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A Probabilistic Approach for Measuring the Fault Tolerance of Robotic Manipulators

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***Abstract*— Fault tolerance is critical for various situations where robotic manipulators are applied, such as for hazardous waste disposal, exploring remote environments, or medical procedures. Metrics that are frequently applied to measure the degree of fault tolerance that a manipulator possesses have been focused on local kinematic properties of the Jacobian matrix, e.g., the worst-case manipulability (or relative manipulability) following a locked joint failure. These measures are useful for characterizing the manipulator's dexterity at a given configuration, however, they do not provide a measure for completion of a task within a specified workspace. This paper extends the use of local fault tolerance measures by using them to compute a global measure using a probabilistic analysis. Specifically, we compute cumulative density functions (CDF) that can be used to specify a desired fault tolerance threshold for completion of a task within a specified workspace region. We further show how this CDF can be improved by utilizing redundancy to optimize manipulator configurations throughout this region, using a modified nine degree of freedom Mitsubishi PA10-7C.**

I. INTRODUCTION

Fault tolerant robots have been considered for various applications such as for hazardous waste disposal and exploring remote environments. Recent developments in robotics technology aim to bring robots into closer contact with humans, for example in robotic surgery [10] or work-partner robots [6]. For these kinds of applications, robots are required to be reliable and safe in order to prevent uncertain or catastrophic behaviors [10]. The design and control of such robots requires a thorough analysis of robot failures and post-failure robot behavior. This has motivated researchers in the robotic community to analyze and design optimal fault tolerant manipulators [4]. The development of fault tolerant manipulators has been possible through the use of kinematic redundancy [2], [5], [13], [14], [19], [18], [22] along with applying fault-tolerant control strategies [7], [11], [13], [16], [15], [20], [21], [23]. The optimal kinematic structures and fault tolerant controllers for the manipulators are commonly obtained by the optimization of the fault tolerance indices [2], [5], [19], [18], [22].

There are different fault tolerance indices that have been used for optimization purposes, such as worst-case relative manipulability [18], [22], [5], worst-case minimum singular value [13], [14], and worst-case condition number [3]. These measures are calculated locally and are useful for tasks that do not require large motions. For example, Lewis et al. [13] have used the worst-case minimum singular value for the design of fault tolerant manipulators and have determined the optimal configuration for the manipulator based on this index whereas Roberts et al. [17], [19], [18], [22] have applied theworst-case relative manipulability. Kazerounian et al. [12] have proposed a global index for analysis of dexterity indices that can be extended for fault tolerance of manipulators. Developing computationally efficient techniques for applying some fault tolerance indices in real time remains a challenge.

In this paper a computationally efficient method for analyzing the global fault tolerance for kinematically redundant manipulators is proposed based on the probability density functions of the local fault tolerance indices. Probabilistic approaches have been previously used for reliability analysis [9] and workspace analysis. For example, reliability of operating at various workspace locations based on the probability of failure in a specific configuration has been studied recently by the authors where reliability and conditional reliability maps were introduced for determining the Reliable Fault Tolerant Workspace [1]. The contribution of this paper is a procedure for determining the probabilistic fault tolerance for a specified workspace region and using it to introduce an index for global fault tolerance analysis. The probability density functions of fault tolerance indices are computed using a Monte Carlo method. The proposed methodology of this paper has two main benefits, (a) it is applicable for global analysis of robotic manipulators and (b) it is computationally efficient.

The remainder of this paper is organized as follows. The common measures of fault tolerance are briefly reviewed in Section II. The limitations of the fault tolerance indices for global applications are discussed in Section III. The stochastic behavior of the fault tolerance indices are obtained by a statistical analysis for a set of configurations in a specified workspace region in Section IV. From the statistical information of the fault tolerance indices a probabilistic model is introduced in Section V. This model provides a computationally efficient framework for global analysis of the fault tolerance of robotic manipulators. The method is applied to a nine degree of freedom modified Mitsubishi PA10-7C robot to analyze its fault tolerance in Section VI. Finally, the concluding remarks are presented in Section VII.

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II. Fault tolerance indices

Different indices for analysis of fault tolerance of robotic manipulators have been used in the robotics community. To review them, consider an n-DoF (Degrees of Freedom) kinematically redundant robotic manipulator with an m by n Jacobian matrix $\mathbf{J}$ where m and n are the workspace and joint space dimensions, respectively. We assume that a locked joint failure can occur to a joint of the manipulator at any configuration of the robot throughout a specified region of the manipulator workspace. For other types of failure an active braking system can lock the joint and the method of this paper will be still valid.

Two common indices to measure the fault tolerance of the manipulator at a given configuration are

1-Worst-case manipulability:

The manipulability of a faulty manipulator with the failure due to the k-th joint locked is

$$^k r = \sqrt{det(^k\mathbf{J}^k\mathbf{J}^T)} \tag{1}$$

when $^k r$ is the manipulability value and $^k\mathbf{J}$ is the k-th reduced Jacobian matrix of the faulty manipulator. The reduced Jacobian matrix $^k\mathbf{J}$ is obtained by removing the k-th column of the original Jacobian matrix $\mathbf{J}$ [2], [14]. Then, the worst-case manipulability as a measure of fault tolerance is given by

$$\min_k {}^k r. \tag{2}$$

This manipulability defines the worst case post failure manipulability of the locked joint faulty robot.

2-Worst-case relative manipulability:

The relative manipulability [18] of a faulty manipulator with the k-th joint locked is

$$\rho_k = \sqrt{\frac{det(^k\mathbf{J}^k\mathbf{J}^T)}{det(\mathbf{J}\mathbf{J}^T)}} \tag{3}$$

where $det(^k\mathbf{J}^k\mathbf{J}^T)$, $det(\mathbf{J}\mathbf{J}^T)$, and ρ_k are the k-th joint locked robot manipulability, original robot manipulability, and the k-th joint locked robot relative manipulability measures, respectively. Then, the worst-case relative manipulability as a measure of fault tolerance is given by

$$\min_k {}^k \rho. \tag{4}$$

In this paper we illustrate our proposed method using the above two fault tolerance indices. However, the method is applicable for other indices that are briefly reviewed below.

The other common fault tolerance indices are defined based on the singular value decomposition (SVD) of the Jacobian matrix of the faulty manipulator. The SVD of the faulty manipulator with the k-th joint locked is denoted by

$$^k\mathbf{J} = \mathbf{U}\Sigma\mathbf{V}^T \tag{5}$$

where $\mathbf{U} \in R^{m\times m}$ and $\mathbf{V} \in R^{(n-1)\times(n-1)}$ are orthogonal matrices and $\Sigma \in R^{m\times(n-1)}$ is a diagonal matrix. For redundant manipulators, the diagonal matrix Σ has the following form

$$\begin{pmatrix} ^k\sigma_1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & ^k\sigma_2 & \ddots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ddots & ^k\sigma_{m-1} & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & ^k\sigma_m & 0 & \cdots & 0 \end{pmatrix} \tag{6}$$

where $^k\sigma_1 \geq^k \sigma_2 \geq \cdots^k \sigma_{m-1} \geq^k \sigma_m$

Some fault tolerance indices based on the SVD are given below.

Algebraic mean of the singular values:

$$\frac{(^k\sigma_1 +^k \sigma_2 + ... +^k \sigma_{m-1} +^k \sigma_m)}{m} \tag{7}$$

Geometric mean of the singular values (related to manipulability):

$$\sqrt[m]{(^k\sigma_1{}^k\sigma_2 ... {}^k\sigma_{m-1}{}^k\sigma_m)} \tag{8}$$

Worst-case minimum singular value (worst case dexterity):

$$\min_k {}^k\sigma_m \tag{9}$$

Worst-case condition number:

$$\max_k \frac{^k\sigma_1}{^k\sigma_m} \tag{10}$$

III. Fault tolerance indices for global applications

A. Overview

All of the above fault tolerance indices are local measures because they are obtained from the Jacobian matrix, which is a function of the configuration of a manipulator. There have been a few global fault tolerance indices that have been proposed, e.g., the "size" of self-motion manifolds [13] and the integral of local indices [12], however, they tend to be computationally expensive to evaluate. In this work, we propose a new index to measure the global fault tolerance of robotic manipulators using the probability density function (PDF) of the fault tolerance indices over a specified operating region of the workspace.

The contribution of this paper is that it provides a method capable of answering the following questions: (1) What is the probabilistic characteristic of the manipulator's fault tolerance behavior for a specified region? and (2) How does such an index correlate the fault tolerance behavior for different configurations with different fault tolerance properties?

B. Assumptions and problem statement

For critical tasks, robotic manipulators need to demonstrate a high level of fault tolerance in a specified region of the manipulator workspace. For example, in a surgical application, fault tolerant operation is required when the robot operates in proximity to the patient to ensure that the patient remains safe while the task is being performed. We assume that the entire workspace of a robotic manipulator is denoted by W and the manipulator is used to work in a specified critical region of the workspace denoted by $\widehat{W}$ where $\widehat{W} \subset W$. It is also assumed that the manipulator consists of n joints with the joint variables being denoted $\theta_1, ..., \theta_n$, the upper and lower bound for the k-th joint limits are given by $\underline{c}_k$ and $\overline{c}_k$, and a locked joint failure can occur to any of the robot's joints.

C. Requirement for global fault tolerance indices

Essentially, failures have inherent probabilistic behavior because of the uncertainty of the configuration and location of the failures. This type of uncertainty results in a probabilistic behavior of the fault tolerance indices. However, the relationship between the probabilistic behavior of the fault tolerance indices and the probabilistic behavior of locked joint failure in most cases of robotic manipulators is not trivial. Mathematically, an unknown locked joint failure causes a perturbation to the Jacobian matrix [3]; this perturbation results in non-deterministic behavior for the fault tolerance indices because the indices are obtained from the Jacobian matrix of the manipulator. For example, because fault tolerant indices are functions of the singular values of the Jacobian, it is difficult to obtain a function for these indices (except in very simple cases). Even in the case of a planar manipulator, the function that can be used to determine the singular values are complex and highly nonlinear. Therefore the behavior of the fault tolerance indices is complex.

In addition to the aforementioned complexity, a global analysis of the fault tolerance of a manipulator requires one to consider the value of the local fault tolerance indices for all possible configurations in all the positions/orientations for the critical workspace $\widehat{W}$. This is computationally inefficient and it is not practical for many applications. This is further complicated by the fact that redundant manipulators possess an infinite number of configurations for each workspace location. Therefore, we use a Monte Carlo method to decrease the number of computations required for global fault tolerance using a probabilistic model for the fault tolerance indices.

IV. Probabilistic model for fault tolerant indices

The probabilistic model for the fault tolerance indices can be obtained by a Monte Carlo analysis of fault tolerance indices. The Monte Carlo analysis needs to be performed for a set of configurations of the robot with random position/orientation in the specified region of the workspace. For generation of random position and orientations, we use a uniform distribution because we assume that task motions are equally important throughout the specified region of the workspace. Other types of distributions may be more suitable in the specified region if the robot mostly operates in a portion of this region where some positions/orientations are more important than others.

TABLE I
D-H parameters of the modified Mitsubishi PA10-7C robot

Joint	a(m)	α (deg)	d (m)	θ (deg)	$\underline{c}$ (deg)	$\overline{c}$ (deg)
1	0.0	-90	0.315	θ_1	-45	45
2	0.0	90	0.0	θ_2	-15	75
3	0.0	-90	0.450	θ_3	-45	45
4	0.0	90	0.0	θ_4	45	135
5	0.0	-90	0.500	θ_5	-45	45
6	0.0	90	0.0	θ_6	-27	63
7	0.0	-90	0.450	θ_7	-45	45
8	0.0	90	0.0	θ_8	-9	81
9	0.0	0	0.080	θ_9	-45	45

The results of the Monte Carlo analysis can be used to obtain a probabilistic model for any of the indices introduced in II. To obtain the model we first use the standard inverse kinematics routine of the Matlab robotic toolbox [8] to generate a set of configuration for each random position/orientation. Then we calculate the Jacobian at that configurationand based on that matrix we calculate the value of the selected fault tolerance index. Then we perform a statistical analysis and obtain frequency distribution and cumulative frequency distribution histograms. The frequency distribution profile is then normalized to obtain a PDF for the fault tolerance index. It is shown that the PDF profile is useful for global analysis of fault tolerance of the manipulator in the specified region of the workspace. In the next section, the probabilistic model is illustrated for the worst-case manipulability and relative manipulability indices of a modified Mitsubishi PA10-7C robot.

V. Case study: Probabilistic model for fault tolerance for a modified Mitsubishi PA10-7C robot

A. Introduction

The Mitsubishi PA10-7C is a portable general-purpose 7-DoF manipulator with an open Visual C++ interface. If a 2-DoF tool with a pitch and a roll motion is attached at the robot tip then the modified Mitsubishi PA10-7C will provide three degrees of redundancy. The Denavit and Hartenberg (D-H) parameters of the modified robot are shown in Table I. A modified model of the Mitsubishi PA10-7C is used due to the limited fault tolerance of the original robot, i.e., the original robot is very close to a singularity after a failure in it's elbow joint. In other words, the original robot is fault intolerant or very close to singularity for most of its configurations. The added 2-DoF attachment improves the robot's fault tolerance. The table includes the parameters a and α that are the length and the twist angle of the link and d and θ that are the joint offset and the joint angle respectively. The last two columns are the joint angle limits.

B. Case study assumptions

Assume the robot is at an initial configuration of $\theta_1 = 0$, $\theta_2 = 30$, $\theta_3 = 0$, $\theta_4 = 90$, $\theta_5 = 0$, $\theta_6 = 18$, $\theta_7 = 0$, $\theta_8 = 36$, $\theta_9 = 0$ (deg) that corresponds to an end-effector pose of $x = 968$ (mm), $y = 0$ (mm), and $z = 041$(mm) and the end effector orientation specified by Euler angles of $roll = 0$ (deg), $pitch = 174$ (deg), and $yaw = 0$ (deg). Consider that the workspace of the manipulator is a cube with a size of 500 (mm) $\times$ 500 (mm) $\times$ 500 (mm) and the orientation of the manipulator remains in a cube of $90(deg) \times 90(deg) \times 90(deg)$. These positional and orientational cubes are centered at ($x = 968$ (mm), $y = 0$ (mm), $z = 041$(mm)) and ($roll = 0$ (deg), $pitch = 174$ (deg), and $yaw = 0$ (deg)). The robot in the central position/orientation is shown in Figure 1 (side view) and 2 (top view).

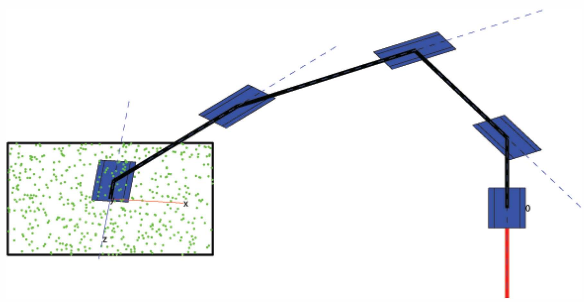

Fig. 1. The modified Mitsubishi PA10-7C robot and the specified region in the workspace (side view).

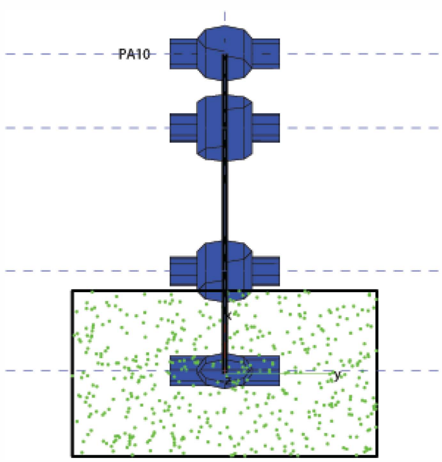

Fig. 2. The modified Mitsubishi PA10-7C robot and the specified region in the workspace (top view).

C. Generating a random set of positions and orientations

A uniformly random set of positions/orientations are selected in the indicated cube around the center point. These points are indicated with green dots both in Figure 1 and 2. Then the standard inverse kinematics routine of the Matlab toolbox was used to generate a configuration for each position/orientation. We used a different initial condition for the inverse kinematics to ensure that the obtained configuration by the inverse kinematic function remains in the joint limits of Table 1.

D. Statistical analysis

For any of the obtained configurations, the Jacobian matrix of the robot is computed using the *jacob*0 command in the Robotics toolbox that gives the Jacobian matrix in the base frame of the robot. We then use this Jacobian matrix (and the associated failure Jacobians) in equations (1)-(4) to compute the worst-case manipulability and worst-case relative manipulability values. Figure 3 shows the value of the worst-case relative manipulability for all selected 500 configurations associated with the 500 random positions/orientations. (The result of the worst-case manipulability are shown in Figure 7 and discussed later.) Statistical analysis for this set of configurations (non optimized) have been performed and the frequency distribution and cumulative frequency distribution of the fault tolerance index were obtained as shown in the Figure 3 middle and bottom graphs.

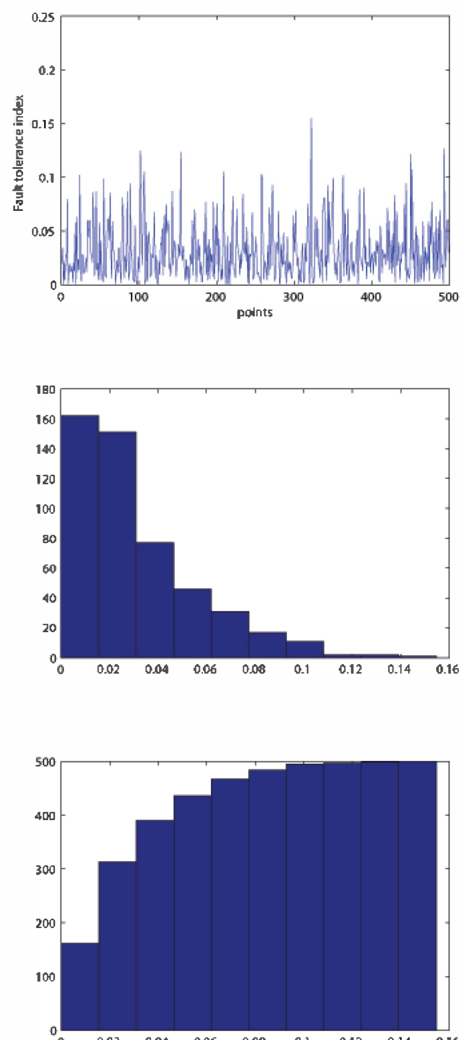

Fig. 3. Worst-case relative manipulability for the non optimized configuration (top), frequency distribution histogram (middle), and cumulative frequency distribution histogram (bottom)

The next study that was performed wass to use the kinematic redundancy to improve the initial configuration in each position/orientation. This is achieved by optimizing the fault tolerance indices under the constraints that robot end effector remains in the same position/orientation. Thus we solved the optimization problem shown below for all the random points in the selected region of the workspace using the constrained optimization command with nonlinear equality constraints, upper and lower joint limits, and the *interior − point* method, i.e.,

$$\max_{\theta} \min_{k} {}^{k}\rho \tag{11}$$

subjected to $x - \mathbf{f}(\theta) = 0$, where x is the position/orientation vector and $\mathbf{f}$ is the forward kinematics equation.

We then obtained the fault tolerant indices for all of these configurations. Figure 4 shows the worst-case relative manipulability of the manipulator for the optimized configurations. A similar statistical analysis for the set of optimized configurations was performed and the frequency distribution and cumulative frequency distribution of the fault tolerance index was obtained as shown in the middle and bottom graphs of Figure 4.

From the results of the statistical analysis of the non-optimized and optimized configurations, the probability density functions (PDFs) of the worst-case relative manipulability were obtained by appropriate scaling of the frequency distributions from the (middle graphs) of Figure 3 and 4. The obtained PDFs are shown in Figure 5 (top graph) and the corresponding CDF (bottom graph).

VI. Global fault tolerance analysis

The PDF and CDF curves can be used for different types of global analysis of fault tolerance. For example, it is possible to calculate the probability of being fault tolerant when operating in the specified region of operation. It is also possible to obtain the probability that a manipulator can operate with more than a specific threshold for its fault tolerance index. The probability that the manipulator is fault tolerant is given by the cumulative distribution functions (CDF). If the CDF curve is denoted by $\mathbf{F}(.)$, then the probability that the manipulator has a fault tolerance index less than a threshold of h is given by $\mathbf{F}(h)$, i.e., the value of the CDF curve at h.

It is also clear that, based on the PDF/CDF curves in Figures 3 and 4, one can greatly improve the probability of being fault tolerant by using configurations that have been optimized for that purpose. To illustrate how much optimization may help, we have sorted the worst-case relative manipulability for the initial configuration and rearranged the worst-case relative manipulability of the optimized case to correspond with the original case. The results of this

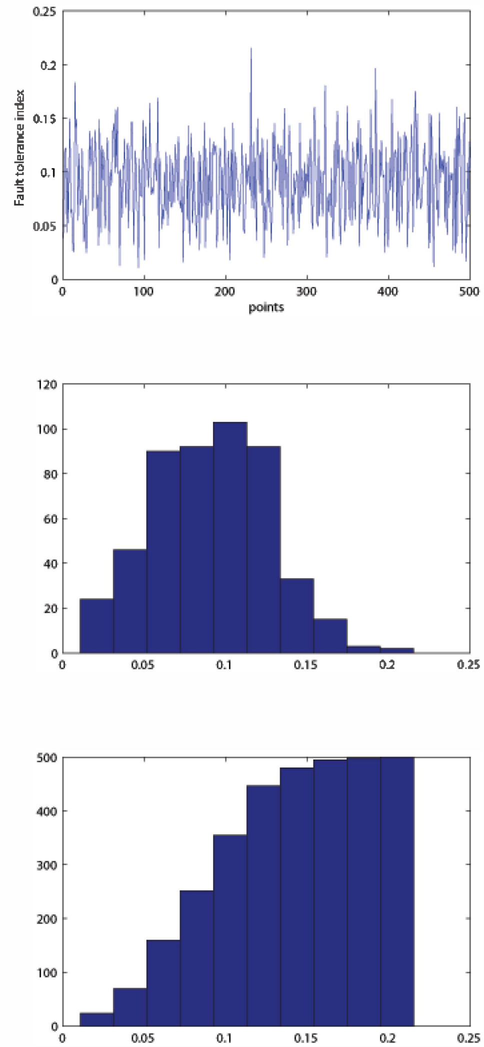

Fig. 4. Worst-case relative manipulability for the optimized configuration (top), frequency distribution histogram (middle), and cumulative frequency distribution histogram (bottom).

rearrangement are shown in Figure 6. This is performed to study the correlation between the fault tolerance of the initial configuration and the optimized configurations. As one might expect, the improvement to an initial good configuration is less than for initial configurations that exhibit a poor fault tolerance index[2].

A Similar study has been performed for worst-case manipulability using the same method that was described in Section V-D. For this case we solved the optimization problem

$$\max_{\theta} \min_{k} {}^{k}r \tag{12}$$

subjected to $x - \mathbf{f}(\theta) = 0$. The final results after sorting the worst-case manipulability values and corresponding optimized worst-case manipulability values are shown in Figure 7. This indicates the applicability of the proposed method

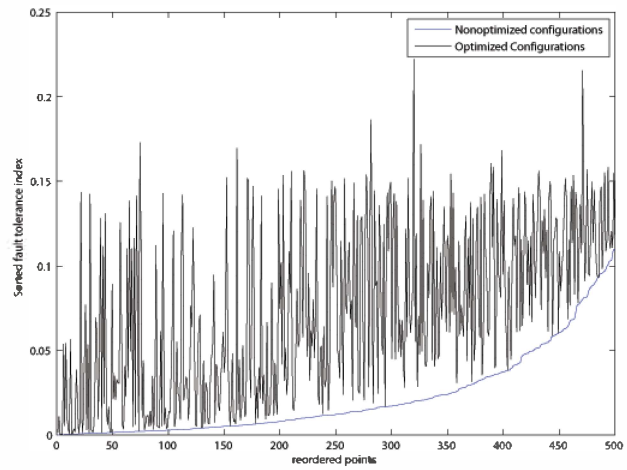

Fig. 7. Sorted worst-case manipulability for non-optimized configurations and corresponding values for optimized configurations.

for other fault tolerance indices.

The proposed method was used for revolute joint, serial manipulators, however, it is also applicable for prismatic joint manipulators and manipulators with combined prismatic and revolute joints. We have also tested the method for up to 2000 configurations with results that were similar to those presented in this paper. For other types of failures e.g. sensor failures and free swing joints, an active braking system can be used to lock the joint so that the method presented in this paper will still be applicable.

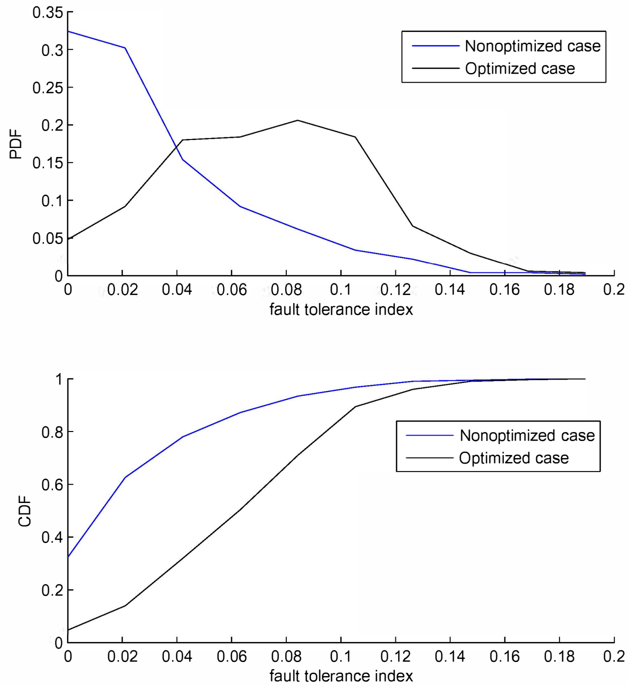

Fig. 5. Probability density function (top graph) and cumulative density function (bottom) of the non-optimized and optimized configuration.

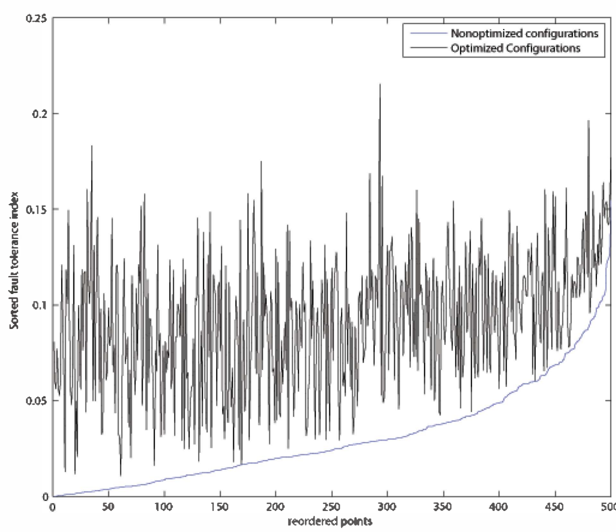

Fig. 6. Sorted worse-case relative manipulability for non-optimized configurations and corresponding values for the optimized configurations.

VII. Conclusion

This work has illustrated a computationally efficient method for analyzing the fault tolerance of kinematically redundant manipulators. This method stemmed from the probabilistic behaviors of failures and the effects of this probabilistic behavior on the fault tolerance indices. The probability density function of the relative manipulability was obtained by using a Monte Carlo analysis with the resulting PDF and CDF used for global analysis of the fault tolerance. The proposed approach of this paper was evaluated for the worst-case relative manipulability and worst-case manipulability of a modified Mitsubishi PA10-7C robot where it was shown that the CDF and PDF curves are sensitive to the change of the fault tolerance property of the manipulator. A global fault tolerance index was proposed based on the CDF curve and the required level of fault tolerance index. The proposed approach is applicable for other indices as well.

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