

# **An Introduction to the Global Circulation of the Atmosphere**

David A. Randall

# Contents

<b>1 Perpetual Motion</b>	<b>3</b>
<b>2 What Makes It Go?</b>	<b>16</b>
2.1 The Earth's radiation budget: An "upper boundary condition" on the global circulation	16
2.2 Meridional energy transports by the atmosphere-ocean system	19
2.3 Surface boundary conditions	25
2.3.1 Temperature	25
2.3.2 Wetness	26
2.3.3 Topography	28
2.3.4 Heat capacity	29
2.3.5 Albedo	31
2.3.6 Roughness	31
2.3.7 Vegetation	31
2.3.8 Sea ice	32
2.3.9 Land ice	32
2.4 Energy and moisture budgets of the surface and atmosphere	35
2.5 Segue	41
2.6 Problems	43
<b>3 What goes on</b>	<b>44</b>
3.1 Introduction	44
3.2 The global distribution of atmospheric mass	44
3.3 Zonal wind	53
3.4 Meridional wind	57
3.5 Vertical velocity and the mean meridional circulation	63
3.6 Temperature	73
3.7 Moisture	78
3.8 Lots of questions	82
3.9 Problems	84
<b>4 The rules of the game</b>	<b>85</b>

4.1	Introduction	85
4.2	Conservation of the mass of dry air	85
4.3	Conservation of atmospheric moisture	90
4.4	Conservation of momentum on a rotating sphere	92
4.5	The equation of motion in spherical coordinates	97
4.6	Conservation of angular momentum	100
4.7	Conservation of kinetic energy and potential energy	101
4.8	Energy transports and dissipation due to small-scale macroscopic motions	104
4.9	Where does mechanical energy come from?	105
4.10	Conservation of thermodynamic energy	107
4.11	Conservation of total energy	109
4.12	Static energies	113
4.13	Entropy	115
4.14	The primitive equations	118
4.15	Potential temperature as a vertical coordinate	120
4.16	Vorticity and potential vorticity	128
4.17	The quasi-geostrophic system	132
4.18	The shallow water equations	137
4.19	Segue	138
4.20	Problems	139
<b>5</b>	<b>Go with the flow</b>	<b>144</b>
5.1	Overview	144
5.2	A way to talk about eddies and their effects on the zonally averaged circulation	146
5.3	An isentropic view of the mass circulation	150
5.4	Water vapor transports	155
5.5	Moist static energy transports	165
5.6	Angular momentum transports	175
5.7	Segue	188
5.8	Problems	189
<b>6</b>	<b>Up moist, down dry</b>	<b>193</b>
6.1	Upward energy transport by deep convection	193
6.2	The basics of dry and moist convection	198
6.3	Radiative-convective equilibrium	203
6.4	Some comments on radiative-convective equilibrium	210
6.5	Representative soundings	212
6.6	The convective mass flux	220
6.7	Why the updraft fractional area is small, and how this simplifies things	226
6.8	A simple cumulus cloud model	230
6.9	Compensating subsidence	231

6.10	The vertical distribution of the mass flux	233
6.11	What determines the intensity of the convection?	235
6.12	Conditional symmetric instability	239
6.13	Segue	243
6.14	Problems	243
<b>7</b>	<b>Heat where it's hot, and cool where it's cold</b>	<b>246</b>
7.1	The vertically integrated enthalpy	246
7.2	Available potential energy	247
7.3	The gross static stability	253
7.4	How available potential energy is converted into kinetic energy	258
7.5	Examples: The available potential energies of three simple systems	259
7.5.1	Example #1: The APE associated with static instability	259
7.5.2	Example #2: The APE associated with meridional temperature gradients	260
7.5.3	Example #3: The APE associated with surface pressure variations	265
7.6	Where do variances come from?	266
7.7	Generation of available potential energy, and its conversion into kinetic energy	272
7.8	The governing equations for the eddy kinetic energy, zonal kinetic energy, and total kinetic energy	278
7.9	Observations of the energy cycle	281
7.10	The role of heating	284
7.11	Moist available energy	285
7.12	Summary	286
7.13	Problems	286
<b>8</b>	<b>A taxonomy of eddies</b>	<b>290</b>
8.1	Not all eddies are waves	290
8.2	Free and forced small-amplitude oscillations of a thin spherical atmosphere with no mean flow	292
8.3	Free and forced oscillations	299
8.3.1	Solution procedure for free oscillations	299
8.3.2	Solution procedure for forced oscillations	299
8.3.3	Analysis of free oscillations	300
8.4	Forced oscillations: The atmospheric tides	308
8.5	The semi-annual oscillation	308
8.6	Propagation of planetary waves	308
8.7	Stationary and transient eddies in middle latitudes	313
8.8	Annular modes	321
8.9	Theory of orographically forced stationary waves	323
8.10	Tropical waves	329

8.11	Tropical cyclones	343
8.12	The response of the tropical atmosphere to stationary heat sources and sinks	344
8.13	Monsoons	350
8.14	The Walker Circulation	357
8.15	The Madden-Julian Oscillation	367
8.16	Segue	374
8.17	Problems	374
<b>9</b>	<b>Tropical cyclones</b>	<b>378</b>
<b>10</b>	<b>What the eddies do</b>	<b>379</b>
10.1	Interactions and non-interactions of gravity waves with the mean flow	379
10.2	Angular momentum transport by Rossby waves	383
10.3	More about the interaction of planetary waves with the mean flow	387
10.4	The Transformed Eulerian Mean (TEM) system of equations	392
10.5	The Eliassen-Palm theorem in isentropic coordinates	398
10.6	Blocking	404
10.7	Stratospheric sudden warmings	412
10.8	The quasi-biennial oscillation	416
10.9	The Brewer-Dobson Circulation	420
10.10	Segue	420
10.11	Problems	421
<b>11</b>	<b>A fluid dynamical commotion</b>	<b>423</b>
11.1	Turbulence is made of vortices.	423
11.2	Nonlinearity and scale interactions	428
11.3	Two-dimensional turbulence	430
11.4	Quasi-two-dimensional turbulence	434
11.5	Dimensional analysis of the kinetic energy spectrum	438
11.6	Observations of the kinetic energy spectrum	442
11.7	Dissipating enstrophy but not kinetic energy	443
11.8	The global circulation as a blender: What does the blender blend?	447
11.9	Sensitive dependence on initial conditions	447
11.10	Quantifying the limits of predictability	455
	11.10.1 The dynamical approach	455
	11.10.2 The empirical approach	457
	11.10.3 The dynamical-empirical approach	459
11.11	Ensemble forecasting	464
11.12	The response of the atmosphere to changes in sea surface temperature	465
11.13	Segue	467
11.14	Problems	468
<b>12</b>	<b>Forcing and response</b>	<b>470</b>

12.1	Chaotic systems respond to changes in external forcing	470
12.2	Lorenz pushes the attractors around	471
12.3	Anthropogenic climate change	475
12.4	Problems	480
<b>Appendices</b>		<b>482</b>
<b>A Vectors, Coordinates, and Coordinate Transformations</b>		<b>482</b>
A.1	Physical laws and coordinate systems	482
A.2	Scalars, vectors, and tensors	482
A.3	Differential operators	485
A.4	Vector identities	487
A.5	Spherical coordinates	489
A.5.1	Vector operators in spherical coordinates	489
A.5.2	Horizontal and vertical vectors in spherical coordinates	490
A.5.3	Derivation of the gradient operator in spherical coordinates	492
A.5.4	Applying vector operators to the unit vectors in spherical coordinates	493
A.6	Solid body rotation	494
A.7	Formulas that are useful for two-dimensional flow	495
A.8	Vertical coordinate transformations	496
A.8.1	Basics	496
A.8.2	Some useful operators	497
A.9	Concluding summary	498
<b>B Dimensional Analysis, Scale Analysis, and Similarity Theories</b>		<b>499</b>
B.1	Dimensions and units	499
B.2	Consistent use of dimensional and nondimensional quantities in equations	501
B.3	The Buckingham Pi Theorem	501
B.4	Scale analysis	506
B.5	Similarity theories	508
B.6	Summary	511
<b>C Why is the Dissipation Rate Positive?</b>		<b>513</b>
<b>D The Moist Adiabatic Lapse Rate</b>		<b>517</b>
<b>E Eddy Kinetic Energy and Zonal Kinetic Energy</b>		<b>521</b>
<b>F Spherical Harmonics</b>		<b>532</b>
<b>G Hermite Polynomials</b>		<b>543</b>
<b>Bibliography</b>		<b>546</b>