbf{v}=\mathbf{p}, \quad (4)$$

where $\mathbf{p}$ denotes a vector representation of a linear functional defining a QoI. A solution for one such adjoint problem is presented in Fig. 1 for scattering from a 1m-radius lossy dielectric sphere with relative permittivity

Submitted On: September 30, 2018
Accepted On: January 1, 2019

ε_r=2.25-1i. Here, the incident wave was chosen to arrive from θ = 90°, ϕ = 0° and was modeled to be θ-polarized at 300 MHz with unit electric field amplitude. Figure 2 shows the magnitude of the adjoint solution.

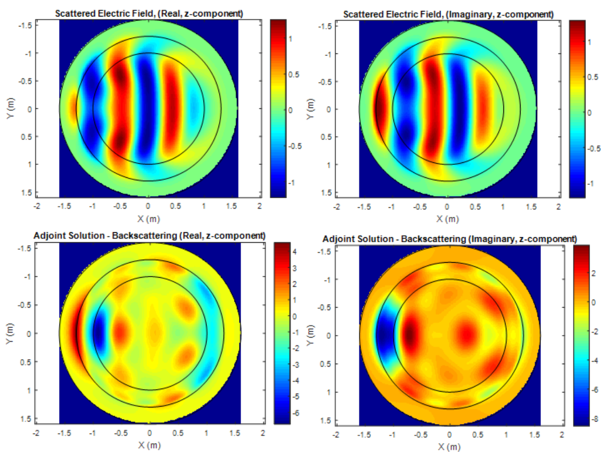

Fig. 1. Solutions to the scattering problem for a lossy dielectric sphere of ε_r=2.25-1i and radius 1 meter in free space. Domain terminated with perfectly matched layer with perfect electric conductor (PEC) exterior boundary. Incident wave was chosen to arrive from θ = 90°, ϕ = 0° and θ-polarized at 300 MHz with unit electric field amplitude. Solutions to the adjoint problem are given below the corresponding forward solution.

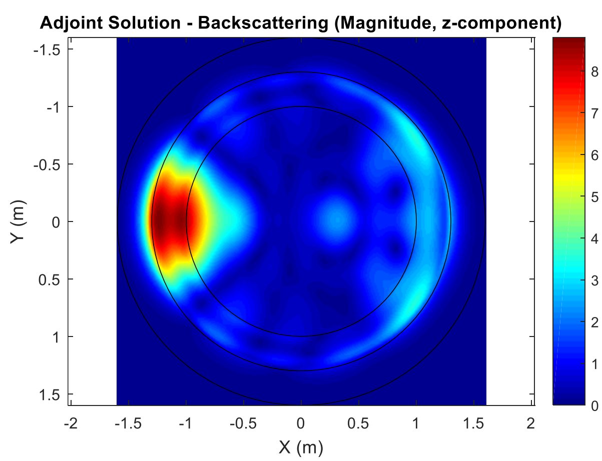

Fig. 2. Magnitude of the *z*-component of the adjoint solution for the problem described in Fig. 1. The areas most affecting the backscattered field lie in the region facing the radar receiver and the Arago spot. Z = 0 cross section is given.

III. FURTHER RESULTS AND DISCUSSION

Given the forward and adjoint solutions at a nominal value of a parameter, values of a desired QoI may be approximated linearly around this parameter value using the adjoint to estimate the derivative of the QoI at the parameter. In [2], this approach is used to construct piecewise linear approximations to the QoI response over the parameter domain in the higher-order parameter sampling method (HOPS). In Fig. 3, we show HOPS results for a 1D analogue of the problem described in Fig. 1. Here, the QoI is the amplitude of the reflected field, and the parameter is the radius of the dielectric scatterer.

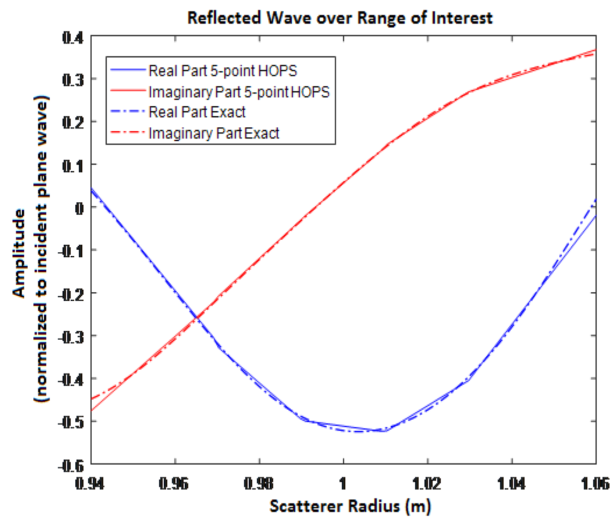

Fig. 3. Amplitude of reflected field calculated using HOPS on adjoint information generated from 1-D higher-order FEM solutions to a lossy dielectric scatterer problem. QoI is amplitude of reflected field and parameter of interest is dielectric scatterer radius.

Figure 4 gives another illustrative example for the same QoI, in this case with respect to the parameter of scatterer conductivity. In both Figs. 3 and 4, we also plot the exact QoI response for which HOPS exhibits excellent agreement. For multiple parameters, we can similarly reconstruct gradient information about the QoI using adjoint solutions. This gradient information allows for faster optimization methods to be employed to optimize a QoI with respect to a set of input parameters. Such information can also be used to reconstruct the underlying relationship between the desired QoI and the given set of input parameters, as shown in Fig. 3 and Fig. 4. An adjoint solution, as given in Fig. 2, may also be used directly to determine areas in the computational domain most affecting some QoI. As expected, for the spherical lossy dielectric scatterer in Fig. 1 the magnitude of the adjoint solution is highest in the region facing the radar receiver and the so-called "Arago Spot" at the center of the opposite face.

Further work will concern utilizing such information for targeted mesh refinement and higher field-order expansion to improve forward solutions.

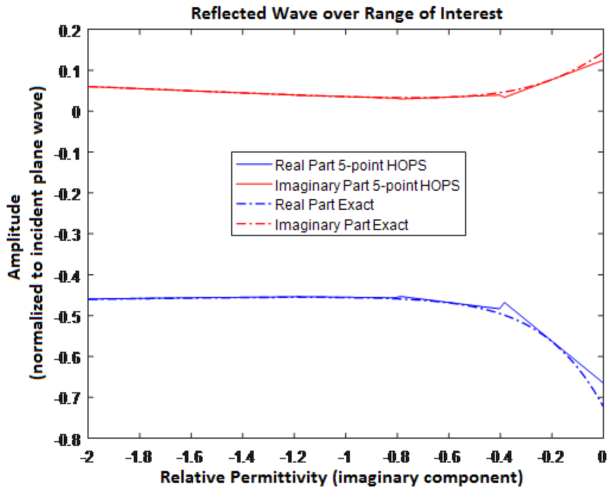

Fig. 4. Amplitude of reflected field calculated using HOPS on adjoint information generated from 1-D Galerkin FEM solutions to a lossy dielectric scatterer problem. QoI is chosen as the magnitude of the reflected field. This is shown with respect to changes in conductivity of the lossy scatterer.

ACKNOWLEDGEMENT

This work was supported by the US Air Force Research Laboratory, CREATE SENTRi, Riverside Research Institute, under contract FA8650-14-D-1725 (6F1957).

REFERENCES

[1] M. M. Ilic and B. M. Notaros, "Higher order hierarchical curved hexahedral vector finite elements for electromagnetic modeling," *IEEE Transactions on Microwave Theory and Techniques*, 51(3), pp. 1026-1033, 2003.

[2] D. Estep and D. Neckels, "Fast and reliable methods for determining the evolution of uncertain parameters in differential equations," *Journal of Computational Physics*, 213(2), pp. 530-556, 2006.